Ocean 420 Physical Processes in the Ocean
Answers to Project 8: Waves

1. A two layer internal wave in the coastal region.

Let the upper layer depth be 100m, the lower layer depth be 300m. The upper layer density is 1028 kg/m3, and the density is 1029 kg/m3 in the lower layer.

a) A wind event generates an upwelling internal wave at 30N with a positive deviation in interface height of size 30m. What would the sea surface height deviation be associated with this wave? Give sign and magnitude.

We use the formula that relates interface offset to sea surface height deviation for internal waves.

$$\frac{\eta}{h} \approx -\frac{g'}{g} = -\frac{\Delta\rho}{\rho}$$

$$\frac{\eta}{30m} = -\frac{1029kg/m^3 - 1028kg/m^3}{1028kg/m^3}$$

$$\eta = -0.0292m$$

b) How long would it take for this internal wave to propagate to 40N?

First we calculate wave speed.

$$C = \sqrt{g'\frac{H_1 H_2}{H_1 + H_2}} = \sqrt{\frac{9.8m/s^2 \cdot 1kg/m^3}{1028kg/m^3} \cdot \frac{100m \cdot 300m}{100m + 300m}}$$

$$C = 0.84m/s$$

Distance to travel.

$$10° \cdot \frac{111km}{1°} \cdot \frac{1000m}{1km} = 1110000m$$

Time to 40N.

$$\frac{1110000m}{0.84m/s} = 1321000s = 15.3days$$

c) At the same time that the wave passes by, there is a storm brewing that has a pressure that is 5 millibars lower than the day before. This results in the sea surface being raised to compensate. You have a tide gauge that measures the total sea level. How would the sea level life from the storm compare to the sea level change due to the passing internal wave? (Hint: the sea level depression owing to the storm would be given by the inverse barometer effect. The sea level rise gives rise to a pressure deviation and this will be equal and opposite to the sea level pressure anomaly).

$$5mb \cdot \frac{100Pa}{1mb} = 500Pa$$

We use the hydrostatic balance, solving for Δz.

$$\frac{\partial p}{\partial z} = -\rho g \Rightarrow \Delta z = -\frac{\Delta p}{\rho g}$$

$$\Delta z = -\frac{-500Pa}{1028kg/m^3 \cdot 9.8m/s^2} = 0.0496m$$

2. Internal Waves in the Coastal Region

Consider a coastal region with potential density that varies continuously from 1028 kg/m^3 at the surface to 1030 kg/m^3 at a depth of 200m (note that you can ignore the effects of rotation for this question).

a) What is the buoyancy frequency?

$$N = \sqrt{-\frac{g}{\rho} \cdot \frac{\partial \rho_\theta}{\partial z}} = \sqrt{-\frac{9.8m/s^2}{1029kg/m^3} \cdot \frac{1030kg/m^3 - 1028kg/m^3}{-200m - 0m}} = 0.00976s^{-1}$$

b) What is the maximum internal wave frequency? What is the period?

Dispersion relation $\omega^2 = N^2 \cos^2 \varphi$

The maximum value of $\cos\varphi$ is 1. This gives us $\omega^2 = N^2$.
Thus, the max internal frequency is N, or 0.00976s^{-1}.
From this, we calculate the period.

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{0.00976s^{-1}} = 644s$$

c) Internal waves are generated at the bottom from interaction of the M2 tide (period of 12.42 hours) with the coastal bathymetry. At what possible angles can the energy propagate with respect to the horizontal? Sketch this.

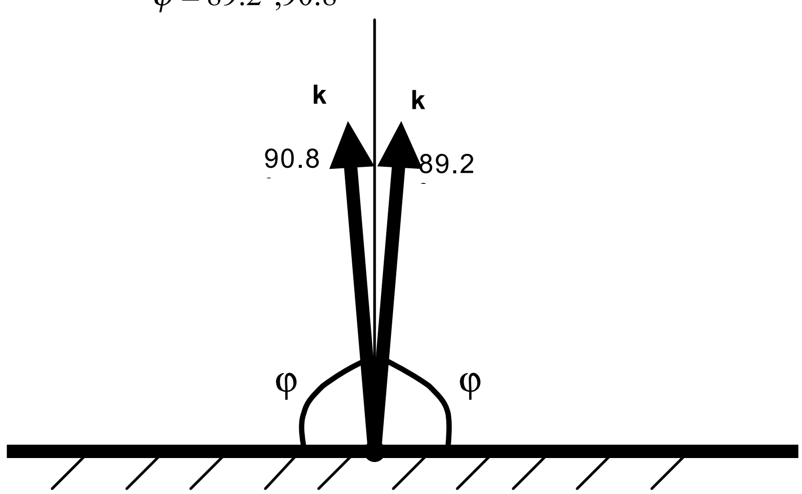
Here, we plug in N and T and solve for the angle φ.

$$\omega^2 = N^2 \cos^2 \varphi$$

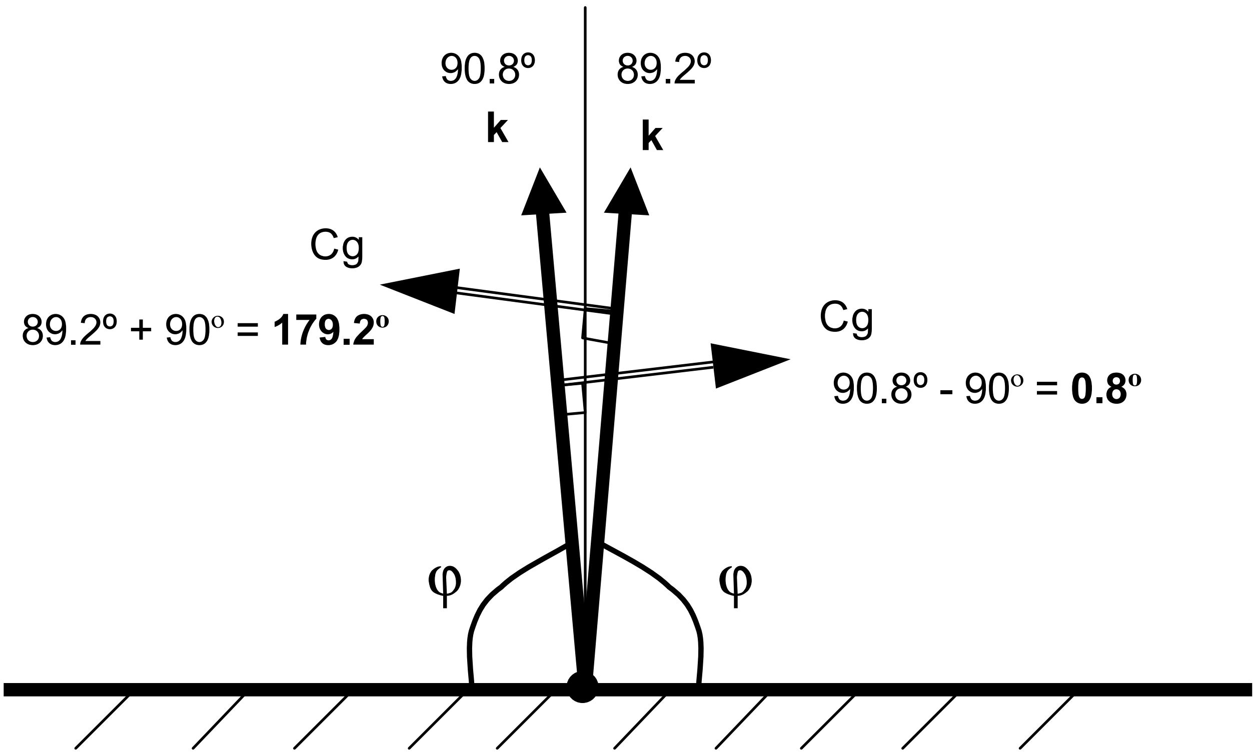
$$\cos^2 \varphi = \frac{\omega^2}{N^2} = \frac{4\pi^2}{N^2 T^2}$$

$$\cos\varphi = \pm\frac{2\pi}{NT} = \pm\frac{2\pi}{0.00976s^{-1} \cdot 644s} = \pm 0.0144$$

$$\varphi = 89.2°, 90.8°$$



Energy, however, is transported in the direction of the group velocity, which is 90° from the angle φ. Thus, energy is transported at angles $180.8°$ and $359.2°$.



d) Consider a storm that generates primarily 20s period waves. Would you expect internal waves to be generated?

For internal wave generation, we need $\omega \le N$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{20s} = 0.314 s^{-1}$$

This is not less than N that we calculated in (a).
No, internal waves are not generated.

e) Now assume that the storm generates 30 min period motion through interactions in the mixed layer. What angle from the horizontal would the energy from the storm propagate?

From part (c) we have that

$$\cos\varphi = \pm\frac{2\pi}{NT} = \pm\frac{2\pi}{0.00976 s^{-1} \cdot 30\,\text{min} \cdot 60 s/\text{min}} = \pm 0.358$$

For our upper rays, this gives

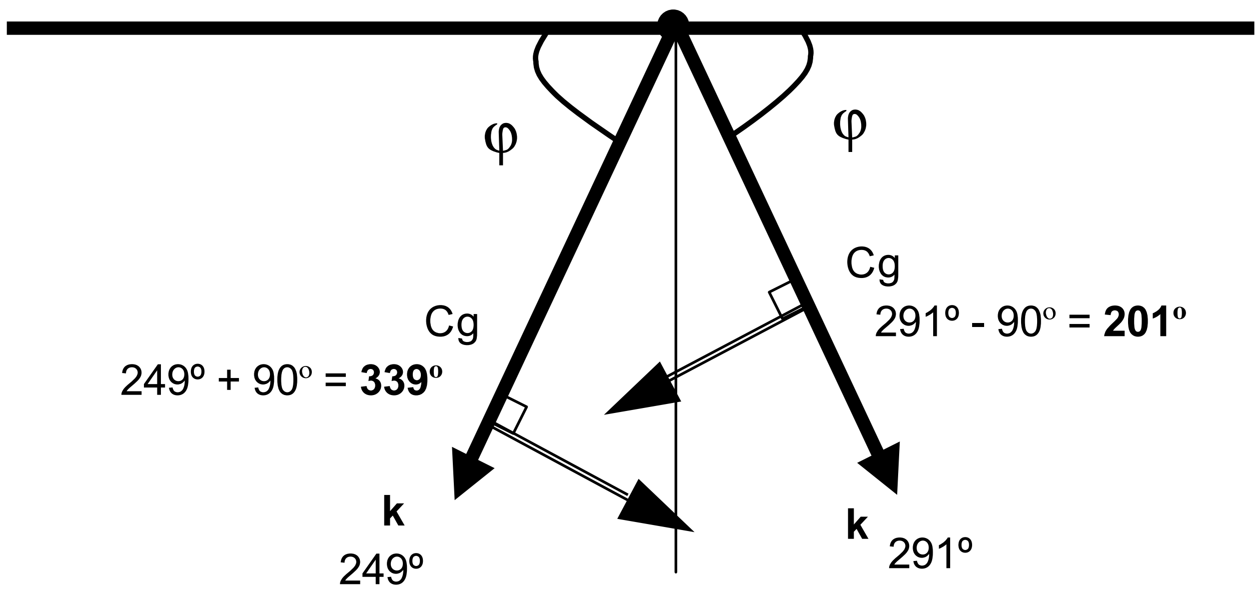
$$\varphi = 69°, 111°$$

Our lower rays are 69º degrees from the vertical, giving us also

$$\varphi = 249°, 291°$$

That these waves are in the mixed layer implies that they are formed at the surface, so we only want to use our lower angles. (Note: If this wave was formed on an internal interface, we would use all four angles!).

Again, energy propagates in the direction of the group velocity, which is 90º from the angle φ. Thus, energy propagates at $21°$ and $159°$.


φ
φ
Cg
291º - 90º = **201º**
Cg
249º + 90º = **339º**
k
249º
k
291º