CEE 342 Final Exam, Aut 2004.

120 pts. possible. Some information that you might find useful is provided below, and a Moody diagram is attached at the end of the exam.

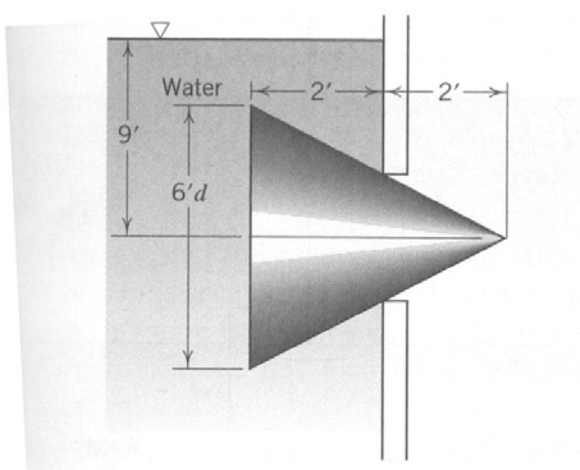
Properties of Water	Constants, Conversions, etc.
ρ = 1000 kg/m^3= 1.94 slugs/ft^3	g = 9.81 m/s^2 = 32.2 ft/s^2
γ = 9800 N/m^3= 62.4 lb/ft^3	1 kW = 1000 N-m/s
μ_{70F} = 1.00 x 10^{-3} N-s/m^2 = 2.05 x 10^{-5} lb-s/ft^2	p_{atm} = 101.5 kPa = 14.7 psi
ν_{70F} = 1.00 x 10^{-6} m^2/s = 1.06 x 10^{-5} ft^2/s	1 hp = 550 ft-lb/s
p_{vap} = 2.34 kPa = 0.036 psi	

	Area	Volume	Centroid	I_c
Rectangle	bh		$h/2$	$bh^3/12$
Triangle	$bh/2$		$h/3$	$bh^3/36$
Circle	$\pi d^2/4$		$d/2$	$\pi d^4/64$
Sphere	πd^2	$\pi d^3/6$	$d/2$	
Cone		$\pi d^2 h/12$	$h/4$ (from flat surface of cone)	

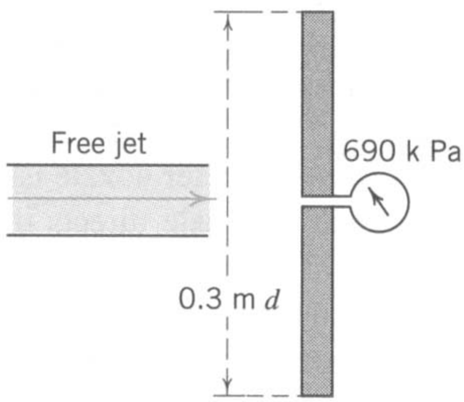
1. (10) Discuss briefly the similarities and differences between the Darcy-Weisbach and Hazen-Williams equations. Focus on when and how the equations are used, not on the mathematical differences.

2. (5) Why is the critical Reynolds number for the transition to turbulent flow only 500 for open channels, but 2000 for pipes flowing full?

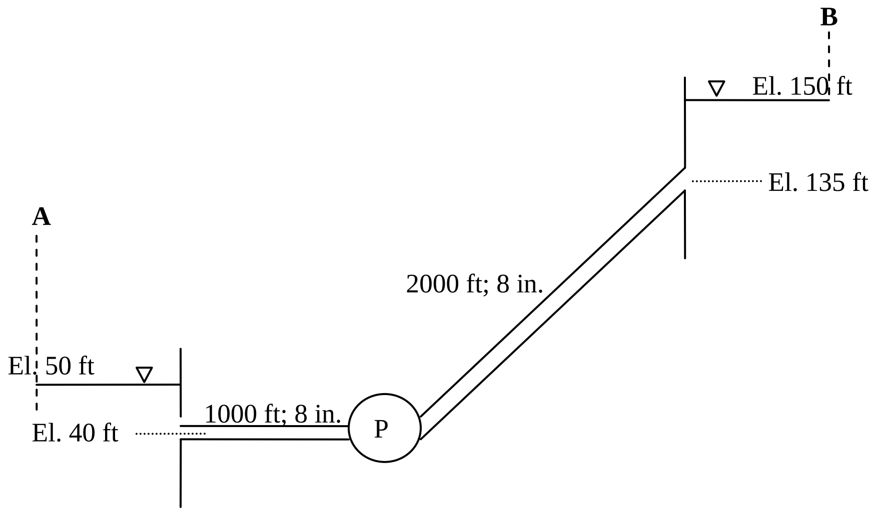
3. (15) Calculate the horizontal component of the force on the solid, conical plug shown below.



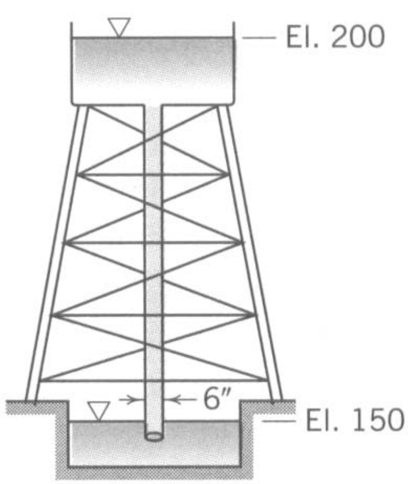
4. (15) Calculate the horizontal force of the free jet on the circular plate shown in the following schematic. The flow rate is 42.5 L/s, the jet is striking the fixed, circular plate, and the pressure gage is measuring the pressure at the centerline of where the flow strikes the plate.



5. (30) Calculate the horsepower that the pump must supply to pump the water at a flow rate of 2.5 ft^3/s in the system shown below, ignoring headlosses other than those through the pipes. The pipes have an absolute roughness of 6.67×10^{-4} ft. Draw the HGL and EL on the diagram, from plane A to plane B. In drawing this sketch, you *should* consider minor headlosses (i.e., headlosses caused by processes other than friction with the pipe walls). Draw and label the lines clearly, and identify all locations where both lines are at the same elevation. Do you expect the pressure to be negative anywhere in the system?

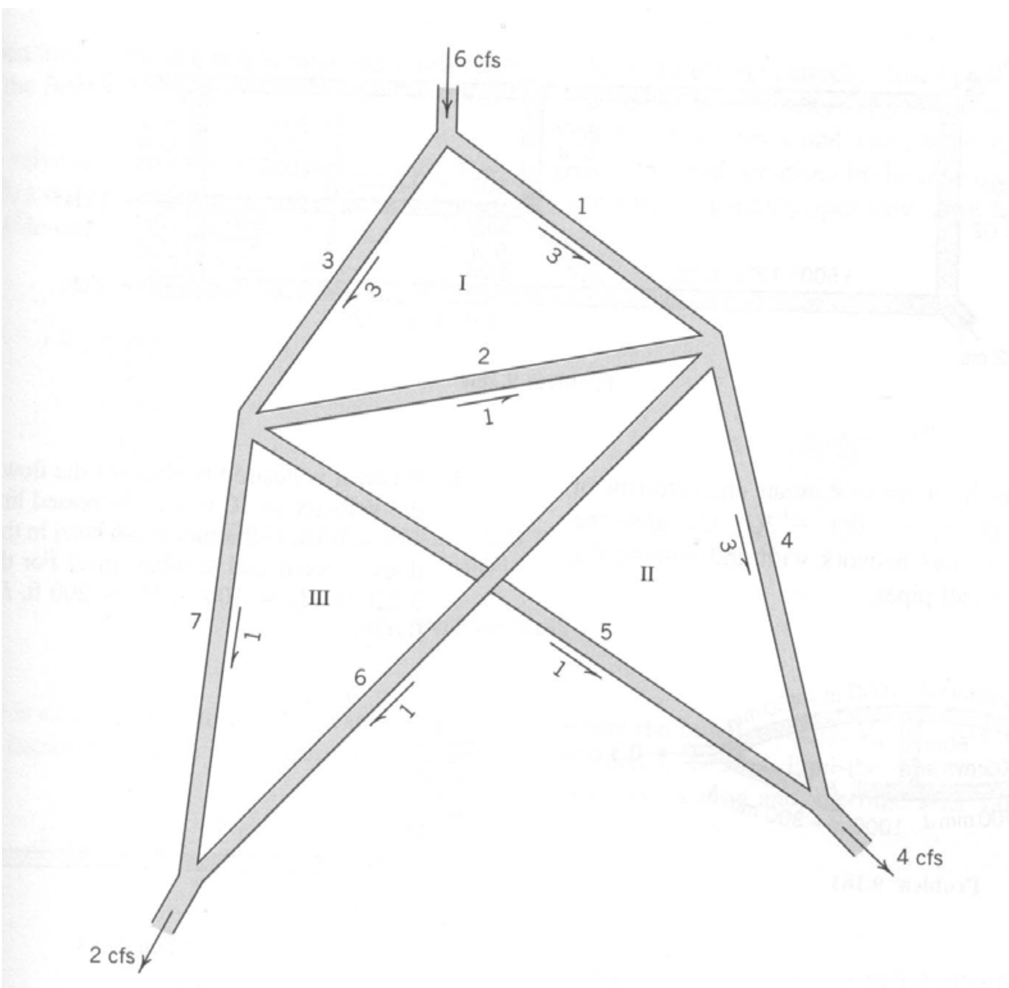


6. (30) Calculate the flow rate from this water tank if the 6-in. pipeline has a friction factor of 0.020 and is 50 ft long, and the water is at 70^{o}F. The water in the tank is 5 ft deep. The minor loss coefficients (*k*) for headlosses at the sharp-edged entrance to the pipe and at the submerged discharge are 0.5 and 1.0, respectively. Where is the location of lowest pressure in the system? Might cavitation be a problem at this location?



7. (15) In the flow network shown below, Loop I includes pipes 1, 2, and 3; Loop II includes pipes 2, 4, and 5; and Loop III includes pipes 2, 6, and 7 (pipe numbers are indicated by # in the diagram). A first guess has been made for the flow rate in each pipe (in cfs) and is shown on the diagram; the corresponding frictional headlosses have been calculated using the Darcy-Weisbach equation and are summarized below. What should the next guesses be for the direction and magnitude of the flows in pipes 2 and 3?

Pipe	$\lvert Q \rvert$	$\lvert h_f \rvert$
1	3	90
2	1	10
3	3	90
4	3	45
5	1	5
6	1	5
7	1	5



1. The DW and HW equations both relate headloss in full-flowing conduits to geometric factors, fluid properties, and operating parameters. The DW equation applies to any fluid flowing with any velocity (i.e., any Reynolds number). On the other hand, the HW equation is specifically for water, in turbulent flow under for a limited range of Reynolds numbers that correspond to conditions typical for water supply pipes.

2. In open channel flow, the Reynolds number is typically defined as $R_h V\rho/\mu$, whereas in circular pipes flowing full, it is defined as $DV\rho/\mu$. Since the hydraulic radius is one-fourth the diameter of a circular pipe, a Reynolds number of 500 in an open channel corresponds to a Reynolds number of 2000 in a full pipe. Thus, the reason for the different Reynolds numbers at the transition from laminar to turbulent flow is the different ways that the Reynolds number is defined; the actual properties of the flow at the transition are (virtually) identical in the two types of systems.

3. The net horizontal force on the cone equals the horizontal force to the right on the flat, vertical (circular) surface minus the horizontal force to the left on the conical sides. The *horizontal* force on any differential area of the conical sides equals the force on a projection of that area on a vertical plane. Therefore, that force exactly balances the force on the corresponding vertical area on the flat surface (i.e., at the same depth). As a result, the *net* horizontal force on the cone is the force on the portion of the flat side of the cone that has only air, and not water, to the right. This area is a circle with a diameter of 3 ft. The average pressure on the circle is γh_c, so:

$$F = p_c A = \gamma h_c A = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)(9 \text{ ft})\pi \frac{(3 \text{ ft})^2}{4} = 3{,}970 \text{ lb}$$

4. At the plate, the velocity head is converted entirely to pressure head. The pressure before the water hits the plate is zero, and is the velocity at the point where the pressure is being measured. In addition, the elevation is identical at those two locations. Therefore, using Bernoulli's equation, we can find the upstream velocity, and then use this value in conjunction with the momentum equation to find the force on the plate:

$$\cancel{\frac{p_1}{\gamma}} + \cancel{z_1} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \cancel{z_2} + \cancel{\frac{v_2^2}{2g}}$$

$$v_1 = \sqrt{\frac{2gp_2}{\gamma}} = \sqrt{\frac{2(9.81 \text{ m/s}^2)(690{,}000 \text{ Pa})}{9810 \text{ kg/m}^3}} = 37.2 \text{ m/s}$$

$$F = Q\rho v = \frac{(42.5\text{ L/s})(1000\text{ kg/m}^3)(37.2\text{ m/s})}{1000\text{ L/m}^3} = 1579\text{ N to the right}$$

5. To find the required horsepower, we need to know the headloss in the pipe. We can find the headloss with the DW equation, if we first estimate the friction factor from either the Moody diagram or the Haaland equation (assuming the flow is turbulent). In either case, to estimate *f*, we need to know the Reynolds number, which we can find as follows:

$$V_1 = \frac{Q}{\pi d^2/4} = \frac{2.5\text{ ft}^3\text{/s}}{\pi(8/12\text{ ft})^2/4} = 7.16\text{ ft/s}$$

$$\text{Re} = \frac{Vd}{\nu} = \frac{(7.16\text{ ft/s})(8/12\text{ ft})}{1.06\text{x}10^{-5}\text{ ft}^2\text{/s}} = 4.50\text{x}10^5$$

The value of e/d can be computed directly from the given information:

$$\frac{e}{d} = \frac{6.67\text{x}10^{-4}\text{ ft}}{8/12\text{ ft}} = 0.001$$

For these values of Re and e/d, the friction factor can be found from the Moody diagram as approximately 0.021. The two pipes have identical diameter, roughness, and velocity, so the headloss per unit length is the same in them, and the total headloss can be computed based on their combined length of 3000 ft. The total headloss through the two pipe sections is therefore:

$$h_L = f\frac{L}{d}\frac{V^2}{2g} = 0.021\frac{3000\text{ ft}}{8/12\text{ ft}}\frac{(7.16\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 75.2\text{ ft}$$

The total head that the pump must supply equals the amount of head lost due to friction and the elevation gain of 100 ft, for a total of 175.2 ft. The power requirement is therefore:

$$P = \gamma Q\Delta h_{tot} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(2.5\frac{\text{ft}^3}{\text{s}}\right)(175.2\text{ ft})\left(\frac{1\text{ hp}}{550\text{ ft-lb/s}}\right) = 49.7\text{ hp}$$

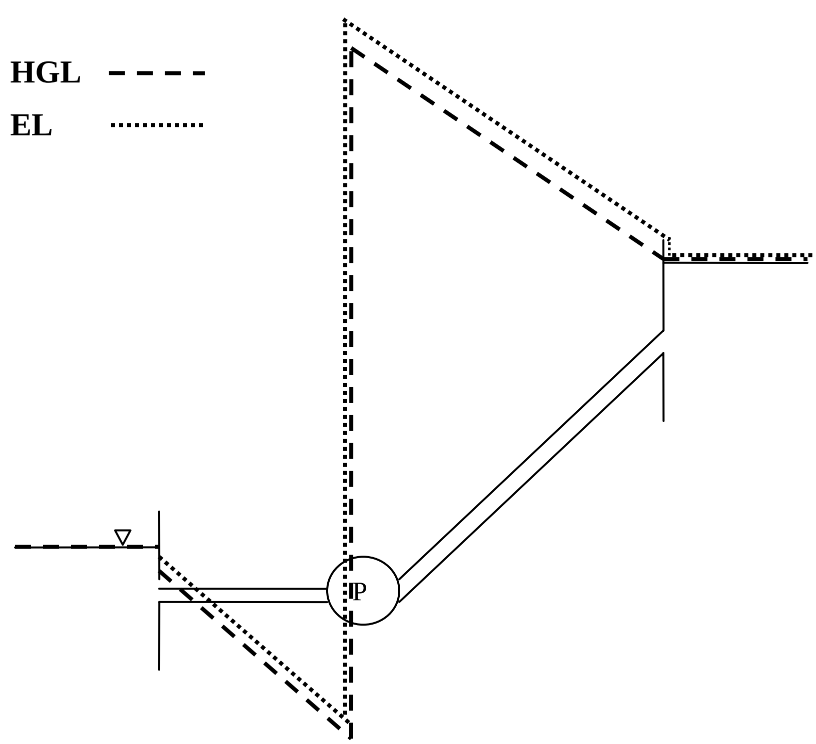
The HGL begins at the water surface and drops below the surface as soon as the water enters the pipe (due to the energy loss at the entrance and the increased velocity). The drop in the HGL due to the velocity increase is $V^2/2g$, or:

$$\frac{V^2}{2g} = \frac{(7.16\text{ ft/s})^2}{2(32.2\text{ ft/s})} = 0.80\text{ ft}$$

If the entrance has a sharp edge, an additional one-half velocity head (~0.4 ft) is lost. From there, the HGL drops farther as the water flows toward the pump, due to friction. The frictional loss in this portion of the pipe is one-third of the total h_L (1000 ft/3000 ft), or 25.1 ft, so the HGL is 26.3 ft (computed as 25.1+0.8+0.4) below the reservoir surface when the water reaches the pump. At the pump, 175.2 ft of head is added to the water, so the HGL rises to approximately 50.1 ft above the second (upper) reservoir. From there, it drops steadily to the surface of the second reservoir.

The EL is at the same level as the HGL in the first reservoir and drops when the water first enters the pipe, but not as much as the HGL (since the only drop in the EL is due to the entrance effect)., so it is 0.80 ft above the HGL at that point. From there on, it remains slightly 0.8 ft above the HGL until the water discharges into the upper reservoir. At that point, energy is dissipated and the EL drops to the level of the reservoir, which is also the level of the HGL. The HGL and EL coincide at both reservoir surfaces, slightly away from the region where the water enters/leaves.

Since the pipe is just 10 ft below the surface of the lower reservoir, the HGL drops substantially below the pipe approaching the pump, so there is negative pressure in approximately the latter two-thirds of this pipe.



6. Applying the energy equation between the surfaces of the two pools of water, we find:

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} = \cancel{\frac{p_2}{\gamma}} + z_2 + \cancel{\frac{V_2^2}{2g}} + h_L$$

$$z_1 - z_2 = h_L$$

The headloss is:

$$\Delta h_L = h_{L,entrance} + h_{L,pipe} + h_{L,exit} = 0.5\frac{V^2}{2g} + f\frac{L}{d}\frac{V^2}{2g} + 1.0\frac{V^2}{2g}$$

so:

$$50 \text{ ft} = \left(0.5 + 0.020\frac{50 \text{ ft}}{6/12 \text{ ft}} + 1.0\right)\frac{V^2}{2(32.2 \text{ ft/s})}$$

$$V = \sqrt{\frac{2(32.2 \text{ ft/s}^2)(50 \text{ ft})}{0.5 + 0.020\frac{50 \text{ ft}}{6/12 \text{ ft}} + 1.0}} = 30.3 \text{ ft/s}$$

The pressure is zero at the top of the upper pool of water and then increases as the elevation decreases. At the entrance to the pipe, some pressure head is lost to friction, and some is converted to velocity head. The amounts of pressure head converted to velocity head and lost due to friction at that point are:

$$\text{Velocity head} = \frac{(30.3 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 14.26 \text{ ft}$$

$$h_{L,entrance} = 0.5\frac{V^2}{2g} = 0.5(14.26 \text{ ft}) = 7.13 \text{ ft}$$

$$\text{Total loss of pressure head} = (14.26 + 7.13)\text{ft} = 21.39 \text{ ft}$$

Because the pressure head is 5 ft before the water enters the pipe, and it drops by 21.39 ft immediately thereafter, the net pressure head is –16.39 ft just inside the pipe, at the top. The headloss due to friction in the pipe is:

$$h_{L,pipe} = f\frac{L}{d}\frac{V^2}{2g} = 0.02\frac{50 \text{ ft}}{0.5 \text{ ft}}(14.26 \text{ ft}) = 28.52 \text{ ft}$$

This frictional headloss corresponds to a loss of a little more than 0.5 ft/ft. Since the water is simultaneously gaining 1 ft of pressure head per ft traveled due to the elevation loss, the net pressure increases as elevation decreases. Therefore, the point of minimum energy is just after the water has entered the pipe. The pressure head at that point is–16.39 ft, as noted above. Converting this value to an absolute pressure, we find:

$$p_{\min,absolute} = p_{\min,gage} + p_{atm} = \gamma \frac{p_{\min}}{\gamma} + p_{atm}$$

$$= 62.4 \frac{\text{lb}}{\text{ft}^3}\left(-16.39\ \text{ft}\right)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) + 14.7\ \text{psi} = +7.60\ \text{psi}$$

Since the vapor pressure of water at 70°F is 0.036 psi, cavitation will not occur.

7. Pipe #2 is a part of all three loops, and pipe #3 is a part of loop *I* only. The corrections to the flow rates in the three loops can therefore be computed as follows:

$$\Delta Q_I = \frac{-\sum_i h_{L,i}}{2\sum_i \frac{h_{L,i}}{Q_i}} = -\frac{90-10-90}{2\left(\frac{90}{3}+\frac{-10}{-1}+\frac{-90}{-3}\right)} = +0.071$$

$$\Delta Q_{II} = \frac{-\sum_i h_{L,i}}{2\sum_i \frac{h_{L,i}}{Q_i}} = -\frac{10+45-5}{2\left(\frac{10}{1}+\frac{45}{5}+\frac{-5}{-5}\right)} = -1.250$$

$$\Delta Q_{III} = \frac{-\sum_i h_{L,i}}{2\sum_i \frac{h_{L,i}}{Q_i}} = -\frac{10+5-5}{2\left(\frac{10}{1}+\frac{5}{5}+\frac{-5}{-5}\right)} = -0.417$$

Defining positive flow in pipe #2 to be from right to left (clockwise for loop #1), the correction to the flow in that pipe equals:

$$\Delta Q_2 = +\Delta Q_I - \Delta Q_{II} - \Delta Q_{III} = +0.071 - \left(-1.250\right) - \left(-0.417\right) = +1.738$$

Defining positive flow in pipe #3 to be from lower left to upper right (also clockwise for loop #1), the correction to the flow in that pipe equals:

$$\Delta Q_3 = +\Delta Q_I = +0.071$$

The new guesses for the flow rates are therefore:

In pipe 2: $-1 + 1.74 = +0.74$ cfs (flow from right to left)

In pipe 3: $-3 + 0.071 = -2.93$ cfs (flow from upper right to lower left)