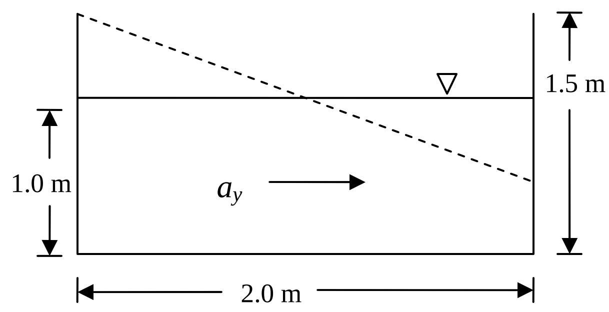
Example calculation for fluid behavior under constant linear acceleration. Munson Problem 2.94.

According to the problem statement, a 1.5-m deep, open tank contains gasoline to a depth of 1 m. The tank is 2.0 m long. We are to find the maximum acceleration in the direction of the tank length that can be maintained without spilling gasoline.



A schematic of the system is shown below. Because of the steady linear acceleration, the surface of the gasoline (which will be at a constant pressure of zero [gage]) will have a constant slope; that is, it will be flat, but not horizontal. The slope of the surface will be given by Equation 2.28:

$$\frac{dz}{dy} = \frac{a_y}{g + a_z}$$

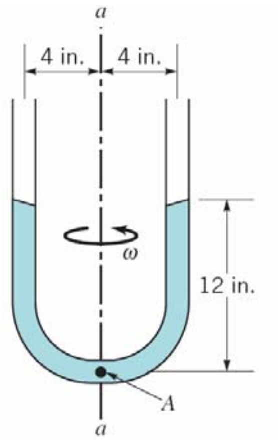
where z is vertical and y is along the long axis of the tank. In the current scenario, $a_z = 0$, and the maximum allowable slope is $dz/dy = (1.0 \text{ m})/(2.0 \text{ m}) = 0.5$. Therefore:

$$a_{y,\max} = g\left(\frac{dz}{dy}\right)_{\max} = 9.81\frac{\text{m}}{\text{s}^2}(0.5) = 4.9\frac{\text{m}}{\text{s}^2}$$

Note that the slope of the fluid surface does not depend on any property of the fluid! The slope would be the same whether the tank was filled with gasoline, water, mercury, or any other liquid.

Example calculation for fluid behavior under constant angular velocity (radial acceleration). Munson Problem 2.99.

We are to determine the angular velocity that will cause the water filling the tube in the schematic shown below to start to vaporize at point *A*.



At equilibrium, the pressure in a rotating fluid is given by Munson Equation 2.33:

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant}$$

Taking the pressure differential between point *A* and a point on the water surface at $r = 4$ in. (which we will refer to as point *B*), we find:

$$p_B - p_A = \frac{\rho\omega^2}{2}\left(r_B^2 - r_A^2\right) - \gamma\left(z_B - z_A\right)$$

The absolute pressure at point *B* is atmospheric (14.7 psi), and for the fluid to boil at point *A*, the absolute pressure at that point must be the vapor pressure of water, which is 0.256 psi (based on Table B.1 and assuming $T = 60^{\mathrm{o}}\mathrm{F}$ [note that a slightly different value is shown in the table on the inside front cover]). All the other terms in the equation other than ω are known, so we can solve for ω (after making appropriate unit conversions) as follows:

$$\omega = \sqrt{2\frac{\left(p_B - p_A\right) + \gamma\left(z_B - z_A\right)}{\rho\left(r_B^2 - r_A^2\right)}}$$

$$= \sqrt{2\frac{\left[(14.7 - 0.256)*144\right]\frac{\text{lb}}{\text{ft}^2} + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1.0\text{ ft})}{\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left(\frac{1\text{ lb}}{\text{slug*ft/s}^2}\right)\left[(0.333\text{ ft})^2 - (0.0\text{ ft})^2\right]}} = 141/\text{s}$$

The required angular velocity is 141 radians/s, or 1348 rpm.