CEE 342 Aut 2004 Midterm #2

Answer all questions. Some data that might be useful are as follows:

ρ_{water} = 1000 kg/m^3 = 1.94 slugs/ft^3

γ_{water} = 9810 N/m^3 = 62.4 lbs/ft^3

1 kW = 1000 N-m/s

1. (10) A 1-in. and a 4-in. hole are drilled next to each other in the side of a reservoir. Through which hole will water exit with a higher velocity? Will the streams get wider, thinner, or remain the same diameter as they fall to the ground? Explain your reasoning for each part *in one or at most two sentences*.

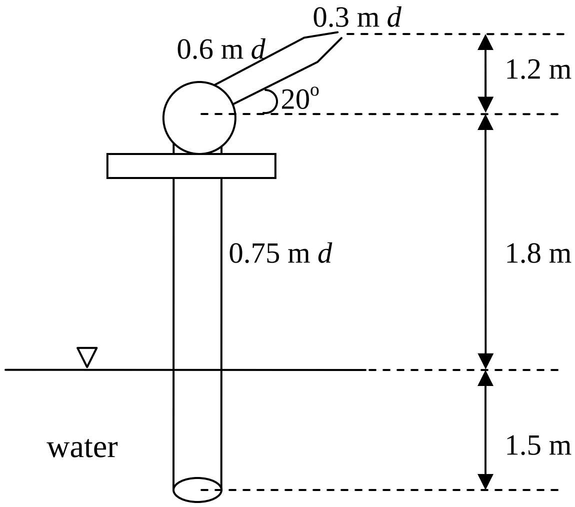
The velocity is generated by the elevation head, which is the same for the two holes. Therefore, the velocities will be the same. This result can also be derived from the Bernoulli equation. Since the pressure is zero at the top of the reservoir and in the exiting stream, the equation is the same when applied between those two locations, for both holes:

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{v_1^2}{2g}} = \cancel{\frac{p_2}{\gamma}} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g\left(z_1 - z_2\right)}$$

As the water falls, the magnitude of its velocity increases. Since the flow rate is constant, continuity requires that the cross-sectional area decrease, so the streams will get thinner.

2. (25) The pump, suction pipe, discharge pipe, and nozzle in the system shown below are all welded together as a single unit. Calculate the horizontal component of the force (magnitude and direction) exerted by the water on the unit when the pump is adding 22.5 m of head to the water that passes through it. Ignore friction.



We can apply the energy equation to determine the velocity of the discharge water. For this calculation, the most convenient control volume is between the water surface and the discharge from the pump, since the pressure is equal (and zero, if expressed as a gage pressure) at those points. The velocity at the water surface is also approximately zero. Since we are ignoring friction, the energy equation is:

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \cancel{\frac{v_1^2}{2g}} + h_{pump} = \cancel{\frac{p_2}{\gamma}} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g\left(h_{pump} + z_1 - z_2\right)} = \sqrt{2\left(9.81\ \text{m/s}^2\right)\left(22.5\ \text{m} + \left(-3\ \text{m}\right)\right)} = 19.56\ \text{m/s}$$

The discharge flow rate is:

$$Q = v_2 A_2 = \left(19.56\frac{\text{m}}{\text{s}}\right)\left(\pi\frac{\left[0.3\ \text{m}\right]^2}{4}\right) = 1.38\frac{\text{m}^3}{\text{s}}$$

The net horizontal force *on* the water can then be computed using the momentum equation. This equation states that the horizontal force equals the rate of change in horizontal momentum. The water has zero horizontal velocity, and therefore zero horizontal momentum, until it goes through the pump and is directed toward the right. When it exits the pump, its horizontal velocity is given by:

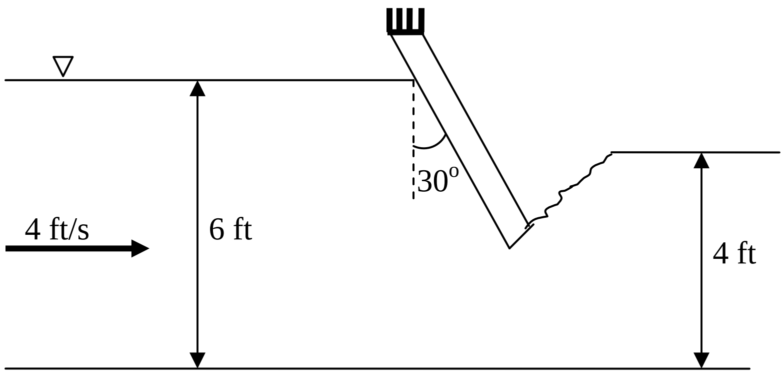
$$v_x = \left(19.56\ \frac{\text{m}}{\text{s}}\right)\cos 20^{\circ} = 18.38\ \frac{\text{m}}{\text{s}}$$

The force on the water is therefore:

$$F_x = \rho Q\left(v_2 - v_1\right) = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(1.38\frac{\text{m}^3}{\text{s}}\right)\left(18.38\frac{\text{m}}{\text{s}} - 0\frac{\text{m}}{\text{s}}\right) = 25,360\text{ N} = 25.36\text{ kN}$$

The force on the water is to the right, so the force that the water exerts on the structure is to the left.

3. (25) Determine the horizontal component of the anchoring force required to hold the sluice gate in the position shown. The gate is 4 ft wide (into the page).



Consider a CV from a point upstream to a point downstream of the gate, where the flow is horizontal at both locations. By continuity, the velocity downstream of the gate is:

$$v_2 = v_1\frac{A_1}{A_2} = \left(4\frac{\text{ft}}{\text{s}}\right)\frac{4\text{ ft x 6 ft}}{4\text{ ft x 4 ft}} = 6\frac{\text{ft}}{\text{s}}$$

The pressure force on the water is γh_c at each location, where $h_c = y/2$. At both locations, the pressure is applied to a vertical plane of the water, so the force is horizontal. Also, both v_1 and v_2 are horizontal velocities. Therefore, defining the force exerted by the gate on the water to be positive to the left, the horizontal force on the water is given by:

$$p_1A_1 - p_2A_2 - F_{gate} = Q\rho\left(v_2 - v_1\right)$$

$$\frac{y_1}{2}\gamma A_1 - \frac{y_2}{2}\gamma A_2 - F_{gate} = \left(v_1A_1\right)\rho\left(v_2 - v_1\right)$$

$$F_{gate} = \frac{y_1}{2}\gamma A_1 - \frac{y_2}{2}\gamma A_2 - \left(v_1A_1\right)\rho\left(v_2 - v_1\right)$$

$$F_{gate} = \frac{6\text{ ft}}{2}\left(62.4\frac{\text{lbs}}{\text{ft}^3}\right)\left(24\text{ ft}^2\right) - \frac{4\text{ ft}}{2}\left(62.4\frac{\text{lbs}}{\text{ft}^3}\right)\left(16\text{ ft}^2\right) -$$

$$\left(4\frac{\text{ft}}{\text{s}}\right)\left(24\text{ ft}^2\right)\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left(6\frac{\text{ft}}{\text{s}} - 4\frac{\text{ft}}{\text{s}}\right)$$

$$F_{gate} = 2124\text{ lbs}$$

The force on the gate is therefore 2124 lbs to the right, and the anchoring force must be 2124 lbs to the left.

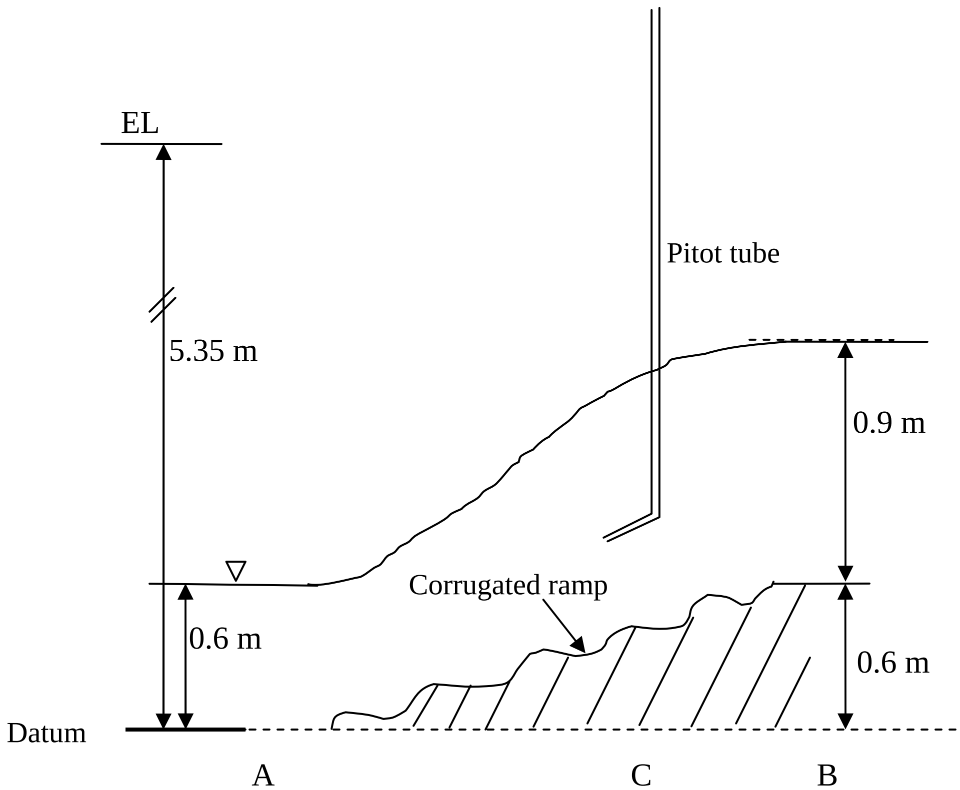
4. (35) The two-dimensional corrugated ramp shown below is being used to generate friction and dissipate energy in the discharge from a cooling tower. The ramp is solid, so that all the water flows over it. The flow rate is 15 m^3/s, and at point A, the energy line is 5.35 m above the base of the water column.

   Note: If you get stuck on one part of the problem and need that value to solve another part of the problem, make a reasonable assumption for the unknown value and continue.

   a. (10) Sketch on the diagram the energy line and the hydraulic grade line from plane A to plane B.

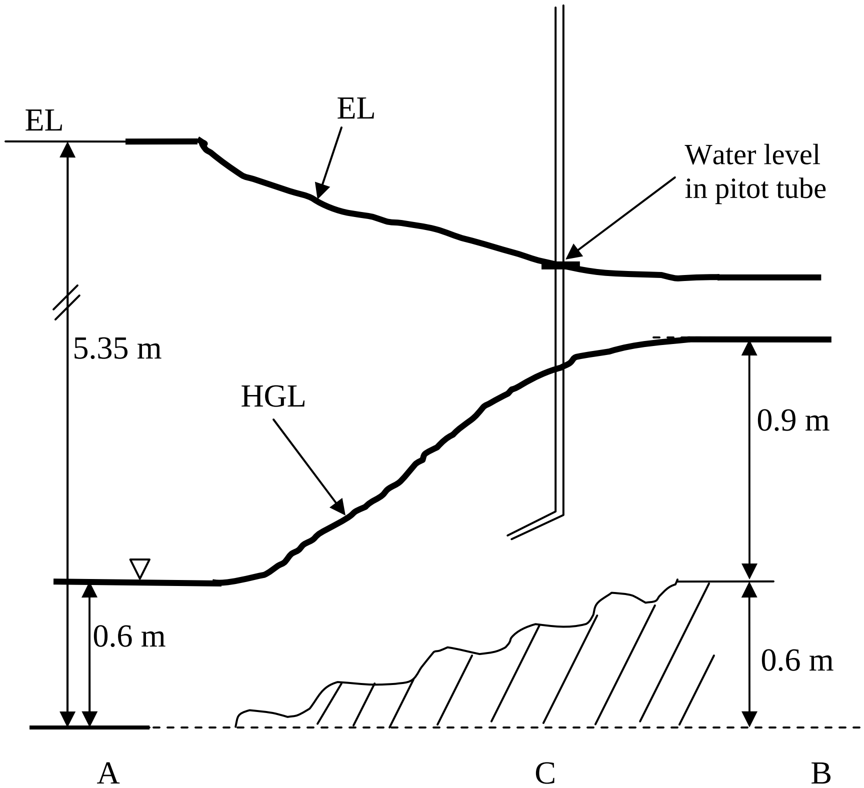
   b. (10) Indicate on the figure how high water would rise in the pitot tube pointing directly into the flow at plane C. Would the water level in the tube rise, fall, or not change elevation if the long part of the tube remained vertical, but the short section at the bottom were bent so that it was horizontal. Explain your reasoning *very briefly*.

   c. (15) How high above the datum is the energy line at point B?



a. As the water goes over the ramp, the energy line drops continuously due to friction. The line would probably drop somewhat more rapidly at the beginning of the ramp because the water velocity is greatest there. Since the (gage) pressure is zero at all locations, the hydraulic grade line follows the surface of the water.

b. If the pitot tube points directly into the flow, the water rises in the pitot tube to the elevation of the energy line at that location. As the tip of the pitot tube is moved away from facing into the flow, the contribution of the velocity head to the pressure at the tip of the tube declines, so the level of the water in the tube falls. If the tip points perpendicular to the flow, the water rises only to the HGL.



c. The velocity at point A can be determined, since the distance from the hydraulic grade line to the energy line is $v^2/2g$:

$$y_{EL,A} - y_{HGL,A} = \frac{v_A^2}{2g}$$

$$v_A = \sqrt{2g\left(y_{EL} - y_{HGL}\right)} = \sqrt{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(5.35\text{ m} - 0.6\text{ m}\right)} = 9.65\frac{\text{m}}{\text{s}}$$

By continuity, the velocity at point B is therefore:

$$v_B = v_A\frac{A_A}{A_B} = 9.65\frac{\text{m}}{\text{s}}\left(\frac{0.6\text{m x } w}{0.9\text{m x } w}\right) = 6.44\frac{\text{m}}{\text{s}}$$

The height of the energy line at point B is therefore:

$$y_{EL,B} = y_{HGL,B} + \frac{v_B^2}{2g} = 1.5\text{m} + \frac{\left(6.44\,\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)} = 3.61\text{ m}$$