Name _____

CEE 342 Final Exam, Aut 2003; Answer all questions; 145 points total. Some information that might be helpful is provided below. A Moody diagram is printed on the last page.

For water at 20° C (68°F):

$$\rho = 1.94 \text{ slugs/ft}^3 = 998 \text{ kg/m}^3;$$

 $\mu = 2.10 \text{ x } 10^{-5} \text{ lb-s/ft}^2 = 1.00 \text{ x } 10^{-3} \text{ N-s/m}^2;$
 $\gamma = 62.3 \text{ lb/ft}^3 = 9.80 \text{ kN/m}^3$

Conversion: 7.48 gal/ft³

Specific gravity of mercury = 13.57

Volume of a cone: $\Psi = \frac{\pi d^2 h}{12}$

1. A large, open tank contains water and oil, and is connected to a square, 6 ft x 6 ft conduit that is sealed by a plug. Determine the magnitude and location of the force on the plug.



2. When the cone-shaped tank shown below is empty and the water level is at point A, the manometer reading (i.e., the difference in Hg levels in the two arms) is 150 mm, and h is 2.04 m. What is the reading when the tank is full?



3. Two identical diagrams are shown below, depicting a pipe exiting from a tank. The pipe has a constant diameter all the way to its outlet (no nozzle).

(a) On the first diagram, draw the HGL and EL between points A and B, if all frictional head losses in the hose are ignored.

(b) Repeat part *a* on the second diagram, this time taking frictional head losses into account, but ignoring the "minor head losses" caused by the bends in the pipe.

(c) Do you expect the gap between the EL and HGL at point B to be larger in scenario *a* or *b* above, or do you expect the gap to be the same in the two cases? Explain briefly.





4. For the scenario shown below, calculate the force required to hold the projectile in place if the mean velocity of the water in the pipe is 6 m/s. The water is flowing through the pipe from left to right toward the projectile and exits as a free jet after flowing by the projectile. Assume that the water behaves as an ideal fluid.



5. During a heavy rainstorm, water completely fills an 18-in diameter, smooth concrete storm sewer with $\varepsilon = 0.001$ ft. If the flow rate is 10 ft³/s and the pipe has a downward slope of 2%, determine the pressure change in a 100-ft section of the pipe.

6. A characteristic curve for a pump is provided below.

- (a) Explain the shape of the system curve, i.e., why is it almost flat at low flow rates, and why does it bend upward at higher flow rates?
- (b) Estimate the brake horsepower if the flow rate is 1600 gal/min.
- (c) In a worst-case scenario, the community that the pump serves could require a flow rate of 2000 gal/min at a head of 65 ft. How many pumps will the community need to meet this demand? Explain your reasoning.
- (d) Would this type of pump be a good choice, based on the efficiency that would be achieved in the worst-case scenario?
- 7. Explain briefly why frictional head loss occurs even in a perfectly smooth pipe.
- 8. Air is flowing through a rectangular, 3 ft x 6 ft duct. What diameter pipe would have approximately the same head loss per unit length, if the air velocities and materials of construction were the same in the two systems?
- 9. Drops of dye are injected simultaneously into the center of a pipe and at a point half-way between the center and the wall, in two different experiments. In one experiment, the flow is laminar, and in the other it is turbulent. A downstream observer notes the spatial relationship between the two drops as they pass her in each experiment. Answer the following, giving a brief explanation for your response:
 - (a) For each experiment, will one drop arrive before the other, or will they arrive together?
 - (b) What difference will the observer note about the appearance of the dye in the two experiments?

CEE 342 Final Exam Solutions, Aut 2003

1. The pressure at each point in the system includes contributions from all the fluid above it. The total force on the plug will be the summation of the forces on the upper half and lower half, which need to be computed separately because of the different specific gravities of the fluids at those levels. The forces can be represented as pressure prisms, as shown below.



 F_1 is the force due to the pressure associated with the 9 ft of oil above the top of the plug; this pressure is constant and is exerted on the whole plug. F_2 is the force on the top half of the plug due to the pressure generated by the oil in that zone; this pressure increases with depth from the top to the middle of the plug. F_3 is the force on the bottom half of the plug, generated by the pressure associated with the oil in the 3 ft above it; this pressure is constant and is exerted on the entire bottom half of the plug. Finally, F_4 is the force on the bottom half of the plug due to the bottom half of the plug due to the bottom half of the plug. Finally, F_4 is the force on the bottom half of the plug due to the pressure generated by the water in that zone; this pressure increases with depth from the middle to the bottom of the plug.

The forces represented by the pressure prisms can then be computed and added, as follows:

$$F_{1} = 0.8 \left(62.3 \frac{\text{lb}}{\text{ft}^{3}} \right) (9 \text{ ft}) (6 \text{ ft}) (6 \text{ ft}) = 16,148 \text{ lb}$$

$$F_{2} = \frac{1}{2} (0.8) \left(62.3 \frac{\text{lb}}{\text{ft}^{3}} \right) (3 \text{ ft}) (6 \text{ ft}) (3 \text{ ft}) = 1,346 \text{ lb}$$

$$F_{3} = 0.8 \left(62.3 \frac{\text{lb}}{\text{ft}^{3}} \right) (3 \text{ ft}) (3 \text{ ft}) (6 \text{ ft}) = 2,691 \text{ lb}$$

$$F_{4} = \frac{1}{2} \left(62.3 \frac{\text{lb}}{\text{ft}^{3}} \right) (3 \text{ ft}) (6 \text{ ft}) (3 \text{ ft}) = 1,682 \text{ lb}$$

$$F_{\text{tot}} = F_{1} + F_{2} + F_{3} + F_{4} = 21,870 \text{ lb}$$

2. When the tank is filled, the increase in pressure is associated strictly with the height of the water, not its volume. As the tank fills and the pressure at the bottom increases, the interface in the right arm of the manometer moves down, while that in the left arm

moves up. Designating the distance moved as Δz , the new pressure at the water/mercury interface will be:

$$P_{\text{water/mercury}} = \gamma_{\text{water}} \left(h_{\text{cone}} + h_{\text{original}} + \Delta z \right) = \left(9800 \frac{\text{N}}{\text{m}^3} \right) \left(3\text{m} + 2.04 \text{ m} + \Delta z \right)$$

The above pressure equals the pressure exerted by the differential heights of the mercury in the two arms:

$$P_{\text{water/mercury}} = \gamma_{\text{mercury}} \left(0.150 \text{ m} + 2\Delta z \right) = 13.57 \left(9800 \frac{\text{N}}{\text{m}^3} \right) \left(0.150 \text{ m} + 2\Delta z \right)$$

Equating the above two expressions, we find:

$$(9800 \frac{N}{m^3}) (3m + 2.04 m + \Delta z) = 13.57 (9800 \frac{N}{m^3}) (0.150 m + 2\Delta z)$$

3.00 m + 2.04 m + $\Delta z = 2.04 m + 27.14 \Delta z$
$$\Delta z = \frac{3.00 m}{26.14} = 0.115 m$$

(Note that the original Δh of 150 mm exactly balances the original water height of 2.04 m, as we would expect.) The manometer reading equals the original reading of 0.150 m, plus $2\Delta z$, for a total of 0.380 m.

3. (a) In the absence of any friction, the energy line (EL) is that the level of the water in the tank for the entire distance from A to B. The hydraulic grade line (HGL) must be at the level of the outlet at B. Since the diameter of the hose is constant, and no energy is lost as the water flows through it, the HGL must be at the level of the outlet throughout the hose. The decline in the HGL between the level of the water in the tank and the level of the hose outlet occurs in the tank, just before the water enters the hose, as the water acquires significant velocity.

(b) When frictional losses are taken into account, the EL drops steadily from the tank to the outlet point, remaining above the outlet point at all locations. As in part *a*, the HGL must go from the level of the water in the tank to the level of the out let as the water passes through the hose. However, in this case, the decline occurs as steadily, in direct proportion to the length of hose through which the water has passed.

(c) The gap between the EL and the equals the velocity head. Friction causes the velocity through the hose to decline, decreasing the velocity head, and decreasing the gap between the EL and the HGL.

4. As the water flows around the projectile, it accelerates, indicating that there is a force being exerted on it from left to right, which occurs because the upstream pressure is

larger than the downstream pressure. This pressure difference is also applied to the projectile, so an external force must be exerted on the projectile from right to left to hold it in place. Defining v_1 as the velocity upstream of the projectile, and v_2 as the velocity as the water flows past the projectile, we have:

$$A_{1} = \frac{\pi}{4} (0.3 \text{ m})^{2} = 0.0707 \text{ m}^{2}$$

$$Q = A_{1}V_{1} = 0.0707 \text{ m}^{2} (6 \text{ m/s}) = 0.424 \text{ m}^{3} / \text{s}$$

$$A_{2} = A_{1} - A_{\text{projectile}} = \frac{\pi}{4} \Big[(0.3 \text{ m})^{2} - (0.25 \text{ m})^{2} \Big] = 0.0216 \text{ m}^{2}$$

$$V_{2} = \frac{Q}{A_{2}} = \frac{0.424 \text{ m}^{3}/\text{s}}{0.0216 \text{ m}^{2}} = 19.64 \text{ m/s}$$

By the work-energy equation for an ideal fluid:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \varkappa_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \varkappa_2$$

$$p_1 = \left(\frac{V_2^2 - V_1^2}{2g}\right)\gamma = \left[\frac{\left(19.64 \text{ m/s}\right)^2 - \left(6.0 \text{ m/s}\right)^2}{2\left(9.8 \text{ m/s}^2\right)}\right] (9.8 \text{ kN/m}^3) = 174.8 \text{ kPa}$$

Then, by the impulse-momentum equation:

$$\sum F_{\text{ext}} = (p_1 A_1 - p_2 A_2) - F = Q\rho(V_2 - V_1)$$

$$F = p_1 A_1 + Q\rho(V_2 - V_1)$$

$$= (174,800 \text{ Pa})(0.0707 \text{ m}^3) + (0.424 \text{ m}^3/\text{s})\left(1000\frac{\text{kg}}{\text{m}^3}\right)((6.0 - 19.64) \text{ m/s})$$

$$= 6570 \text{ N}$$

5.
$$V = \frac{Q}{A} = \frac{10 \text{ ft}^3 / \text{s}}{\pi (0.75 \text{ ft})^2} = 5.66 \text{ ft/s}$$

$$R = \frac{VD\rho}{\mu} = \frac{(5.66 \text{ ft/s})(0.75 \text{ ft})(8.94 \text{ slug/ft}^3)}{2.1 \text{x} 10^{-5} \text{ lb-s/ft}^2} = 7.84 \text{x} 10^5$$

Thus, the flow is turbulent.

$$\frac{\varepsilon}{D} = \frac{0.001 \text{ ft}}{0.75 \text{ ft}} = 6.67 \text{ x} 10^{-4}$$

From the Moody Diagram, we find f = 0.0182.

$$h_L = f \frac{L}{D} \frac{V^2}{2g} = 0.0182 \frac{100 \text{ ft}}{0.75 \text{ ft}} \frac{(5.66 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 0.60 \text{ ft}$$

We can now apply the Bernoulli equation, noting that the velocity is constant and that the elevation decreases by 2 ft over a 100-ft length of pipe:

$$\frac{p_1}{\gamma} + \frac{V_1^{2'}}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^{2'}}{2g} + z_2 + h_L$$

$$p_2 - p_1 = \gamma \left((z_1 - z_2) - h_L \right)$$

$$= \left(62.3 \text{ lb/ft}^3 \right) \left(2 \text{ ft} - 0.60 \text{ ft} \right) = 87.2 \text{ lb/ft}^2 = 0.61 \text{ lb/in}^2$$

6. (a) and (b) The head required to service the needs of the community is indicated by the system curve. This head includes contributions from an elevation gain, the velocity head, and the head loss. Although the elevation gain is constant, both the velocity head and the frictional losses (which are proportional to the velocity head) increase with the square of the velocity. The system curve is flat at low flow rates because the head associated with the elevation gain is much greater than the other two contributions. Of these latter two contributions, the head loss is likely to be much more important than the velocity head. Note that the shape of the system curve has nothing to do with the pump itself, or the pump's efficiency.

(c) The brake horsepower is the horsepower provided to the pump. This power includes both the power that is actually transmitted to the water and the power that generates the waste heat. When the pump operates at a flow rate of 1600 gal/min, it develops a head of 65 ft and has an efficiency of 85%. The brake horsepower can therefore be calculated as follows:

BHP =
$$\frac{\gamma QE_p}{\eta} = \frac{(62.3 \text{ lb/ft}^3)(1600 \text{ gal/min})(65 \text{ ft})}{0.85(7.48 \text{ gal/ft}^3)(60 \text{ s/min})} = 16,984 \frac{\text{ft-lb}}{\text{s}}$$

BHP = 16,984 $\frac{\text{ft-lb}}{\text{s}} \left(\frac{1 \text{ ft-lb/s}}{550 \text{ hp}}\right) = 30.9 \text{ hp}$

7. Friction occurs even in perfectly smooth pipes because of the no-slip condition at the pipe wall. Because the fluid at the wall is not moving, and that in the center of the pipe is moving, friction is generated as the parcels of fluid slide past one another.

8. A full flowing pipe will have the same head loss as the channel described if the two have equal hydraulic radii (which would cause the two systems to have equal Reynolds numbers). The hydraulic radius is defined as the area contacted by the fluid divided by the perimeter in contact with the fluid. Thus, for the duct:

$$R_{H} = \frac{\text{Duct area}}{\text{Duct perimeter}} = \frac{6 \text{ ft x 3 ft}}{6 \text{ ft} + 2(3 \text{ ft})} = 1.5 \text{ ft}$$

For a circular pipe, the hydraulic radius is one half of the actual radius. Therefore, for a pipe to have a hydraulic radius of 1.5 ft, it must have a true radius of 3.0 ft, or a diameter of 6.0 ft.

9. (a) In the laminar flow system, the fluid (and therefore the droplet) at the center of the pipe has a considerably higher velocity than the fluid nearer the wall. Therefore, the droplet in the middle of the pipe will reach the observer sooner. On the other hand, in turbulent flow, the velocity profile is almost flat across the pipe. In this case, the two droplets would reach the observer at almost the same instant.

(b) The droplets in the laminar flow system would retain their shape and still look like distinct droplets when they pass the observer. The droplets in the turbulent system, by contrast, would disperse into the fluid and might not be distinguishable as droplets at all by the time they reach the observer.