CEE 342 Aut 2005 Final Exam Solutions

1. Loop L4 consists of pipes P3, P4, P6, and P13. Positive flow in these pipes is defined as being to the right, downward, to the left, and downward, respectively. For these directional definitions, a positive (clockwise) correction to the flow in Loop L4 increases the flows in P3, P4, and P6, but decreases it in P13. P2 and P3 are part of Loop L4 only, but P6 is also a part of loop L3, and P13 is also a part of loop L2. Positive (clockwise) corrections to the flows in those loops represent a decrease in the flow in P6 and an increase in the flow in P13, respectively, so the overall equations for the next set of flow estimates for the pipes in loop L4 are:

$$\mathrm{P3}_{i+1} = \mathrm{P3}_i + \Delta\mathrm{L4} = 1.0 + (-0.4) = 0.6 \ \ (\text{to the right})$$

$$\mathrm{P4}_{i+1} = \mathrm{P4}_i + \Delta\mathrm{L4} = -1.0 + (-0.4) = -1.4 \ \ (\text{upward})$$

$$\mathrm{P6}_{i+1} = \mathrm{P6}_i + \Delta\mathrm{L4} - \Delta\mathrm{L3} = -2.5 + (-0.4) - (-0.8) = -2.1 \ \ (\text{to the right})$$

$$\mathrm{P13}_{i+1} = \mathrm{P13}_i - \Delta\mathrm{L4} + \Delta\mathrm{L2} = 1.0 - (-0.4) + (-1.4) = 0.0 \ \ (\text{no flow})$$

2. The head loss represents a loss of total head, not just velocity head. The total head is much greater than the velocity head, so a loss of twice the velocity head might represent only a small fraction of the total head. If the pipes upstream and downstream of the valve have the same diameter, the fluid doesn't lose any velocity head at all as it passes through the valve; all the loss of energy is manifested as a loss of pressure head.

3. (a) The model system (the small-scale, test system) should be run with identical values for the independent dimensionless groups as the prototype system (the full-scale system in winter). Since $L_m/L_p = 1/3$ (where m and p stand for model and prototype, respectively), d_m/d_p must also equal 1/3, so d_m should be (1/3)(4 mm), or 0.133 mm. Similarly, to make $Vd\rho/\mu$ (i.e., the Reynolds number) identical in the two systems, the velocity in the model system must be:

$$V_m = \frac{V_p d_p \cancel{\rho_p}}{\mu_p}\frac{\mu_m}{d_m \cancel{\rho_m}} = \frac{d_p}{d_m}\frac{\mu_m}{\mu_p}V_p = (3)\left(\frac{8.9\text{x}10^{-4}}{1.5\text{x}10^{-3}}\right)\left(0.30\frac{\text{m}}{\text{min}}\right) = 0.53\frac{\text{m}}{\text{min}}$$

(b) If the independent Π groups have the same values in the model and prototype systems, the dependent Π groups must also have the same value. Therefore, the expected pressure drop in the full-scale system in the winter is:

$$\Delta p_p = \frac{\Delta p_m}{\cancel{\rho_m} V_m^2}\left(\cancel{\rho_p} V_p^2\right) = \frac{V_p^2}{V_m^2}\Delta p_m = \frac{(0.30\text{ m/min})^2}{(0.53\text{ m/min})^2}(80\text{ kPa}) = 25.9\text{ kPa}$$

4. (a) The velocities and pressures are both zero at the surfaces of the reservoirs. Also, since the velocity drops to zero after the water exits the pipe, the loss coefficient at the exit is 1.0. As a result, when we apply the energy equation between the surfaces of the reservoirs, we find:

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} = \cancel{\frac{p_2}{\gamma}} + z_2 + \cancel{\frac{V_2^2}{2g}} + \sum h_L$$

$$z_1 - z_2 = \Delta z = h_{L,pipes} + h_{L,entry} + h_{L,bends} + h_{L,exit} = \left(f\frac{L}{d} + 0.8 + 2*0.2 + 1.0 \right)\frac{V^2}{2g}$$

The friction factor and pipe diameter are given, and the total pipe length is 4.8 m. The elevation difference Δz is 0.3 m, so:

$$0.3\text{ m} = \left(0.020\frac{4.8\text{ m}}{0.1\text{ m}} + 0.8 + 2*0.2 + 1.0 \right)\frac{V^2}{2\left(9.8\text{ m/s}^2\right)}$$

$$V = \sqrt{\frac{\left(0.3\text{ m}\right)\left(2\right)\left(9.8\text{ m/s}^2\right)}{0.020\frac{4.8\text{ m}}{0.1\text{ m}} + 0.8 + 2*0.2 + 1.0}} = 1.36\frac{\text{m}}{\text{s}}$$

$$Q = VA = \left(1.36\frac{\text{m}}{\text{s}}\right)\frac{\pi\left(0.1\text{ m}\right)^2}{4} = 0.011\frac{\text{m}^3}{\text{s}}$$

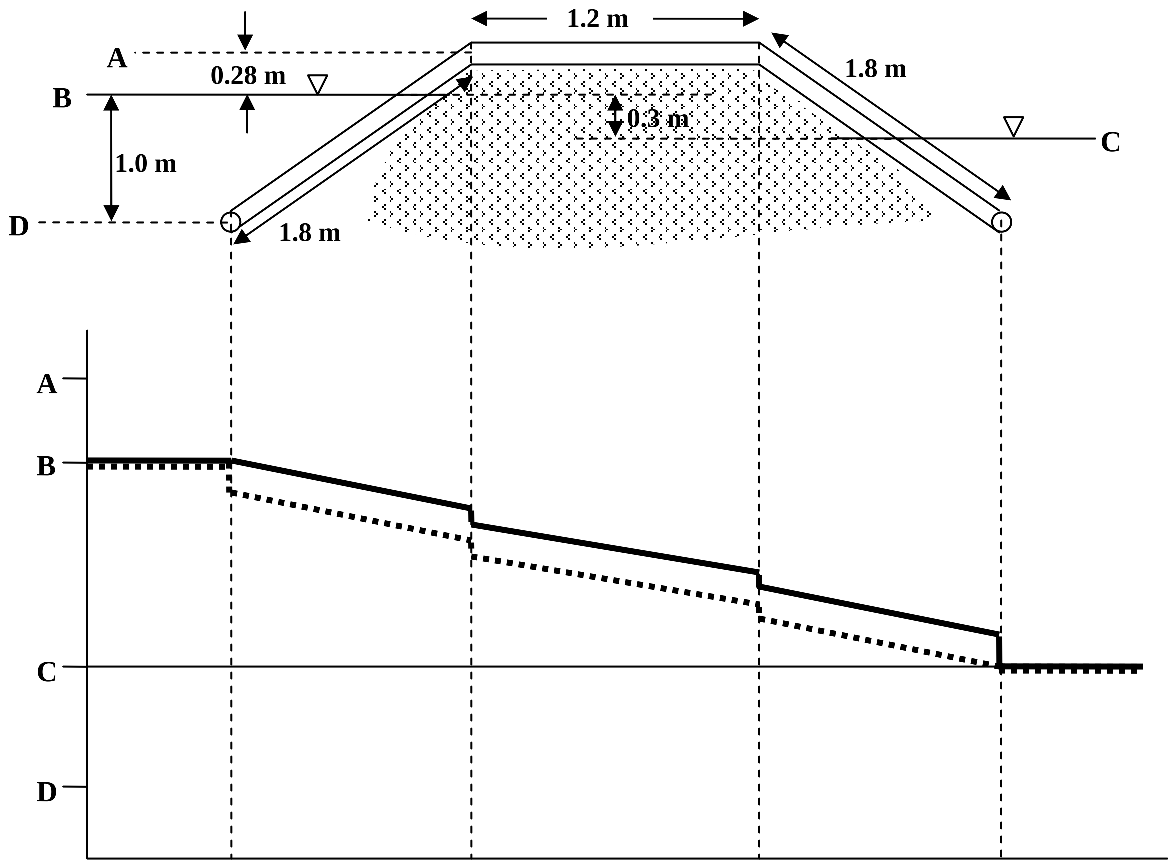
(b) The absolute roughness of the pipes can be estimated from the Moody diagram, given that the friction factor is 0.020. The Reynolds number is:

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{\left(0.1\text{ m}\right)\left(1.36\frac{\text{m}}{\text{s}}\right)\left(1000\frac{\text{kg}}{\text{m}^3}\right)}{\left(1.002\text{x}10^{-3}\text{ N-s/m}^2\right)\left(1\frac{\text{kg-m/s}^2}{\text{N}}\right)} = 1.36\text{x}10^5$$

The curve that passes through the point where $\text{Re} = 136{,}000$ and $f = 0.020$ has ε/D of approximately 0.0006. Therefore, we can estimate the absolute roughness to be:

$$\varepsilon = \frac{\varepsilon}{D}D = \left(0.0006\right)\left(0.1\text{ m}\right) = 6\text{x}10^{-5}\text{m} = 0.06\text{ mm}$$

(c) Since the water at the surfaces of the reservoirs has zero velocity and zero pressure, the EL and HGL are both zero at both of those surfaces. The EL drops steadily as the water passes through the pipes, due to friction. It also drops abruptly at the entrance, at each bend, and at the exit, due to the minor losses at those locations. The HGL drops below the EL by an amount equal to $V^2/2g$ when the water enters the pipe, and the gap between the two lines stays exactly the same until the water exits. At that point, the water loses velocity head, but not elevation or pressure head, so the HGL remains constant while the EL drops, allowing the two lines to coincide once again. These ideas are shown in the diagram below.



5. (a) At the upstream location prior to the placement of the obstacle, the water depth is 1.9 m and the flow is sub-critical. The specific energy can therefore be found by identifying the value of E that corresponds to $y = 1.9$ m on the upper leg of the curve; this value is $E = 1.95$ m. When the obstacle is put into place, the bottom of the water column overlying the obstacle is 1.1 m higher than upstream. For ideal flow, this means that the specific energy must decrease by 1.1 m as the water passes over the obstacle, to a value of 0.85 m. Since this value of E is less than E_{min}, the given value of q will not be able to pass over the obstacle if the depth of the upstream water remains at 1.9 m. As a result, water will back up upstream of the obstacle, flow over the obstacle will be critical (once steady flow is re-established), and a transition to critical flow is possible.

(b) When the obstacle is put into place, the upstream water will back up. This backup will cause the water depth both upstream and over the obstacle to increase until q over the obstacle equals q approaching the obstacle. The minimum depth of water which will allow steady flow equal to the given q is y_c, which can be read from the graph as 0.75 m. Therefore, the water depth must be at least 0.75 m everywhere in the channel for steady flow to be maintained. In particular, the water will be at this critical depth, and E will equal E_{min} (i.e., 1.15 m), when the water passes over the obstacle.

When flow over the obstacle is critical and $E = E_{min} = 1.15$ m at that location, the specific energy upstream will be $E_{up} = E_{min} + \Delta z_{up} = E_{min} + 1.1 \text{ m} = 2.25$ m. Since the flow is sub-critical upstream, the corresponding depth is the depth that yields $E = 2.25$ m on the upper leg of the curve, or $y_{up} = 2.20$ m.

Downstream of the obstacle, the specific energy will equal $E_{down} = E_{min} + \Delta z_{down} =$ 1.15 m + 0.7 m, or 1.85 m. Since the downstream flow is super-critical, the depth will correspond to $E = 1.85$ m, on the lower leg of the diagram; this depth is 0.39 m.

6. (a) In both laminar and turbulent flow, momentum is transferred from the fluid in the middle of the pipe toward the exterior, and is eventually lost. This loss of momentum accounts for the shear force exerted on the fluid by the walls. The net momentum of the flowing fluid does not decrease, because an equal and opposite force is applied to the fluid due to the pressure differential between the upstream and downstream locations.

In laminar flow, the momentum transfer toward the pipe walls is primarily by exchange of individual molecules, whereas in turbulent flow, it is primarily by exchange of "packets" of fluid, in eddies. Because much more material is transferred by exchange of eddies, the rate of momentum exchange between adjacent layers of fluid is much greater in turbulent than in laminar flow.

(b) Because pipes are radially symmetric, the shear stress in the middle of a pipe is zero regardless of the flow regime. (In addition, in both laminar and turbulent flow, the shear stress increases linearly from zero in the pipe center to the maximum value at the pipe wall.)