Example 1

Determine the magnitude of the force as well as the depth of application on the circular gate shown in the figure below.



<u>Given</u>: h = 5 ft, D = 4 ft

First calculate the force on the gate using FF Eq. 3.16:

$$F = \gamma h_c A = \gamma \left(h + \frac{D}{2} \sin 60^\circ \right) \left(\pi \left(\frac{D}{2} \right)^2 \right)$$
$$= 62.4 \frac{lb}{ft^3} \left(5 ft + 2 ft \sin 60^\circ \right) \left(\pi \left(4 ft^2 \right) \right) = 5276 lb$$

The distance along the sloping plane from the surface to the point of application, y_p , is related to the location of the centroid and the moment of inertia via:

$$y_p = y_c + \frac{I_c}{y_c A}$$

where y_c is the distance along the sloping plane from the surface to the centroid. This value can be determined by summing the distance of the sloping plane down to depth h and the distance to the center of the circular gate:

$$y_c = \frac{5}{\sin 60} + 2 = 7.78 \, ft$$

Also, for a circle:

$$I_c = \left(\frac{\pi D^4}{64}\right) = \left(\frac{\pi \left[4ft\right]^4}{64}\right) = 12.56ft^4$$

Inserting the above values for y_c and I_c into the equation for y_p , we find:

$$y_{p} = y_{c} + \frac{I_{c}}{y_{c}A}$$
$$= 7.78 ft + \frac{12.56 ft^{4}}{(7.78 ft) \left(\pi \left(\frac{4 ft}{2}\right)^{2}\right)} = 7.78 ft + 0.13 ft = 7.91 ft$$

Finally, we can use geometry to determine the height of water above the center of pressure:

$$h_p = y_p (\sin 60) = 6.85 \, ft$$

Example 2

Determine the minimum value of *b* in the figure below necessary to keep the rectangular masonry wall from sliding if it weights 160 lb/ft³, a = 14 ft, c = 16 ft, and the coefficient of friction is 0.45. With this minimum *b* value, will it also be safe against overturning? Assume that water does not get underneath the block.



First, determine the force per unit length (into the paper) applied by the water against the wall. Defining the length of the wall as L, the area of contact between the water and the wall is La. The centroid of this rectangular area is in its middle, so:

$$\frac{F_{x,water}}{L} = \frac{\gamma h_c A}{L} = \gamma h_c a$$
$$= \left(64.2 \frac{lb}{ft^3}\right) \left(\frac{14 ft}{2}\right) (14 ft) = 6120 \frac{lb}{ft}$$

We can relate the weight of the wall, *W*, to its dimensions:

$$W = \gamma abL = \left(160 \frac{lb}{ft^3}\right) (16 ft) bL$$
$$\frac{W}{L} = 2560 (b) \frac{lb}{ft^2}$$

The frictional resistance opposing the force applied by the water equals the coefficient of friction times the weight of the wall. This force is in the opposite direction from the force of the water on the wall. Therefore:

$$F_{x,friction} = -0.45 \left(2560(b) \frac{lb}{ft^2} \right) = -1152(b) \frac{lb}{ft^2}$$

To determine the minimum value of *b* to avoid sliding of the wall, we sum the horizontal forces and set them equal to zero.

$$\sum F_{x} = 0 = F_{x,water} - F_{x,friction} = 6120 \frac{lb}{ft} - 1152(b) \frac{lb}{ft^{2}}$$
$$b = \frac{6120 \frac{lb}{ft}}{1152 \frac{lb}{ft^{2}}} = 5.31 ft$$

To determine whether the wall is safe from overturning, we consider the moments around the toe of the wall. The force of the water is pushing the wall in a clockwise direction, which is counteracted by the force due to the weight of the wall. If the moment of the force in the clockwise direction is greater than the moment in the counter clockwise direction, then the wall will overturn. The moment due to friction acts through the center of the toe, and is therefore equal to zero.



To compute these moments, we need to identify the locations where they act. The water pressure on the wall increases linearly with depth, so the corresponding pressure profile is a triangle, as shown in the picture above. The surface on which this triangular pressure distribution acts is rectangular, so the center of pressure everywhere along the wall is at the same height, corresponding to one-third of the distance from the base to the top. Thus, the center of pressure for the force of water on the wall will be (14)/3 ft. The center of pressure for the weight of the wall will be at half the width of the wall, or (5.31)/2 ft. The moments associated with these forces, computed around the toe per unit length of wall are as follows:

$$\frac{Moment_{water}}{L} = \frac{14\,ft}{3} \left(6120 \frac{lb}{ft} \right) = 28560 \frac{lb-ft}{ft}$$
$$\frac{Moment_{wall}}{L} = \frac{5.31\,ft}{2} \left(2560 \frac{lb}{ft^2} \right) 5.31\,ft = 36100 \frac{lb \cdot ft}{ft}$$

The (counter-clockwise) moment due to the weight of the wall is greater than the (clockwise) moment due to the force of the water, so the wall is safe from overturning.