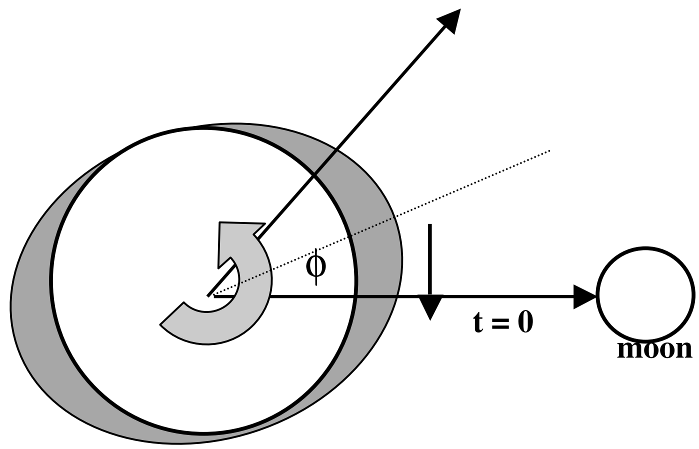
Tide Dynamics

Dynamic Theory of Tides.

In the equilibrium theory of tides, we assumed that the shape of the sea surface was always in equilibrium with the forcing, even though the forcing moves relative to the Earth as the Earth rotates underneath it. From this Earth-centric reference frame, in order for the sea surface to "keep up" with the forcing, the sea level bulges need to move laterally through the ocean. The signal propagates as a surface gravity wave (influenced by rotation) and the speed of that propagation is limited by the *shallow water wave speed*, $C = \sqrt{gH}$, which at the equator is only about half the speed at which the forcing moves. In other words, if the system were in equilibrium at a time t = 0, then by the time the Earth had rotated through an angle ϕ, the bulge would lag the equilibrium position by an angle $\phi/2$.



Laplace first rearranged the rotating shallow water equations into the system that underlies the tides, now known as the ***Laplace tidal equations***. The horizontal forces are:

acceleration + Coriolis force = pressure gradient force + tractive force.

As we discussed, the tide producing forces are a tiny fraction of the total magnitude of gravity, and so the vertical balance (for the long wavelength appropriate to tidal forcing) remains *hydrostatic*. Therefore, the relevant force, the *tractive* force, is the projection of the tide producing force onto the local *horizontal* direction. The equations are the same as those that govern rotating surface gravity waves and Kelvin waves. The tide is forced by the sum of the pressure gradient force (due to the bulging sea surface) and the tractive force, and propagates like a Kelvin wave.

Tides can be predicted using a numerical ocean tidal model that incorporates these equations. Observational data in the form of sea level at the coast and altimeter data in the open ocean are often assimilated into the numerical model. The model results, such as the one at Oregon State University that you will look at in your problem set, are presented in the form of maps of phase and amplitude for each tidal component. Tidal components are added together to form the total tidal sea level variation.

Tidal parameters

For each tidal constituent (or component), several quantities are defined:

- **Co-range lines** link places having the same tidal range (amplitude).
- **Co-tidal lines** link all points having the same phase. Numbers are hours of lag of high tide after the moon's transit over the Greenwich meridian (0°) or phase of the tide relative to Greenwich (e.g., a phase of 0° has high tide at the same time as the moon is passing over Greenwich, 180° has low tide at this time)
- **Amphidromic point**s are points where the tidal range is zero. Wave crests move around the amphidrome once per tidal cycle. Near the coast, progression around amphidromes tends to be cyclonic (i.e., in the sense of a Kelvin wave) keeping the coast on the right (N Hem).

Shown below are the co-range and co-tidal lines for the M2 tide. Note: "cophase" here is the same as "co-tidal" above. Note the amphidrome in the center of the North Atlantic and several amphidromes in the Pacific. Note the southward propagation along the west coast of South America and the northward propagation along the west coast of North America and also Africa.

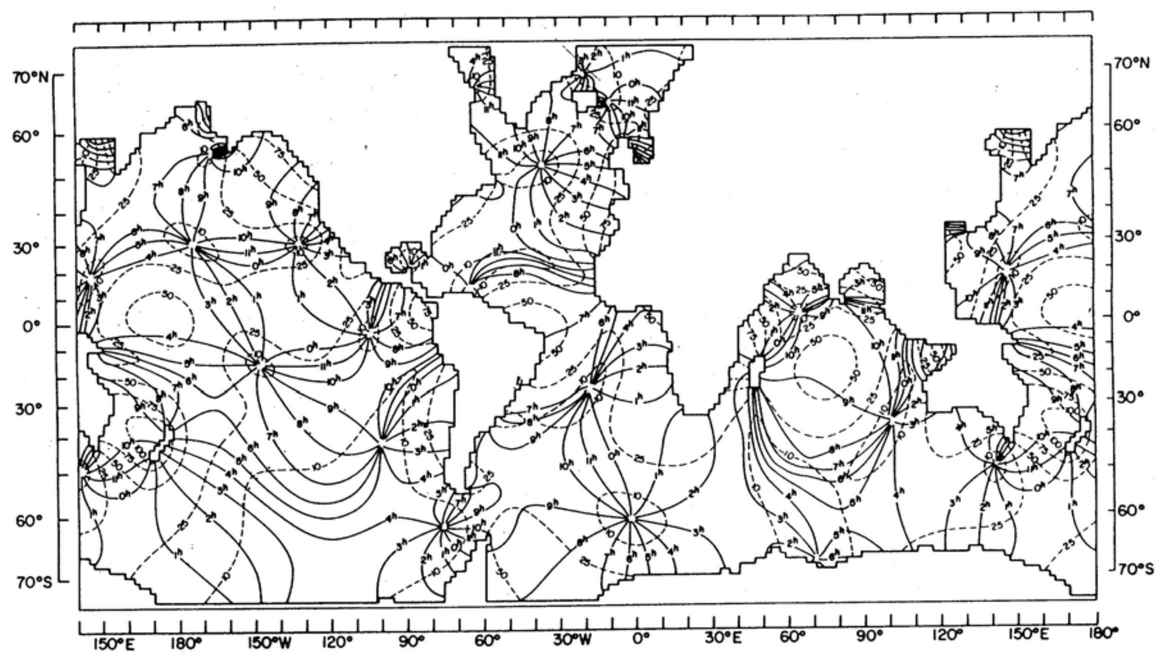


FIG. 13.8 *Global M_2 tide calculated from tidal potential including effects of self-attraction and of tidal loading, 2° grid. Full lines = cophase lines (Greenwich hours), dashed lines = corange lines (cm). (From Accad and Pekeris, 1978.)*

Tides in bays and channels

The tides in the open ocean and along boundaries tend to take the form of a progressive Poincaré or Kelvin wave. For example, within the English Channel, we see propagation from the North Atlantic eastward into the channel, as indicated by the co-tidal lines (solid). The time of the high tide increases going eastward into the Channel. The tidal range (indicated by the dashed lines) is generally higher on the right hand side of the channel owing to the Kelvin wave character of the tide.

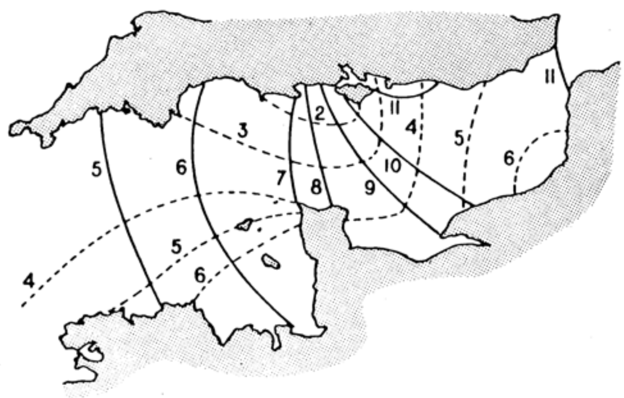
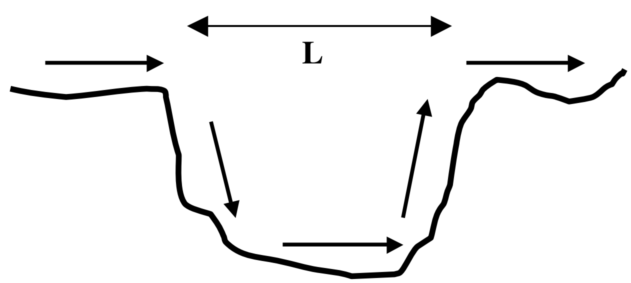


Fig. 10.5. Co-tidal lines (solid) with time in lunar hours, and co-range lines (dotted with values in meters) for the English Channel. [From Proudman (1953, p. 262); after Doodson and Corkan (1931).]

The response of a bay or inlet depends on its width L, compared with the wavelength of the tidal response.

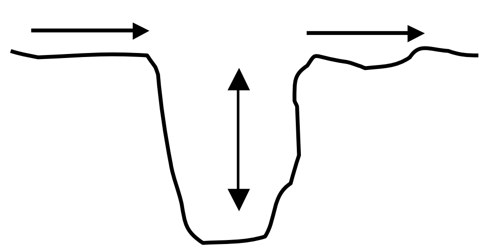
Wide bays. Tides will propagate around the boundaries of a ***wide bay.*** How wide does the bay need to be to have tidal propagation around it? Kelvin waves have an offshore scale given by the barotropic Rossby radius,

$$L_R = \sqrt{gH}/f .$$



The bay must be wide enough ($L > 2 L_R$), relative to its depth H, so that the incoming and outgoing waves do not significantly overlap and interfere.

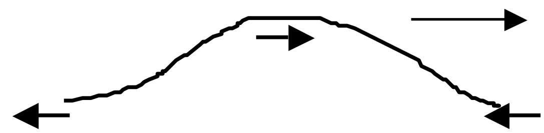
Narrow Bays. If the bay is narrow the wave entering from the open ocean will be *reflected* to set up a standing wave. At this extreme, when the width of the bay is small with respect to Rossby radius of deformation, *rotation is unimportant* to the dynamics. As the tide passes the opening of the inlet, it uniformly raises and lowers sea level there (because the length scale of the open ocean tide is much greater than the width of the opening.
As the wave propagates into the inlet, it encounters reflections off the landward end from previous



wave cycles, and interference occurs, leading to the possibility of standing waves.

Narrow inlets (or fjords) and narrow channels allow the possibility of standing waves. Each fjord or harbor has a set of natural frequencies. (The physics of this problem were originally studied in Lake Geneva, a closed basin, where the natural frequency of the basin is called the ***seiche mode***.) To understand these dynamics we first examine the structure of tidal currents.

Tidal currents. Propagating tides have maximum current speeds (in opposite directions) at high and low tide, i.e., at the crest and trough of the Kelvin wave that is carrying the signal. Vertical velocity is always small (shallow water waves).



When the propagating tide meets a reflection there is the possibility of a ***standing wave***. Standing waves are composed of two waves of the same amplitude and wavelength travelling with the same speed in opposite directions.

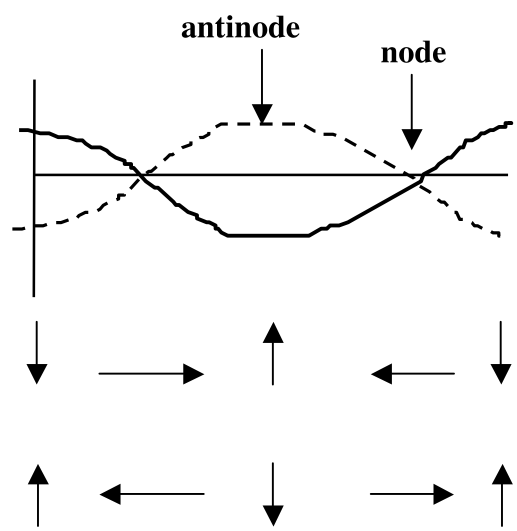
$$\eta(x,t) = a\cos(kx - \omega t) + a\cos(kx + \omega t) = 2a\cos(kx)\cos(\omega t)$$

Within the standing wave, there are two extreme conditions:

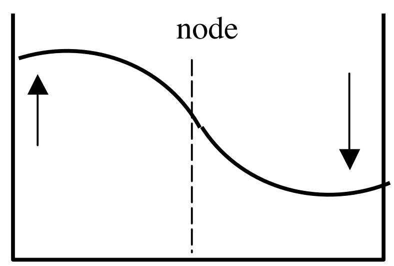
1: An ***antinode*** is the location where the standing wave has maximum amplitude (constructive interference of the incoming and reflected waves).

2: A ***node*** is the location where the standing wave has zero amplitude (destructive interference).

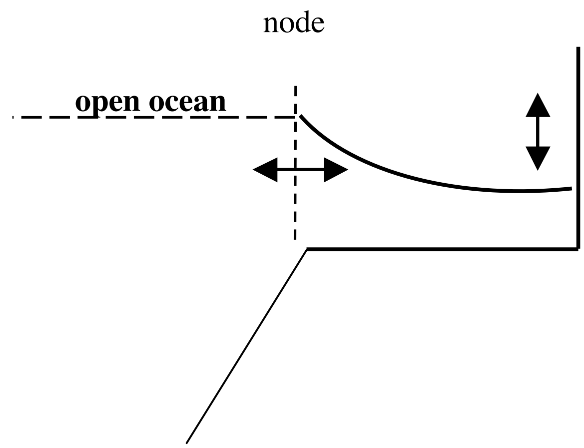
Tidal currents are different for standing waves compared with progressive waves. Maximum *vertical* velocities occur at the antinodes, while velocities at the nodes are *horizontal*.



The location of nodes and antinodes is determined by the *geometry* of the basin. In a closed harbor (or your bathtub!) there are solid sidewalls where the velocity can only be vertical. So there are antinodes at the solid walls and a node in the center – this is the basic structure of the bathtub seiche. It is also the structure of the standing wave in a channel (viewed in cross-section)



Harbor seiches. An open harbor or inlet, on the other hand, has one end open to the ocean, rather than two solid walls. An easy way to picture the boundary conditions on the open harbor is to think of it as *half a bathtub*. Like the closed basin, there is an antinode at the inner (land) end of the inlet and a node where the harbor meets the open ocean (like the middle of the bathtub. The largest vertical velocity (the largest amplitude of the seiche) is at the land or closed end. At the open end, the sea level needs to match the open ocean and its tidal modulation.



Maximum horizontal velocities are found at the nodes, here the opening of the inlet. Temporally, maximum tidal currents are found in-between the extremes of sea level, as water rushes in and out of the fjord to fill or empty the *tidal prism*. The ***average tidal prism*** is defined as the volume difference between mean high water and mean low water. The period when sea level is rising (filling the tidal prism) is the ***flood tide*** and the period when sea level is falling is the ***ebb tide***.

Resonance.

Examining the geometry of the closed harbor, one can see that the simplest standing wave structure that can have antinodes at each wall is exactly *one-half wavelength* long, that is, L = Λ/2, where L is the distance between the walls. Similarly, one can see that the simplest wave to fit the open harbor is *one-quarter wavelength* long, L = Λ/4, where L is the length of the harbor.

Combining these special spatial scales with the phase speed of the shallow water wave, $C = \sqrt{gH}$ = Λ/T, one can determine the *period* for this wave, $T = \frac{2L}{\sqrt{gH}}$

Actually, there can be more than one node in the harbor, so the ***natural periods*** for n nodes are

$$T = \frac{2L}{n\sqrt{gH}}$$, n=1,3,5,…, for a closed harbor

For the open harbor with n nodes

$$T = \frac{4L}{n\sqrt{gH}}$$ n=1,3,5,…, for an open harbor

Resonance (extremely large amplitudes) occurs when the harbor is forced at these special periods.

In-class exercise

Consider a harbor 40m deep and 220 km long . The natural period of oscillation for the bay will be given by

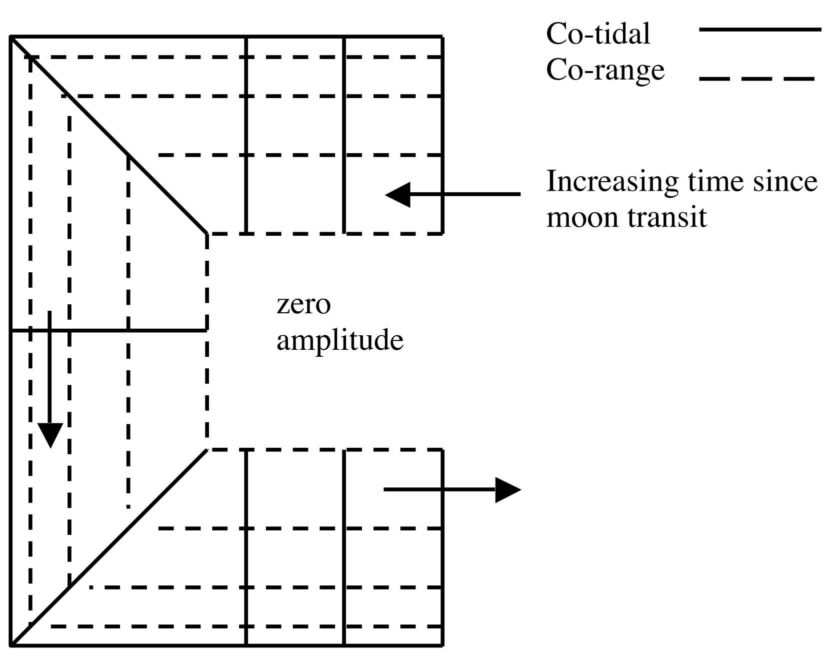
$$T = \frac{4L}{\sqrt{gH}} = \frac{4 * 220000m}{\sqrt{9.8 * 40}} = 44400s == 12.35hr$$

The natural period of oscillation is very close to the period of the M2 tide. This suggests that the bay will be very near resonance, and the tide will be quite high at the head of the bay.

L

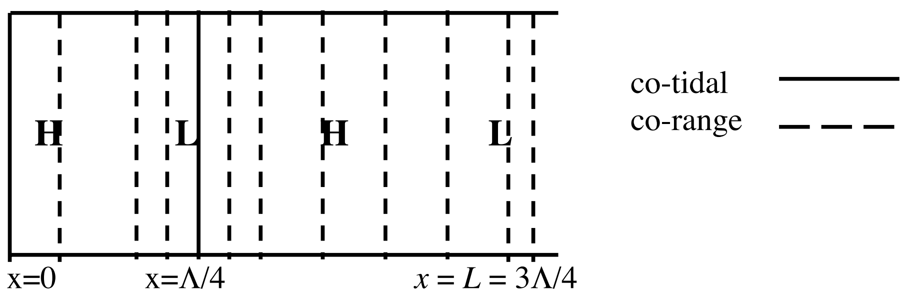
Mixed geometries

Real bays and inlets will have something of the character of both of these idealized extremes, the open and the closed harbors. Imagine the situation for a tide propagating into a **wide** bay, wide enough so that the tide propagates around the edges as a Kelvin wave (don't worry about the corners too much!). Let's sketch the **co-tidal** and **co-range** lines. (northern hemisphere)



The co-tidal lines show the progression of the Kelvin wave around the bay keeping the coastline on the right (N Hem). The co-range lines show the exponential offshore decay of the Kelvin wave over the Rossby radius, $L_R = \frac{\sqrt{gH}}{f}$. To determine if the bay will show rotational effects and the decay of the tide away from the coast, we calculate the deformation radius relative to the wide of the bay. For instance, if the bay were 40 m and we are at 45N, then the deformation radius would be about 400 km. Thus, the bay would have to be wider than that to see the full effects of rotation.

Now imagine that the bay is very narrow and that the M_2 tide sets up a resonant standing wave for which rotation is unimportant. There is a node at $x = \Lambda/4$. Let's sketch the **co-tidal** and **co-range** lines.



Everything to the left of $x = \Lambda/4$ experiences high tide at the same time. Everything to the right experiences high tide 6.125 hours (6 lunar hours) later. So there is one co-tidal line at the node to separate these two regions. The maximum range is seen at the landward end of the channel (at x=0) and halfway between the nodes at $x = \Lambda/2$. Minimum range is near the nodes.

If the inlet width is *intermediate,* the tide will have the character of both cases. On the boundaries, the tide will most strongly resemble the Kelvin wave case. The incoming wave from the ocean will tend to be amplified towards the coastline on the right (N Hem), and the outgoing wave will also be amplified on the right (the opposite coastline). The center axis of the channel will most strongly resemble the standing wave case, with low amplitude tide at the node and high amplitude tide on either side with an out of phase relationship. The nodal line, which stretches across the channel when rotation is not important will turn into a nodal point in the center of the channel around which the tide progresses (renamed an **amphidrome**).

Tides in the Adriatic show an example of this effect.

10.2 Seiches and Tides in Narrow Channels and Gulfs **375**

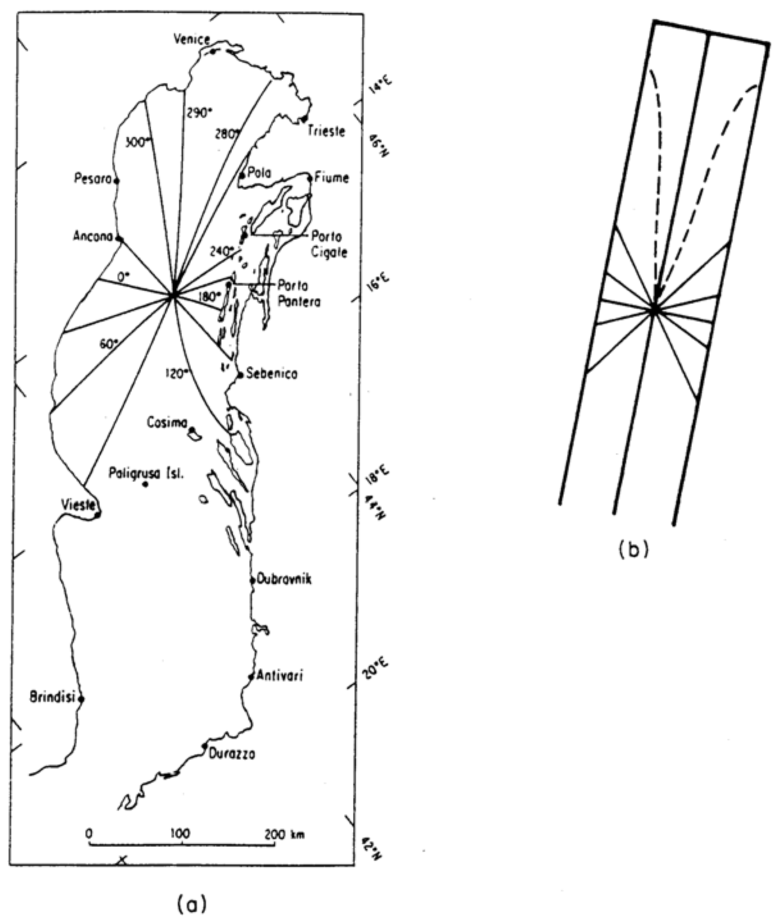


Fig. 10.1. (a) Co-tidal lines for the northern Adriatic. [After Polli (1960); from Hendershott and Speranza (1971, Fig. 7).] (b) Co-tidal lines for a simple model with depth increasing quadratically with distance from the end. The phase difference between the solid lines is 30°. The phase on the broken lines differs by 10° from that on the axis.