CEE 342 AUT 2003 MIDTERM EXAM #2 SOLUTIONS

1. Water in a fountain always seems to spread out as it rises. Explain this observation in 1-2 sentences or equations.

By continuity, the mass flow rate of water must be constant at all elevations. Since the density of the water is constant, the volumetric flow rate, equal to vA, must also be constant. As the water rises, its velocity decreases, so its cross-sectional area must increase; hence, the water spreads out.

2. The end of a water hose is lying in a randomly curved pattern on a flat sidewalk. When the water is turned on, the hose tends to straighten out. Explain the fluid mechanical principles behind in this observation, in 1-2 sentences or equations.

As the water flows through the hose, the hose causes the water's direction to change. To do this, the hose must exert a force on the water in the direction of the curve. The water exerts a reaction force on the hose in the opposite direction, thereby straightening out the hose.

3. The general form of the Reynolds Transport Theorem is as follows. *CV* stands for control volume, and *CS* stands for control surface.

$$\frac{\partial}{\partial t} \int_{CV} i\rho d\Psi = -\int_{CS} i\rho \vec{\mathbf{V}} \bullet d\mathbf{A} + \dot{E}$$

- (a) In general, what is the difference between an extensive property (*E*) and an intensive property (*i*)? Give one example of each type of property.
- (b) Why is E always zero when the RTT is applied to mass, but not necessarily zero when it is applied to energy or momentum?
- (c) The term $\int_{CS} i_{en} \rho \vec{\mathbf{V}} \cdot d\mathbf{A}$ appears in the RTT when it is applied to energy.

Explain the meaning of this term, and why the integration is around the control surface rather than throughout the control volume. Be very brief, and don't try to explain every term in the integral; just describe what the <u>overall</u> integral means.

(a) An extensive property is one that characterizes the total amount of something in a sample. Therefore, if the sample size or mass increases the value of the extensive property increases. Examples include total mass, total energy, total momentum, weight, etc. Intensive properties, on the other hand, have the same value no matter how large the sample size is. Examples include temperature and pressure.

(b) E represents the rate at which the property of interest is entering the system by processes other than advection. Energy and momentum can enter or leave a system by processes other than advection, specifically by the use of pumps or turbines, or by applying a force to the fluid. Mass, on the other hand, can only enter or leave a system by advection.

(c) The integral describes the rate at which energy leaves the system via advection. The integral is evaluated around the control surface, because advection occurs only across that surface.

4. (a) Draw the energy line and hydraulic grade line for the system shown below, for all points between A and B, assuming the fluid is ideal.

(a) The HGL and EL are shown on the diagram below as the thick solid and dashed lines, respectively. Both lines are at the level of the reservoir at point A. The HGL drops slightly as the water gains velocity and enters the pipe. It then remains flat (because the velocity is constant) until it reaches the turbine, at which point it drops significantly. It then remains flat again until the beginning of the nozzle, where the velocity increases and the grade line falls to meet the water at the discharge point. From there on, the pressure is constant at atmospheric, so the grade line follows the water. The EL stays flat at the level of the reservoir surface until the water reaches the turbine, at which point the EL drops by an amount exactly equal to the drop in the HGL. Thereafter, the EL remains flat to point B.

(b) What significant change in fluid properties would occur if the hydraulic grade line dropped below the elevation of the pipe? (This situation might or might not occur in the given system.)

(b) When the hydraulic grade line is below the elevation of the pipe, the pressure in the pipe is less than atmospheric, i.e., the pipe is under vacuum.



5. A rectangular trough is 10 ft long, 2 ft wide, and 4 ft deep. Water is entering the trough from above at a steady rate of 200 gal/min. The only outlet is a rectangular, 0.2-inch x 2 ft gap at the bottom of the tank on one of the ends. Will the tank overflow? If so, at what rate? If not, at what level will the water stabilize? Ignore friction.



This problem can be approached in several ways. One way is determine the depth of water required for the outflow rate to reach 200 gal/min. We begin by computing the velocity of the water at the top of the tank and in the outlet, for the given volumetric flow rate:

$$V_{1} = \frac{(200 \text{ gal/min})(1 \text{ ft}^{3} / 7.48 \text{ gal})(1 \text{ min/60 s})}{(2 \text{ ft})(10 \text{ ft})} = 0.022 \text{ ft/s}$$
$$V_{2} = \frac{(200 \text{ gal/min})(1 \text{ ft}^{3} / 7.48 \text{ gal})(1 \text{ min/60 s})}{(0.2/12 \text{ ft})(2 \text{ ft})} = 13.4 \text{ ft/s}$$

We can then apply the Bernoulli equation between the water surface (point 1) and the middle of the gap (point 2):

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

(Keep in mind that point 1 is defined here as the elevation that would cause the flow rate out to be 200 gal/min, so it is not necessarily the geometric top of the tank.) The pressures at both locations are zero. Defining the elevation datum as the bottom of the tank, we find:

$$\frac{p_1'}{\gamma} + \frac{(0.022 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + z_1 = \frac{p_2'}{\gamma} + \frac{(13.4 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + z_2$$
$$z_1 - z_2 = \frac{(13.4 \text{ ft/s})^2 - (0.022 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 2.79 \text{ ft}$$

The result indicates that, once the water surface reaches an elevation 2.79 ft above the middle of the gap, the outflow will equal 200 gal/min. Therefore, the tank will not overflow.

6. A tank is being weighed as it is filled with water through a 5-cm diameter inlet on the bottom. The flow rate is $0.5 \text{ m}^3/\text{min}$. An instant before in a valve on the flow is a shut, the scale records a weight of 5,000 N. What weight will show on the scale after the flow stops? (Note: this scenario is related to the one we observed in the lab when we weighed the water entering the tank.)

The entering water exerts a force upward on the tank, equal to the loss of momentum of the water as its velocity decreases to zero, i.e.:

$$F_{upward} = \stackrel{\bullet}{m} \left(v_{entering} - \not{p}_{in tank} \right) = Q \rho v_{entering}$$
$$v_{entering} = \frac{Q}{A} = \frac{0.5 \text{ m}^3/\text{min}}{\pi \left(0.05 \text{ m} \right)^2 / 4} = 255 \text{ m/min} = 4.24 \text{ m/s}$$

$$F_{upward} = \left(0.5 \frac{\text{m}^3}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{1000 \text{ kg}}{\text{m}^3}\right) \left(\frac{4.24 \text{ m}}{\text{s}}\right) = 35.4 \text{ N}$$

once the flow is stopped, and this upward flick force is eliminated, the weight recorded by the scale will be 35.4 N greater than before, i.e., it will be 2035.4 N.