Ocean 420 Physical Processes in the Ocean
Project 2: Force Balance and Geostrophy
Answers

3. Geostrophic flow in the Southern Ocean

On the next page is a density section from the southern ocean, with an idealization of the density section on the following page. You may assume that the latitude of the section of 55S. Because the isopycnals never really become flat, the reference level is not necessarily at the bottom.

a) If the reference level is at the bottom (layer 4 has no velocity), what direction is the geostrophic flow?

Out of the page. u is positive.

c) If the reference level is at the bottom, what is the velocity in each layer? Be quantitative. Make a drawing of the velocity profile.

The equations we will use for this are as follows.

$$f(u_1 - u_2) = g_1' i_1$$

$$f(u_2 - u_3) = g_2' i_2$$

$$f(u_3 - u_4) = g_3' i_3$$

with $g_1' = \frac{(\rho_2 - \rho_1)}{\rho_1} g$, and $i = \frac{\Delta z}{\Delta y}$.

We have to solve this set of equations for each of the three sections from left to right. The first and last section (where we have zero slopes) are trivial.

$$i_1 = i_2 = i_3 = 0 .$$

Since $u_4 = 0$, we must also have $u_3 = u_2 = u_1 = 0$ to satisfy the equations.

For the middle section, more work is required. The reference velocity is zero at the bottom, so $u_4 = 0$. This gives us the equation for u_3.

$$u_3 = \frac{g_3' i_3}{f} = \frac{(\rho_4 - \rho_3)}{\rho_3} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y}$$

$$u_3 = \frac{(1027.85 kg/m^3 - 1027.8 kg/m^3)}{1027.8 kg/m^3} \cdot \frac{9.8 m/s^2}{2 \cdot {}^{2\pi}\!/_{86400s} \cdot \sin(-55°)} \cdot \frac{-2500m}{1000000m}$$

$$u_3 = 0.01 m/s .$$

Determining u_2 follows the same pattern.

$$u_2 = \frac{g_2' i_2}{f} + u_3 = \frac{(\rho_3 - \rho_2)}{\rho_2} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y} + u_3$$

$$u_2 = \frac{(1027.8 kg/m^3 - 1027.7 kg/m^3)}{1027.7 kg/m^3} \cdot \frac{9.8 m/s^2}{-1.19 \times 10^{-4} /s} \cdot \frac{-1600m}{1000000m} + 0.01 m/s$$

$$u_2 = 0.013 m/s + 0.01 m/s = 0.023 m/s .$$

And again for u_1.

$$u_1 = \frac{g_1' i_1}{f} + u_2 = \frac{(\rho_2 - \rho_1)}{\rho_1} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y} + u_2$$

$$u_1 = \frac{(1027.7kg/m^3 - 1027.1kg/m^3)}{1027.1kg/m^3} \cdot \frac{9.8m/s^2}{-1.19 \times 10^{-4}/s} \cdot \frac{-1000m}{1000000m} + 0.023m/s$$

$$u_1 = 0.048m/s + 0.023m/s = 0.071m/s .$$

b) If the reference level is at the bottom, draw a quantitative picture of the sea surface height as a function of distance from the beginning of the section. Gives units and sizes.

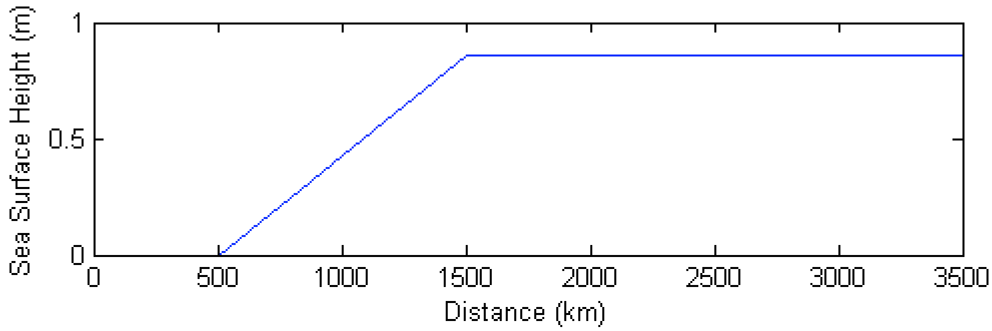
The work done above to find the velocities allows us to find the sea surface height easily. We use the following equation and apply it to the complicated middle section:

$$fu_1 = gi_{surf}$$

$$\Rightarrow i_{surf} = \frac{fu_1}{g} = \frac{-1.19 \times 10^{-4}/s \cdot 0.071m/s}{9.8m/s^2} = -8.62 \times 10^{-7} .$$

$$\Delta z = i_{surf} \cdot \Delta y = -8.62 \times 10^{-7} \cdot -1000000m = 0.862m .$$

Since the surface velocities (u_1) are zero in the other sections, their slope is also zero. We realize that there can't be a step in sea surface height and so connect the sections continuously. The picture looks like this:



d) If the reference level is at the bottom, what is the total transport of the current (units of m^3/s)? (hint to find the area of a trapezoid, go to http://argyll.epsb.ca/jreed/math9/strand3/trapezoid_area_per.htm)

The only sections that can transport water are ones with non-zero velocities, allowing us to neglect the edge sections and focus on the middle. The first step is to find the areas of each of the layers, using the formula to find the area of a trapezoid,

$A_{trapezoid} = \frac{1}{2} \cdot (a+b) \cdot h$, where a and b are the lengths of the parallel sides and h is the perpendicular distance between them.

Layer 1: $A_1 = 0.5 \cdot (0m + 1000.862m) \cdot 1000000m = 5.00 \times 10^8 m^2$

Layer 2: $A_2 = 0.5 \cdot (150m + 750m) \cdot 1000000m = 4.50 \times 10^8 m^2$

Layer 3: $A_3 = 0.5 \cdot (350m + 1250m) \cdot 1000000m = 8.00 \times 10^8 m^2$

Layer 4: $A_4 = 0.5 \cdot (2500m + 1000.m) \cdot 1000000m = 1.75 \times 10^9 m^2$

The transport in each layer is the velocity times the area.

Layer 1: $T_1 = u_1 A_1 = 0.071 m/s \cdot 5.00\times10^8 m^2 = 3.55\times10^7 m^3/s = 35.5Sv$

Layer 2: $T_2 = u_2 A_2 = 0.023 m/s \cdot 4.50\times10^8 m^2 = 1.04\times10^7 m^3/s = 10.4Sv$

Layer 3: $T_3 = u_3 A_3 = 0.01 m/s \cdot 8.00\times10^8 m^2 = 8.0\times10^6 m^3/s = 8.0Sv$

Layer 4: $T_4 = u_4 A_4 = 0.0 m/s \cdot 1.75\times10^9 m^2 = 0 m^3/s = 0Sv$

Add these all together to get the total transport:

$T_{total} = 3.55\times10^7 + 1.04\times10^7 + 8.0\times10^6 = 5.39\times10^7 = 53.9Sv$

e) Now assume that the flow in the bottom layer is 5 cm/s. What is the velocity in each of the other layers?

We follow the same procedure as before, but for the left and right sections, there is flow. However, the same argument applies as before.

$i_1 = i_2 = i_3 = 0$.

Because of this, the velocities must all be the same to satisfy the equations. Since $u_4 = 0.05 m/s$, we must also have $u_3 = u_2 = u_1 = 0.05 m/s$.

For the middle section, the flow becomes 0.05m/s stronger as we will see. The reference velocity is not zero at the bottom, so $u_4 = 0.05 m/s$. This gives us the equation for u_3.

$$u_3 = \frac{g_3' i_3}{f} + u_4 = \frac{(\rho_4 - \rho_3)}{\rho_3} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y} + u_4$$

$$u_3 = \frac{(1027.85 kg/m^3 - 1027.8 kg/m^3)}{1027.8 kg/m^3} \cdot \frac{9.8 m/s^2}{2 \cdot \frac{2\pi}{86400s} \cdot \sin(-55°)} \cdot \frac{-2500m}{1000000m} + 0.05 m/s$$

$u_3 = 0.01 m/s + 0.05 m/s = 0.06 m/s$.

Determining u_2 follows the same pattern.

$$u_2 = \frac{g_2' i_2}{f} + u_3 = \frac{(\rho_3 - \rho_2)}{\rho_2} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y} + u_3$$

$$u_2 = \frac{(1027.8 kg/m^3 - 1027.7 kg/m^3)}{1027.7 kg/m^3} \cdot \frac{9.8 m/s^2}{-1.19\times10^{-4}/s} \cdot \frac{-1600m}{1000000m} + 0.06 m/s$$

$u_2 = 0.013 m/s + 0.06 m/s = 0.073 m/s$.

And again for u_1.

$$u_1 = \frac{g_1' i_1}{f} + u_2 = \frac{(\rho_2 - \rho_1)}{\rho_1} \cdot \frac{g}{f} \cdot \frac{\Delta z}{\Delta y} + u_2$$

$$u_1 = \frac{(1027.7 kg/m^3 - 1027.1 kg/m^3)}{1027.1 kg/m^3} \cdot \frac{9.8 m/s^2}{-1.19\times10^{-4}/s} \cdot \frac{-1000m}{1000000m} + 0.073 m/s$$

$$u_1 = 0.048m/s + 0.023m/s = 0.121m/s\,.$$

f) Draw a quantitative picture of the sea surface height assuming a 5 cm/s flow in the bottom layer.

Because there is flow in the left and right surface sections, the sea surface is not just flat there. We use the same equation as before to determine slope and height.

$$fu_1 = gi_{surf}$$

$$\Delta z = \frac{fu_1}{g} \cdot \Delta y\,.$$

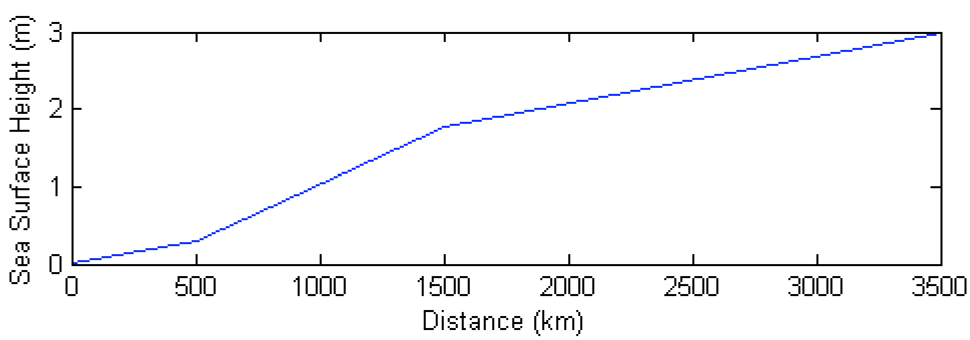
For the left section, $\Delta z = \frac{fu_1}{g} \cdot \Delta y = \frac{-1.19\times10^{-4} \cdot 0.05m/s}{9.8m/s^2} \cdot -500000m = 0.304m\,.$

For the middle section, $\Delta z = \frac{fu_1}{g} \cdot \Delta y = \frac{-1.19\times10^{-4} \cdot 0.121m/s}{9.8m/s^2} \cdot -1000000m = 1.47m\,.$

For the left section, $\Delta z = \frac{fu_1}{g} \cdot \Delta y = \frac{-1.19\times10^{-4} \cdot 0.05m/s}{9.8m/s^2} \cdot -2000000m = 1.21m\,.$

Since these are only the change in height, the absolute height at a given point has to include the previous height in it.

Distance from left	0 km	500 km	1500 km	3500 km
Sea surface height	0 m	0.304 m	1.77 m	2.98 m



g) What is the total transport of the current with flow of 5 cm/s in the bottom layer?

For transport, again we find area and multiply it by velocity.

Velocity:

0.05 m/s	0.121 m/s	0.05 m/s
0.05 m/s	0.073 m/s	0.05 m/s
0.05 m/s	0.06 m/s	0.05 m/s
0.05 m/s	0.05 m/s	0.05 m/s

Area:

0 m^2	5.0 x 10^8 m^2	2.0 x 10^9 m^2
7.5 x 10^7 m^2	4.5 x 10^8 m^2	1.5 x 10^9 m^2
1.75 x 10^8 m^2	8.0 x 10^8 m^2	2.5 x 10^9 m^2
1.25 x 10^9 m^2	1.75 x 10^9 m^2	2.0 x 10^9 m^2

Transport:

0 m^3/s	6.05 x 10^7 m^3/s	1.00 x 10^8 m^3/s
3.75 x 10^6 m^3/s	3.29 x 10^7 m^3/s	7.50 x 10^7 m^3/s
8.75 x 10^6 m^3/s	4.80 x 10^7 m^3/s	1.25 x 10^8 m^3/s
6.25 x 10^7 m^3/s	8.75 x 10^7 m^3/s	1.00 x 10^8 m^3/s

Adding all of these together, we get

$$T_{total} = 7.06\times10^8 m^3/s = 706Sv$$