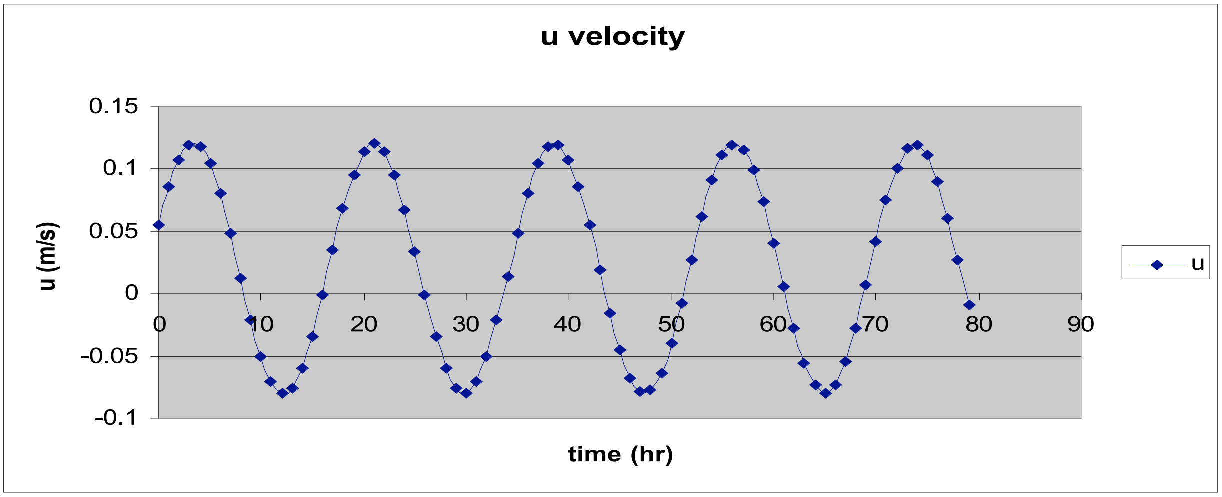
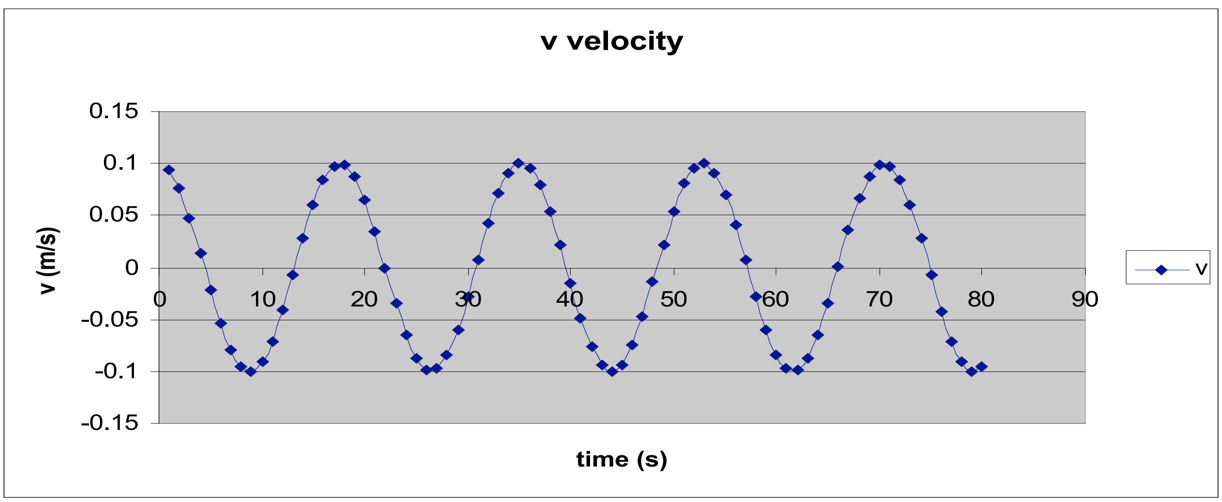
Ocean 420 Physical Processes in the Ocean
Answers to Project 9: Internal waves

2. A mysterious oscillation…

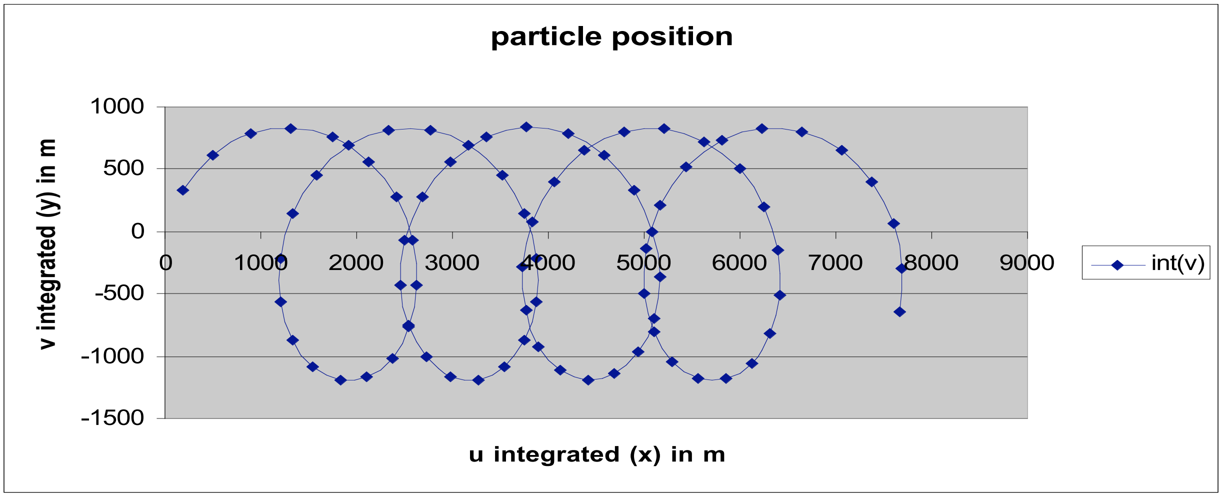
You suspect that the observations will show a nearly constant current. Instead you find significant variability.

a) To begin your analysis, plot u versus time and v versus time.





b) Next, plot a progressive vector diagram. A progressive vector diagram shows the displacement that a parcel would experience if it were in the velocity field. To do this, integrate u and v with respect to time, and plot the results against each other. To integrate, start the integration at x=y=0, then for t=1 hour, the location is 1 hour times the velocity at 1 hour. For hour two, the location will be the location at hour 1 plus the velocity at hour 2 times one hour etc.



c) You decide that the time series represents a constant current superimposed on an oscillation with a fixed period. Estimate both the constant current and the period of the oscillation.

The current can be calculated by looking at the u vs. time plot and figuring out how much it needs to be shifted to have the peaks and the crests the same magnitude.

max – min = 0.12-0.08 = 0.04m/s.

However, we only want to shift half this distance to make each side even.

Thus, our constant current is 0.02m/s.

The period of the oscillation can be determined from finding the time between peaks on either of the u vs. time or v vs. time plots. Reading from the graph, we get ~18 hour period.

d) What is the oscillation?

$$f = 2\Omega \sin\theta = 2 \cdot \frac{2\pi}{86400s} \cdot \sin 47° = 10.6 \times 10^{-5} s^{-1}$$

Our period taken from the above graphs is 18 hours.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{18hr} \cdot \frac{1hr}{3600s} = 9.7 \times 10^{-5} s^{-1}$$

Since $f \approx \omega$, it is an inertial oscillation.

3. A two layer internal wave in the coastal region.

We are going to examine a similar internal wave that you studied in problem set 8, but this time with rotational effects. Let the upper layer depth be 100m, the lower layer depth be 1000m. The upper layer density is 1028 kg/m3, and the density is 1029 kg/m3 in the lower layer. We are at 30N.

a) A wind event generates an upwelling internal wave at 30N with a positive deviation in interface height of size 30m at the coast. What would the sea surface height deviation be associated with this wave? Give sign and magnitude.

$$\frac{\eta}{h} \approx -\frac{g'}{g} = -\frac{\Delta\rho}{\rho}$$

$$\frac{\eta}{30m} = -\frac{1029kg/m^3 - 1028kg/m^3}{1028kg/m^3}$$
$$\eta = -0.0292m$$

b) If the period of the wave were 24 hours, what would the wavelength of the internal (baroclinic) wave be?

$C = \frac{\omega}{k} = \frac{\Lambda}{T}$ and $C = C_g$ since a Kelvin wave is non-dispersive.

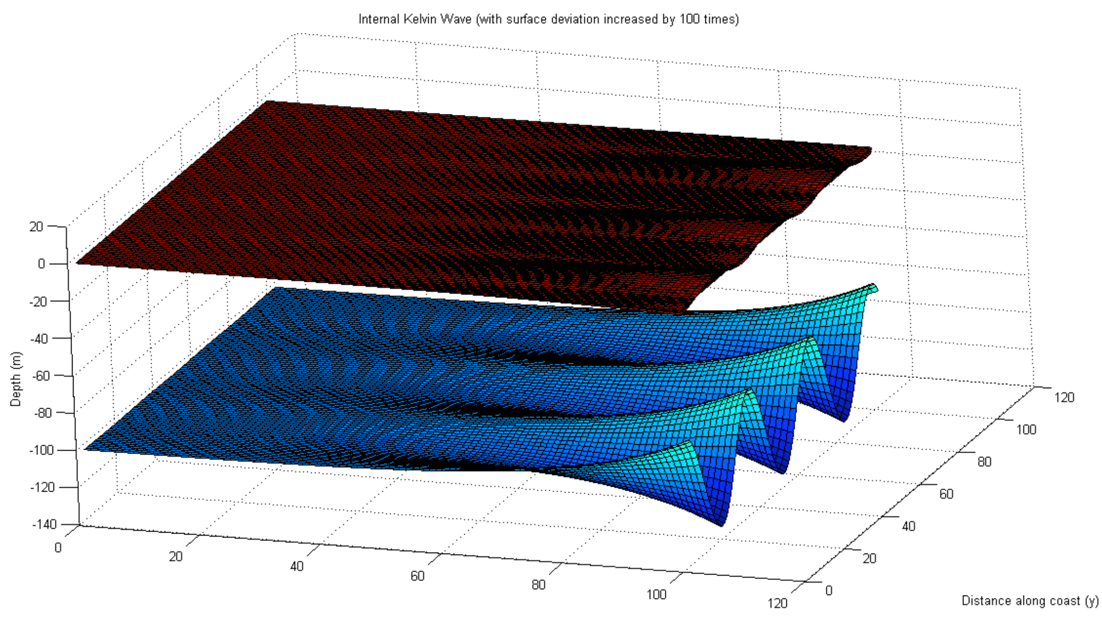
$$C = \sqrt{g'\frac{H_1 H_2}{H_1 + H_2}} = \sqrt{\frac{9.8m/s^2 \cdot 1kg/m^3}{1028kg/m^3} \cdot \frac{100m \cdot 1000m}{100m + 1000m}}$$
$$C = 0.931m/s$$
$$\Lambda = CT = 0.931m/s \cdot 86400s = 80.4km$$

c) Calculate the internal deformation radii at 30N.

$$L_R = \frac{\sqrt{g'H_1}}{f} = \frac{1}{f}\sqrt{\frac{9.8m/s^2 \cdot 1kg/m^3 \cdot 100m}{1028kg/m^3}} = 13430m = 13.4km$$

d) Sketch the sea surface height and interface height associated with this wave. Write down a formula for the sea surface height as a function of distance from the coast.

$$\eta = \eta_0 e^{\frac{x}{L_R}} \cos(ky - \omega t)$$



e) Write down a formula for the velocity of the wave as a function of distance from the coast. What would the velocity perturbation in the upper layer be at the coast associated with the internal wave? Give the direction and magnitude.

First, we find the formula for the velocity of the wave.

$$-fv = -g\frac{\partial \eta}{\partial x} \Rightarrow v = \frac{g}{f}\frac{\partial \eta}{\partial x}$$

Next we differentiate the formula from (d).

$$\frac{\partial\eta}{\partial x} = \eta_0 \cos(ky - \omega t) \cdot \frac{1}{L_R} e^{\frac{x}{L_R}}$$

$$v = \frac{\eta_0 g}{f L_R} e^{\frac{x}{L_R}} \cos(ky - \omega t)$$

To find the velocity perturbation at the coast $(x = 0)$, we take cos=1.

$$v = \frac{\eta_0 g}{f L_R} = \frac{-0.0292m \cdot 9.8m/s^2}{7.27 \times 10^{-5} \cdot 13400m} = -0.294m/s$$