Ocean 420 Winter 2007
Project 7 Answers

2. Surfs up!!
Your answers will vary depending on the day. So: **be sure to write down the wave height, period and direction of incoming North and South Pacific waves as they are listed on the day you surf this site.** This information appears in a table in the upper right hand corner of the predicted wave height map.

b) If all these waves were generated by a single storm, do you expect the period of the waves to be longer or shorter tomorrow (i.e. 24 hours after you access the site)? Why?

The period will be shorter. In dispersive waves, the longer period waves travel faster than the shorter period waves.

c) Assume that the storm that generated these waves was 5000 km away in the Pacific. How long ago were the waves generated? What would you predict for the wave period tomorrow at the same time? (Choose the North or South Pacific case.)

First, we calculate the wave number and frequency using the period.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{14.4s} = 0.436s^{-1}$$

We use the dispersion relation for deep water waves to calculate wave number.

$$\omega^2 = gk \Rightarrow k = \frac{\omega^2}{g} = \frac{(0.436s^{-1})^2}{9.8m/s^2} = 0.019m^{-1}$$

We have enough information to calculate group velocity.

$$C_g = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}\sqrt{\frac{9.8m/s^2}{0.019m^{-1}}} = 11.36m/s$$

Now we can determine how long the wave took to reach us.

$$D = C_g t \Rightarrow t = \frac{D}{C_g} = \frac{5000km}{11.36m/s} \cdot \frac{1000m}{1km} \cdot \frac{1day}{86400s} = 5.09days$$

We use the same formula, though we add time, to figure out a new group velocity.

$$C_{g,new} = \frac{D}{t} = \frac{5000km}{5.09days + 1day} \cdot \frac{1000m}{1km} \cdot \frac{1day}{86400s} = 9.5m/s$$

Now we go back to our equation for group velocity to get a new wave number.

$$C_{g,new} = \frac{1}{2}\sqrt{\frac{g}{k}} \Rightarrow k = \frac{g}{4C^2_{g,new}} = \frac{9.8m/s^2}{4 \cdot (9.5m/s)^2} = 0.0271m^{-1}$$

To find the new period, we need the new frequency, which we get from the dispersion relation.

$$\omega = \sqrt{gk} \Rightarrow \frac{2\pi}{T} = \sqrt{gk}$$

$$T = \frac{2\pi}{\sqrt{gk}} = \frac{2\pi}{\sqrt{9.8m/s^2 \cdot 0.0271m^{-1}}} = 12.2s$$

3. Waves at the beach.

A storm occurs off of the Washington coast in deep water and it generates 10s waves.

a) What is the energy flux carried by a 1 m length of wave crest if it has amplitude of 50cm?

$$E = \frac{1}{2}\rho g a^2 ,\ E_{flux} = E \cdot C_g \cdot s = \frac{1}{2}\rho g a^2 \cdot C_g s$$

$$\omega^2 = gk \Rightarrow k = \frac{\omega^2}{g} = \frac{(2\pi/10s)^2}{9.8m/s^2} = 0.0403m^{-1}$$

$$C_g = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}\sqrt{\frac{9.8m/s^2}{0.0403m^{-1}}} = 7.8m/s$$

$$E_{flux} = \frac{1}{2}\cdot 1025kg/m^3 \cdot 9.8m/s^s \cdot (0.5m)^2 \cdot 7.8m/s = 9793W$$

b) At what depth will the wave start to behave like a shallow water wave?

Since the energy flux remains constant during the transition to a shallow water wave, we can solve H from this equation.

For a shallow water wave, $C_g = C = \sqrt{gH}$.

$$E_{flux} = EC_g s = \frac{1}{2}\rho g a^2 \cdot \sqrt{gH}$$

We know the energy flux from (a), so we solve for H.

$$H = \frac{4E_{flux}^2}{g\cdot(\rho g a^2)^2} = \frac{4\cdot(9793W)^2}{9.8m/s^2\cdot(1025kg/m^3\cdot 9.8m/s^2\cdot(0.5m)^2)^2} = 6.2m$$

Alternately, we can compute wavelength from the deep water dispersion relation using the rough estimate that $\Lambda > 20H$.

$$\omega = \sqrt{gk} \Rightarrow (\frac{2\pi}{T})^2 = g\frac{2\pi}{\Lambda} \Rightarrow \Lambda = \frac{gT^2}{2\pi} = \frac{9.8m/s\cdot(10s)^2}{2\pi} = 156m$$

$$H < \frac{\Lambda}{20} = \frac{156m}{20} = 7.8m$$

c) The wave approaches shore to a point where the depth is only 2 m. What would its amplitude be there?

We know that the energy flux from deep water to shallow water along a wave ray is the same. From this we can find a ratio of amplitudes that compare the deep water wave to the shallow water wave.

$E_1 C_{g,1} s = E_2 C_{g,2} s$, with $E = \frac{1}{2}\rho g a^2$

$$\frac{E_2}{E_1} = \frac{C_{g,1}}{C_{g,2}} \Rightarrow \frac{a_2^2}{a_1^2} = \frac{C_{g,1}}{C_{g,2}} = \frac{\frac{1}{2}\sqrt{\frac{g}{k_1}}}{\sqrt{gH}}$$

$C_{g,1}$ is the deep water group velocity that we calculated in part (a). H is our 2m depth.

$$\frac{a_2^2}{a_1^2} = \frac{7.8m/s}{\sqrt{9.8m/s^2\cdot 2m}} = 1.76$$

$$a_2 = \sqrt{1.76} \cdot a_1 = 1.33 \cdot 0.5m = 0.665m$$

d) At what depth would you expect the wave to break?

From the notes, we have that a wave will break when the ratio of its amplitude to the depth of the water is about 0.7 or 0.8. We can then calculate the depth it will break, assuming a ratio of 0.7.

$$\frac{a_3}{H} = 0.7 \Rightarrow a_3 = 0.7H$$

We use our relationship between velocity and amplitude from above to solve for H.

$$\frac{a_3^2}{a_1^2} = \frac{C_{g,1}}{C_{g,3}} \Rightarrow \frac{(0.7H)^2}{(0.5m)^2} = \frac{7.8m/s}{\sqrt{gH}}$$

$$0.49\sqrt{g}H^{5/2} = 7.8m/s \cdot 0.25m^2$$

$$H = 1.1m$$

4. Wave spectrum.

Take a look at the following three figures from Knauss, Figures 9.11, 9.12, and 9.13 and answer the following questions

a) We are thinking of going out on Puget Sound for a day on our boat. We know that the wind speed will be about 7 m/s. The wind is coming from the south. I know that I get seasick when we go out on Lake Washington when the wind is that strong. Using figure 9.13, estimate how much higher the waves will be in Puget Sound. Lake Washington is about 20 km long, while Puget Sound is about 100 km long. You may assume that the wind has been blowing for several days.

To answer this question, we look at Figure 9.13 which gives spectral energy for different fetch lengths for a wind of 7m/s. From this, we can find that Lake Washington, with a fetch of 20km, has an energy of ~2.3 (we are ignoring units because we will use a ratio and they won't be important if they are the same). There is no explicit line for a 100km, but careful reading of the figure text tells us that the solid line represents a fully developed sea which will happen above 40km. This gives an energy max of ~7.1 for Puget Sound. To compare amplitudes, we take a ratio of these energies.

$$E = \frac{1}{2}\rho g a^2$$

$$\frac{E_{LW}}{E_{PS}} = \frac{a_{LW}^2}{a_{PS}^2} = \frac{2.3}{7.1} \Rightarrow a_{PS} = 1.76a_{LW}$$

The waves will be 1.76 times stronger in Puget Sound.

b) Once again we are going out on Puget Sound, and we want to study a fully developed sea. Using Figure 9.12, how long should we wait after the beginning of a storm with wind speeds of 20 knots before we go out to study the wave spectrum?

First we need to check that the fetch is long enough to create a fully developed sea. Puget Sound is 100km, which is ~50 nautical miles. Looking on the graph, this is roughly the point that coincides with the 20 knot point. Now that we know we have a long enough fetch, we just read off the graph that it will take ~10 hours for it to develop fully.

c) The next storm is huge and the winds are blowing at 30 knots. Will the sea on Puget Sound be fully developed in this case?

Remembering from (b) that Puget Sound is ~50nm, this is not a long enough fetch for 30 knot winds to fully develop. Looking at the figure, it would need a fetch of ~250nm to develop fully.