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Beginner's Mathematica Tutorial

Introduction

This document is designed to act as a tutorial for an individual who has had no prior experience with Mathematica. For a more advanced tutorial, walk through the Mathematica built in tutorial located at Help > Tutorial on the Mathematica Task Bar. For any questions or concerns, please contact

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Starting the Program

1. Start Mathematica. After the program starts, you should see something similar to that shown in Figure 1.

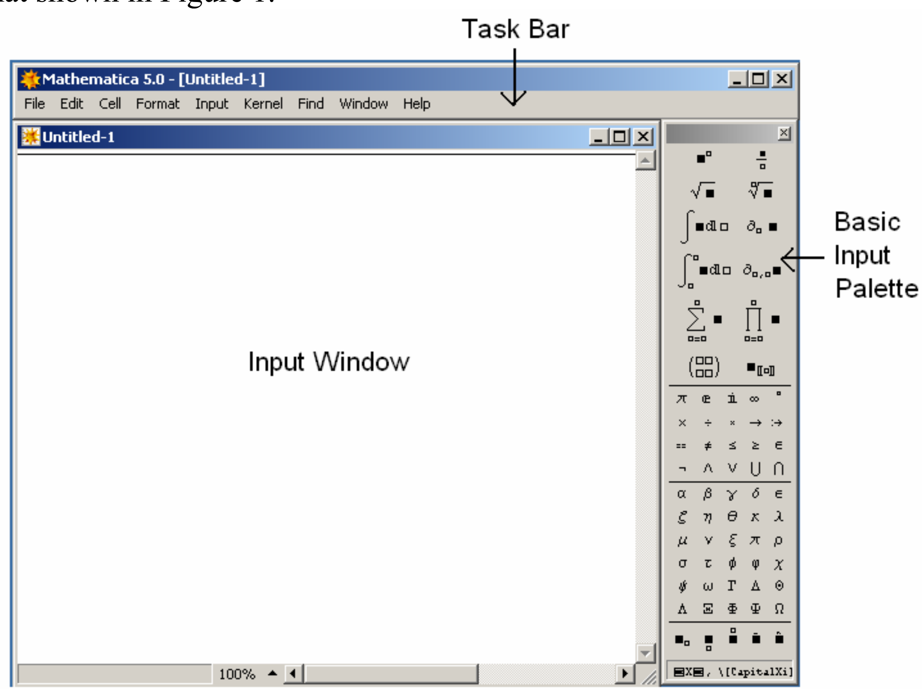


Figure 1: Basic Mathematica interface

2. It is possible that the Basic Input Palette is not visible at startup. To activate this window, go File > Palettes > Basic Input

Using Mathematica

1. Mathematica is a symbolic manipulator. To assign a variable, simply type it in the Input Window. The enter in the command, you need to hit "Shift + Enter".

    1. Type in "x = 1" then hit "Shift + Enter"
    2. Type in "y = a+b;" then hit "Shift + Enter"   (note the semicolon here!)
    3. Type "z = x + y" then hit "Shift + Enter"

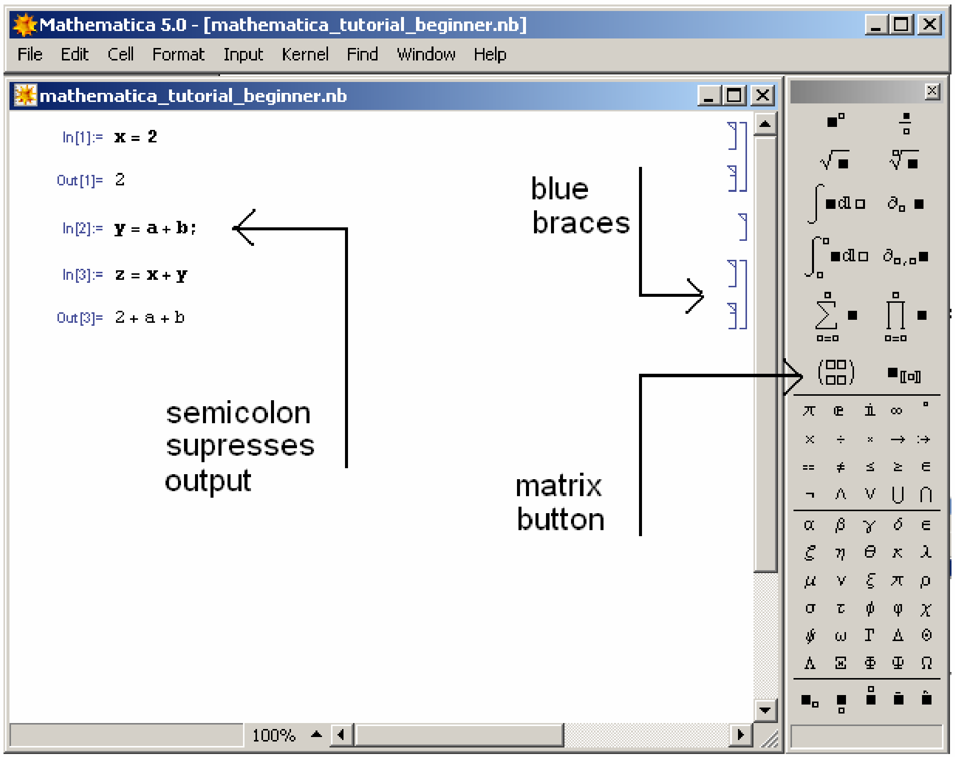


Figure 2: Entering in variables into Mathematica

As can be seen, the semi-colon suppresses the outputs to the screen. Also notice that a and b do not need to be defined as numerical values.

2. Mathematica is nice because it can be used to compose documents which have text and code interlaced with each other. To add a line of text, first type in the line. Notice that this is in bold font just like the rest of the commands. Also notice the blue braces that appear to the right of the Input window (see Figure 2). Click on this brace to select it. Then select Format > Style > Text on the Task Bar or simply hit "Alt + 7".

3. Mathematica stores variables until they are explicitly cleared or you quit the kernel. Therefore, it is good practice to clear variables when they are no longer needed. To do this, we use the Clear function. Clear the variables x, y, and z.

Functions:	*Clear*
Note:	*All Mathematica functions begin with a capital letter and their arguments are enclosed in square braces. For example to clear x, y, and z, we need to type* `Clear[x,y,z]` *Remember that you need to hit "Shift + Enter" to execute the command.*

4. Recall that the state space representation of the mass/spring/damper system is given by

$$\dot{\bar{x}} = A\bar{x} + Bu \qquad \textbf{Equation 1}$$

$$y = C\bar{x} + Du$$

where $A = \begin{pmatrix} 0 & 1 \\ -k/m & -c/m \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 1/m \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$ $D = 0$

For this situation, let k = 2, c = 0.5, and m = 1. Enter in these constants into Mathematica now. A sample code is shown below.

Part 4

Enter in the constants of the spring /mass/damper system.

```
In[9]:= k = 2;

In[73]:= c = 1 / 2;

In[10]:= m = 1;
```

Notice here that c is entered as ½, not 0.5. This is important in Mathematica because there is no numerical round off when using fractions.

5. Now define the A, B, C, and D matrices. Note: You cannot assign a value/object to the letter "C" because this is a protected Mathematica symbol. Therefore, use a different name for the matrices.

To enter a matrix, click on the Matrix Button as shown in Figure 2. This generates a blank 2x2 matrix to be filled in. To add rows, select where you would like to insert a row and press "Ctrl + Enter". To add columns, select where you would like to insert a column and press "Ctrl + ," A sample code is shown below.

Part 5

Now define the A, B, C, and D matrices

In[11]:= $\text{Amat} = \begin{pmatrix} 0 & 1 \\ -k/m & -c/m \end{pmatrix};$

In[15]:= $\text{Bmat} = \begin{pmatrix} 0 \\ 1/m \end{pmatrix};$

In[16]:= $\text{Cmat} = \begin{pmatrix} 1 & 0 \end{pmatrix};$

In[19]:= $\text{Dmat} = (0);$

6. To check the internal stability of the system, calculate the eigenvalues and of the A matrix. Also obtain the eigenvectors. You may want to use the Simplify function to simplify the expressions.

Functions:	*Eigenvalues, Eigenvectors, Simplify*
Note:	*Greek symbols can be inserted using the Basic Input Palette of by using the Esc, letter, Esc key combination. For example to quickly insert the λ symbol. Type "Esc, l, Esc". A sample code is shown below.*

Part 6

Calculate the eigenvalues and eigenvectors of the A matrix

In[101]:= `EigenvaluesA = Eigenvalues[Amat] // Simplify`

Out[101]= $\left\{\frac{1}{4} i \left(i+\sqrt{31}\right), -\frac{1}{4} i \left(-i+\sqrt{31}\right)\right\}$

In[100]:= `EigenvectorsA = Eigenvectors[Amat] // Simplify`

Out[100]= $\left\{\left\{-\frac{1}{8} i \left(-i+\sqrt{31}\right), 1\right\}, \left\{\frac{1}{8} i \left(i+\sqrt{31}\right), 1\right\}\right\}$

Seperate these into two individual eigenvalues and eigenvectors.

In[102]:= `λ1 = EigenvaluesA[[1]] // Simplify`

Out[102]= $\frac{1}{4} i \left(i+\sqrt{31}\right)$

In[103]:= `λ2 = EigenvaluesA[[2]] // Simplify`

Out[103]= $-\frac{1}{4} i \left(-i+\sqrt{31}\right)$

Be careful, Mathematica 5.0 gives eigenvectors in row form.

In[104]:= `v1 = ( EigenvectorsA[[1, 1]] ; EigenvectorsA[[1, 2]] )`

Out[104]= $\left\{\left\{-\frac{1}{8} i \left(-i+\sqrt{31}\right)\right\}, \{1\}\right\}$

In[105]:= `v2 = ( EigenvectorsA[[2, 1]] ; EigenvectorsA[[2, 2]] )`

Out[105]= $\left\{\left\{\frac{1}{8} i \left(i+\sqrt{31}\right)\right\}, \{1\}\right\}$

7. Verify that the eigenvectors and eigenvalues obtained satisfy the definition

$$(A-\lambda_i I)v_i = 0 \qquad \textbf{Equation 2}$$

Functions:	*IdentityMatrix,*
Note:	*Mathematica may give a number which appears to not be zero. Once again, just use the Simplify function to simplify the expression.*

8. Calculate the characteristic equation for this system, p(λ). Recall that the characteristic equation is given by

$$p(\lambda) = \det(\lambda I - A) = 0 \qquad \textbf{Equation 3}$$

This is a function of λ. It is simple to define your own function in Mathematica. For example, Equation 3 can easily be defined using the command

```
In[88]:= p[λ_] = Det[λ*IdentityMatrix[2] - Amat]

Out[88]= 2 + λ/2 + λ^2
```

Notice the λ_ . This defines that the function p is a function with λ as its argument. Multiple arguments would be separated by commas.

Functions:	*Det*
Note:	*None*

9. Verify that the eigenvalues obtain in part 6 are roots of the characteristic equation. A sample code is shown below

Functions:	*None*

Note:	**Part 9**
	Verify that the eigenvalues we obtained previously are roots of the characteristic equ
	In[90]:= p[λ1] // Simplify
	Out[90]= 0
	In[91]:= p[λ2] // Simplify
	Out[91]= 0

10. Calculate the transfer function between x_1 and y. Recall that this is given by

$$G(s) = \frac{y(s)}{u(s)} = C(sI - A)^{-1} B + D \qquad \textbf{Equation 4}$$

Functions:	*Inverse*
Note:	*A sample code is shown below*

<table>
<tr><td></td><td>

Part 10

We can calculate the transfer function between the input and output of this system.

In[92]:= `temp = Cmat.Inverse[s IdentityMatrix[2] - Amat].Bmat + Dmat`

Out[92]= $\left\{\left\{\frac{1}{2+\frac{s}{2}+s^2}\right\}\right\}$

Notice that this returns a matrix, if we want to extract the transfer function from the matrix, we pull out the 1,1 element of the matrix.

In[93]:= `G[s_] = temp[[1, 1]]`

Out[93]= $\frac{1}{2+\frac{s}{2}+s^2}$

</td></tr>
</table>

11. Now calculate the response of the system to a unit step input in the Laplace domain. Recall that the in the Laplace domain, a step input is $u(s)=1/s$.

    After you have the response in the Laplace domain, use the inverse Laplace transform to calculate the response in the time domain.

<table>
<tr><td>Functions :</td><td>InverseLaplaceTransform</td></tr>
<tr><td>Note:</td><td>

A sample code is shown below

Part 11

We can now calculate the response of the system to a step input in the Laplace dom

In[94]:= `Y[s_] = G[s] * 1 / s // Simplify`

Out[94]= $\frac{2}{4s+s^2+2s^3}$

Inverse Laplace transform this to obtain the response of the system in the time dom

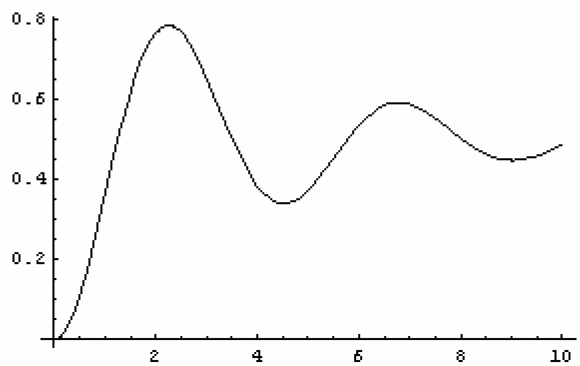
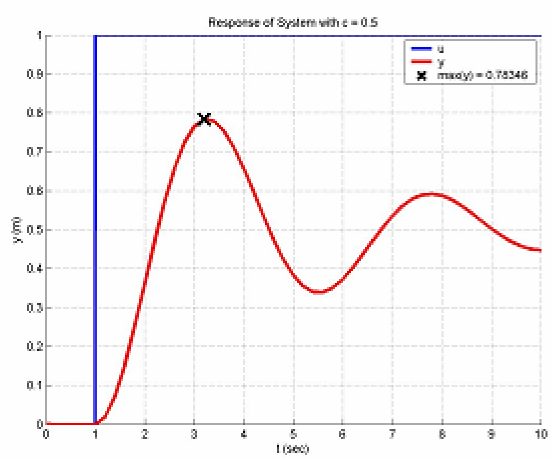
In[98]:= `y[t_] = InverseLaplaceTransform[Y[s], s, t] // FullSimplify`

Out[98]= $\frac{1}{2}-\frac{1}{62}e^{-t/4}\left(31\,\mathrm{Cos}\left[\frac{\sqrt{31}\,t}{4}\right]+\sqrt{31}\,\mathrm{Sin}\left[\frac{\sqrt{31}\,t}{4}\right]\right)$

</td></tr>
</table>

12. Plot the response of the system in the time domain over the range 0 < t < 10 seconds.

In addition, import the response of the system that was calculated using Matlab/Simulink. You should have created this figure during the Simulink Tutorial and saved it as a .jpg. To import a jpg into Mathematica, select Edit > Insert Object > Microsoft Word Picture. Microsoft Word should open. Simply insert the picture into the box and then close Microsoft Word.

Functions:	*Plot*
Note:	*A sample code is shown below* Part 12 Now let's plot the response of the system In[99]:= `Plot[y[t], {t, 0, 10}]`  Out[99]= - Graphics - Let's compare to see how this compares to the results that we obtained with Simulink 

Version History:	09/29/04: Created:	
	11/23/05: Updated:	Made this format match other to-do documents and removed references to AA547.
	12/01/05: Updated:	Changed headers to match how-to template
	12/09/05: Updated:	Made changes to layout and added footer.