Ocean 420 Physical Processes in the Ocean
Project 1: Hydrostatic Balance, Advection and Diffusion
Answers

1. Hydrostatic Balance
a) Set all of the levels on one of the columns to the lowest possible density. Click the *Calculate Pressure* button. What is the pressure at the bottom?

   501.7815 db

b) Set all of the densities on one of the columns to the highest possible density and find their pressures. What is the pressure at the bottom?

   504.234 db

   A common approximation in oceanography is that one decibar of pressure is about equal to one meter of water. In this case, how many meters off would you be and in which direction?

   Using this approximation, we think that we are 504.234 meters deep when we are only 500 meters deep. This means we think we are about 4 meters deeper than we actually are.

c) Experiment with the densities and find two different stable profiles that have the same bottom pressure. Record the densities in all levels for both of them.

   There were many correct answers—one is given below. The point of the exercise was to realize that knowing the pressure at a given depth tells you nothing about the stratification.

1023 kg/m^3	1025 kg/m^3
1024 kg/m^3	1025 kg/m^3
1025 kg/m^3	1025 kg/m^3
1026 kg/m^3	1025 kg/m^3
1027 kg/m^3	1025 kg/m^3

d) Given the following two profiles, calculate the pressure gradient between them (at each level) if they are 100 kilometers apart.

   The pressures from the given profile are:

100.3563 db	100.4053 db
200.7617 db	200.9579 db
301.216 db	301.7065 db
401.8176 db	402.5043 db
502.5663 db	503.3511 db

   The horizontal pressure gradient is given by $\dfrac{\Delta p}{\Delta x} = \dfrac{p_2 - p_1}{\Delta x}$. To calculate this based on our data, we subtract the right pressure from the left pressure and divide by 100,000 meters. To put this in standard units, convert db into Pa.

   Layer 1: $\dfrac{100.4053 db - 100.3563 db}{100,000 m} = 4.9*10^{-7}\dfrac{db}{m}*10^4\dfrac{Pa}{db} = 0.0049\dfrac{Pa}{m}$ or $\dfrac{N}{m^3}$

Layer 2: $\dfrac{200.9079db-200.7617db}{100,000m}=1.962*10^{-6}\dfrac{db}{m}*10^4\dfrac{Pa}{db}=0.01962\dfrac{Pa}{m}$ or $\dfrac{N}{m^3}$

Layer 3: $\dfrac{301.7065db-301.216db}{100,000m}=4.905*10^{-6}\dfrac{db}{m}*10^4\dfrac{Pa}{db}=0.04905\dfrac{Pa}{m}$ or $\dfrac{N}{m^3}$

Layer 4: $\dfrac{402.5043db-401.8176db}{100,000m}=6.867*10^{-6}\dfrac{db}{m}*10^4\dfrac{Pa}{db}=0.06867\dfrac{Pa}{m}$ or $\dfrac{N}{m^3}$

Layer 5: $\dfrac{503.3511db-502.5663db}{100,000m}=7.848*10^{-6}\dfrac{db}{m}*10^4\dfrac{Pa}{db}=0.07848\dfrac{Pa}{m}$ or $\dfrac{N}{m^3}$

If you left the pressure gradients in db/km, the answers are 10 times smaller. Note: if you calculated the pressure based on density, the hydrostatic balance must be taken into account. That is, to calculate the pressure correctly, you must add up the different densities multiplied by the depth that they occur. If you have more questions, come by and ask me.

e) Click on the *Perturbation* radio button. This shows the densities as a perturbation from the standard value of 1025 kg/m³. Input the two profiles in d) by subtracting 1025 kg/m³ from each of the values. Now again calculate the pressure gradient at each level assuming 100 kilometer separation. How are these different from the answers to d)?
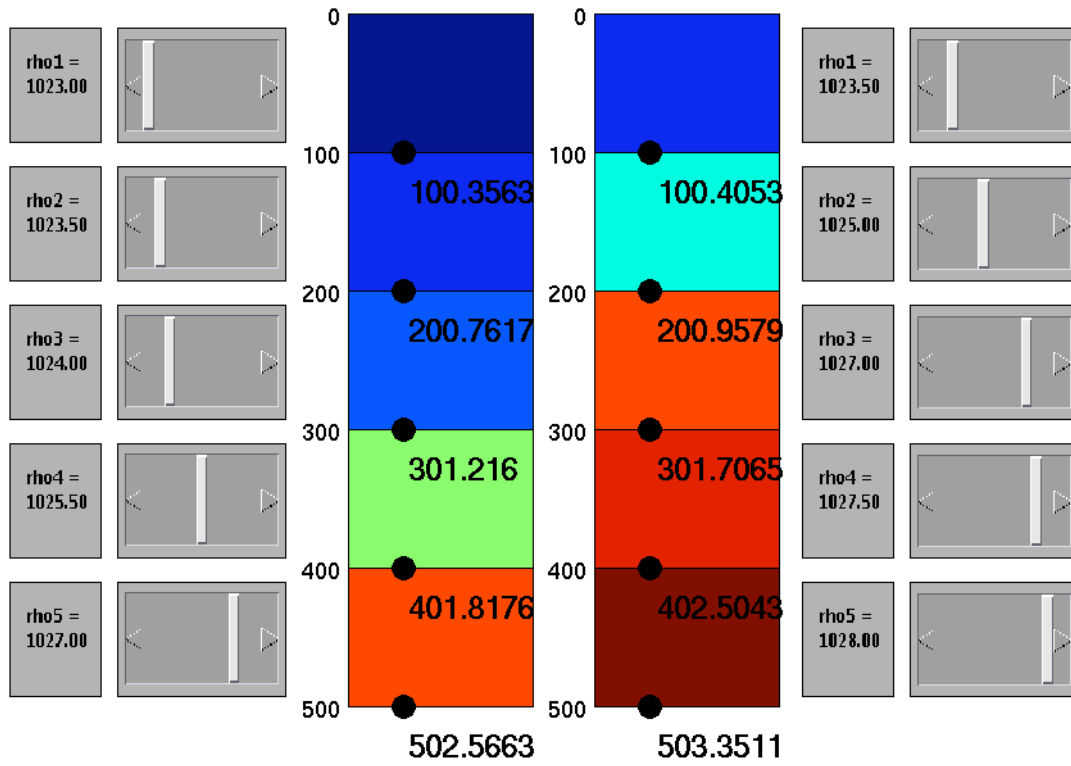
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These are no different than those in part d). If you found differences, it was likely due either to rounding or slight offsets when inputting the densities (i.e. 1023.51 instead of 1023.5).
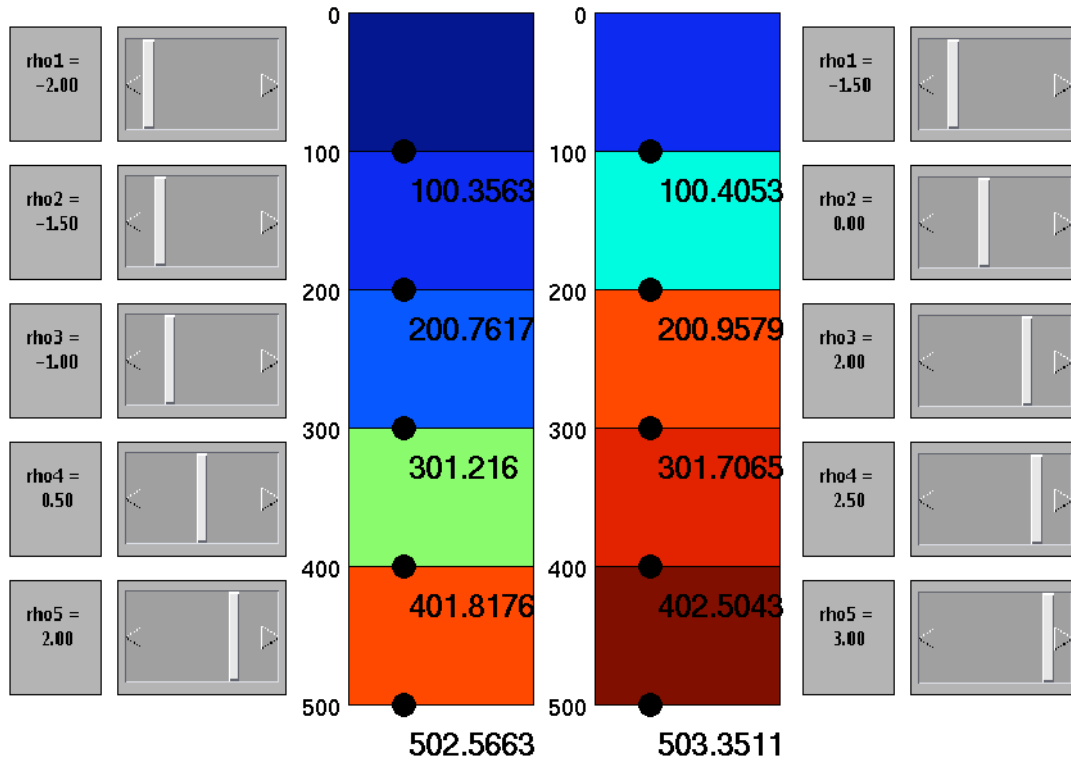
The perturbation does not change the density; it is just a different way of expressing it. When using the hydrostatic balance to compute the pressure, you use the total density which is a sum of the reference density and the perturbation.
The total density is $\rho=\rho_0+\rho'$, with ρ_0 being the reference density of $1025\,kg/m^3$ and ρ' the density perturbation.

Pressures in decibars

rho1 = 1023.00				100.3563		100.4053		rho1 = 1023.50	
rho2 = 1023.50				200.7617		200.9579		rho2 = 1025.00	
rho3 = 1024.00				301.216		301.7065		rho3 = 1027.00	
rho4 = 1025.50				401.8176		402.5043		rho4 = 1027.50	
rho5 = 1027.00				502.5663		503.3511		rho5 = 1028.00	

Pressures in decibars

rho1 = −2.00				100.3563		100.4053		rho1 = −1.50	
rho2 = −1.50				200.7617		200.9579		rho2 = 0.00	
rho3 = −1.00				301.216		301.7065		rho3 = 2.00	
rho4 = 0.50				401.8176		402.5043		rho4 = 2.50	
rho5 = 2.00				502.5663		503.3511		rho5 = 3.00	

f) For the pressure gradients that you calculated in d), calculate the size of the current at each level that would result after one day of the pressure gradient being applied.

The most common mistake in the problem was with the units. Using db/km does not give the current in m/s without converting! The equation we use, in simplified form is

$$\frac{\Delta u}{\Delta t} = -\frac{1}{\rho_0}\frac{\Delta p}{\Delta x}.$$

Solving for Δu, $\Delta u = -\frac{1}{\rho_0}\frac{\Delta p}{\Delta x}\Delta t$. We know the current is in m/s, so let's check that the units work out right.

$$\frac{1}{kg\Big/m^3}\frac{N}{m^3}s = \frac{m^3}{kg}\frac{kg\cdot m}{s^2\cdot m^3}s = \frac{m}{s}.$$

2. Advection

In this exercise we will examine the effects of advection. Go to the *Advection-Diffusion* exercises, and choose *Simple Advection*.

a) Initially set the velocity to be uniform across the channel with speed 2.0. Describe how the movement of the particles evolves. Does the distance between particles change over time?

The particles move uniformly across the page at a constant rate of 2.0. No, the distance between particles does not change.

b) The initial set up has a velocity that is linearly sheared across the channel. Examine the particle movement over time. How does the distance between particles change over time? Does the change in distance depend on location?

The particles stay in a straight line, but this line slopes more and more from the vertical. The distance between particles increases over time, but this does not depend on location, which can be seen in the equal spacing of the final line.
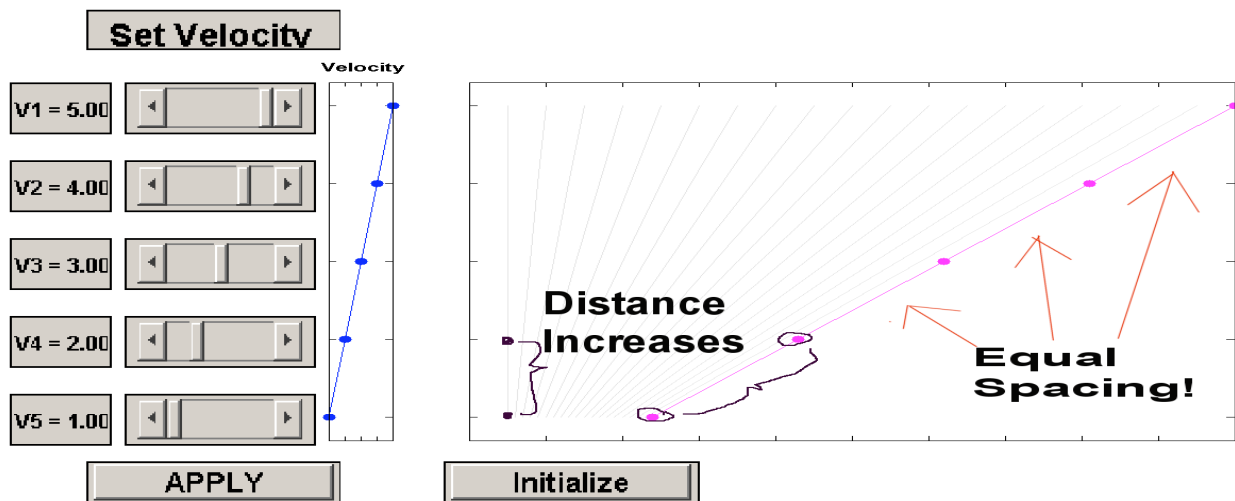
c) Now try a velocity profile with a large velocity in the middle of the channel, and smaller velocities on the edges. What happens to the line of particles over time?

The particles go from being in a straight line to a v-shape on its side. This v gets steeper and steeper as time goes on. The distance between particles increases over time, but whether it depends on location depends on the exact profile.
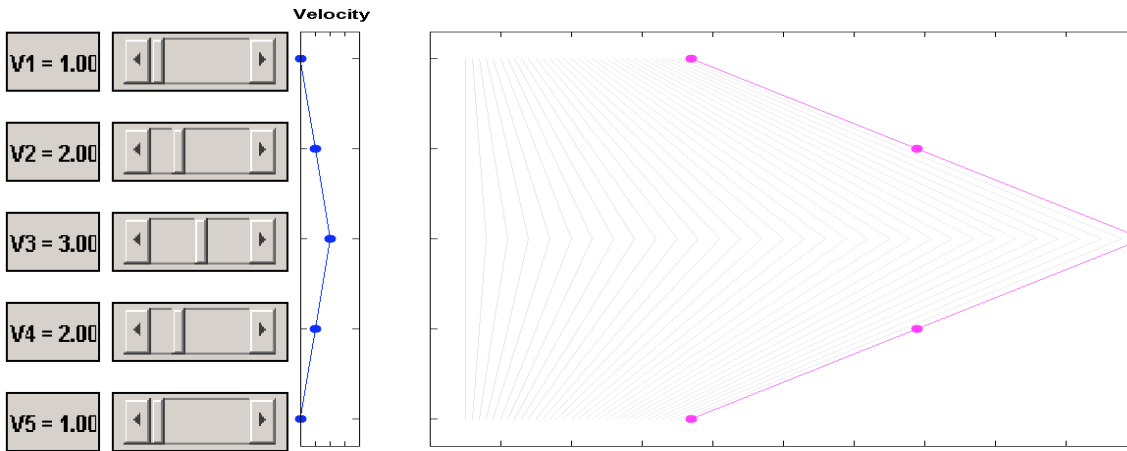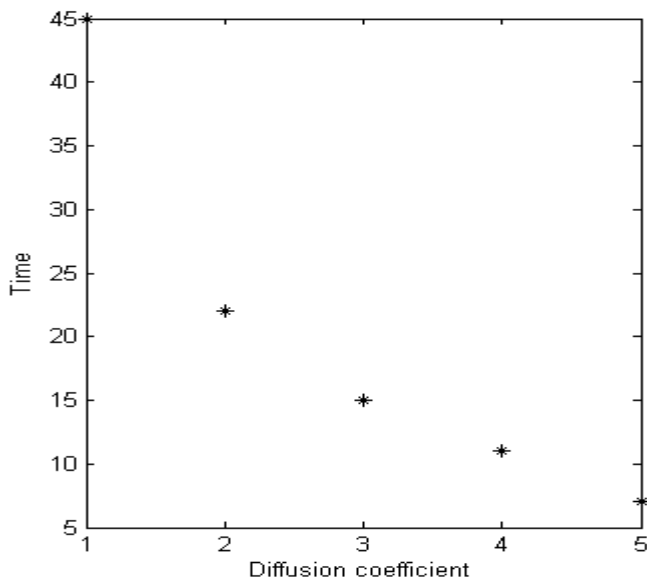

3. Diffusion

Now choose the *Simple Diffusion* exercise. In this case, we have as an initial condition a uniform temperature fluid. At $t=0$, the temperature at the surface immediately rises. Over time, the heat diffuses into the interior.

a) Do five simulations of the temperature field using values of diffusivity from 1 through 5. Record the time when the temperature at depth 6 reaches 1 unit. Graph this as a function of the diffusion coefficient. What is the relationship between time and the diffusivity?

Diffusivity (m²/s)	1	2	3	4	5
Time (s)	45	21	15	11	7

The relationship is roughly exponential decay.

b) Compare the traces for diffusivities 1 and 5. When does the 5-diffusivity run look like the time 50 trace of the 1-diffusivity run?

> 10 seconds.

Use dimensional analysis to explain why this is so. (Remember that the units of the diffusivity are length squared per second).

$$\kappa_1 = \frac{L^2}{T_1}, \ \kappa_2 = \frac{L^2}{T_2}$$ Dimensions of κ.

$$L^2 = \kappa_1 T_1 = \kappa_2 T_2$$ Solving for L2 for both and setting them equal since the length is the same for both (both started and ended in the same trace).

$$T_2 = \frac{\kappa_1}{\kappa_2} T_1$$ Solve for the time it will take for κ_2.

$$10s = \frac{1\,{}^{m^2}\!/_{s}}{5\,{}^{m^2}\!/_{s}} \cdot 50s$$ Plug in numbers to see that it works.

4. Advection and Diffusion

Now choose the *2-D Diffusion-Advection* exercise. In this case, you have three things you can vary: 1) Where the initial patch of tracer is located (in the fast Gulf Stream or in the slow interior flow), 2) whether the diffusivity is small or large and 3) whether you want a simulation with advection only, diffusion only, or advection with diffusion.

a) Examine the simulations with advection only. For which simulation (fast or slow flow) does the tracer spread out more?

> The tracer spreads out more with faster flow.

b) Which velocity profile from part 2 does the tracer most resemble for the fast flow?

> The profile from 2c) most resembles the fast flow motion.

c) Now try with diffusion only in the Gulf Stream and interior regions. Try both large and small values of diffusivity. Does the tracer spread out more or less as compared to the simulations in part (a)?

> The tracer spreads out more, that is, it covers more area as compared to part a).

d) Imagine that an oil spill has occurred in the Gulf Stream and you are consulted to determine whether the spill will reach Newfoundland. You perform simulations with both large and small diffusivities. Under what conditions will the oil spill reach Newfoundland?

> With advection and high diffusivity the oil spill will reach Newfoundland. If you thought the question was asking only about diffusion, the spill never reaches the coast of Newfoundland.

e) In the slow current region, do five simulations, one with advection only, two with diffusion only for both values of diffusion, and two with advection and diffusion with both values of diffusivity. Record the time that the tracer first crosses 30°N. For which simulation does this happen fastest and why?

Run	A	D small	D large	A & D small	A & D large
Time (days)	260	--	60	180	40

This happens fastest for the run with advection and the large diffusion coefficient. Since the current is moving towards 30N, advection pushes the water towards its goal. The large diffusion coefficient spreads the tracer out more, again helping speed the water to 30N. These two together combine to make the tracer reach 30N first.

5. Getting quantitative with advection and conservation

a) We have measured the oxygen concentration in the water on a float that moves with the fluid at 100 m depth. We observe oxygen concentration starts at a values of 4 ml/liter and decreases by over the course of 10 days to 2 ml/liter and that the float moves a distance of 50 km. If we assume that there has been no discernable mixing over the 10 days, what would be the oxygen respiration rate?

Since we are moving with the fluid, the only change in concentration is due to respiration. To calculate the respiration rate (ml/l/day) we use

$$\frac{\Delta C}{\Delta t}, \text{ with C the concentration and t the time.}$$

Thus, we have

$$\frac{\Delta C}{\Delta t} = \frac{C_2 - C_1}{t_2 - t_1} = \frac{2ml/l - 4ml/l}{10days} = -0.2ml/l/day$$

This represents a sink of oxygen. Respiration rate is defined as the positive value, so it is 0.2ml/l/day. Both were accepted as correct.

b) Next we measure oxygen at two fixed moorings that have sensors at 100 m depth. The oxygen levels at the beginning of the experiment are 7 ml/liter at 40N and 140W and 3 ml/liter at 39N and 140W. We also measure a southward current of 5 cm/s. After 10 days, we measure an oxygen concentration of 2 ml/l at the southern mooring. Estimate the respiration rate (ml/l/day) for the southern mooring.

We want to complete this problem in two steps. First, figure out what the concentration will be after 10 days due only to advection. Then, any difference between that and the given value of 2 ml/l must be due to respiration.

First, the advection part. We go about this like the example in class.

$$\frac{\Delta C_{time}}{\Delta t} = -v \cdot \frac{\Delta C_{space}}{\Delta y}, \text{ with C the concentration and v the current.}$$

We want to know the final concentration, so we solve for C_{final},

$$C_{final} = C_{initial} - v \cdot \Delta t \cdot \frac{\Delta C_{space}}{\Delta y}.$$

We plug in our values for these, making sure the units agree: $v = -0.05m/s$, $\Delta y = 111,000m$, and $\Delta t = 864,000s$.

$$C_{final} = 3ml/l + 0.05m/s \cdot 864,000s \cdot \frac{7ml/l - 3ml/l}{111,000m} = 4.56ml/l$$

As above, $\frac{\Delta C}{\Delta t} = \frac{C_2 - C_1}{t_2 - t_1} = \frac{2ml/l - 4.56ml/l}{10days} = -0.256ml/l/day$

Again, respiration rate is the positive of this. If you used ΔC_{time} instead of C_{final}, you still received full credit.