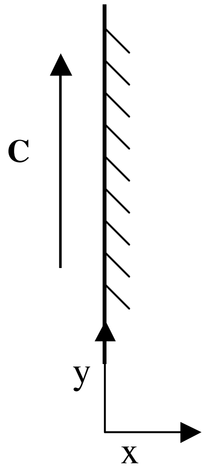
Kelvin Waves

Next we will look at ***Kelvin waves***, waves for which *rotation* is important and that depend on the existence of a *boundary*. They are important in tidal wave propagation along boundaries, in wind-driven variability in the coastal ocean, and in El Niño. The mathematical derivation for Kelvin waves can be found in Knauss. Consider the eastern boundary of a sea. The basic dynamics of Kelvin waves the same as for Poincare waves

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$



For the Kelvin wave, we need a solution such that the velocity into the wall is identically zero everywhere

$$u = 0$$

which reduces the force balance to

$$-fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}$$

Notice that the cross-shore (x) momentum balance is *geostrophic,* while the along shore momentum balance (y) is the same as that for shallow water gravity waves. The solution for the sea surface height becomes

$$\eta = \eta_o \exp(x/L_D)\cos(ky - \omega t)$$

Look at figure 10.4 in Knauss for the structure of the Kelvin wave. The wave propagates along the boundary (*poleward* along an eastern boundary, *equatorward* along a western boundary) with its maximum amplitude at the boundary.

The wave amplitude decays offshore with a scale equal to the deformation radius $L_D = \frac{\sqrt{gH}}{f}$. The wave is *trapped* to the boundary.

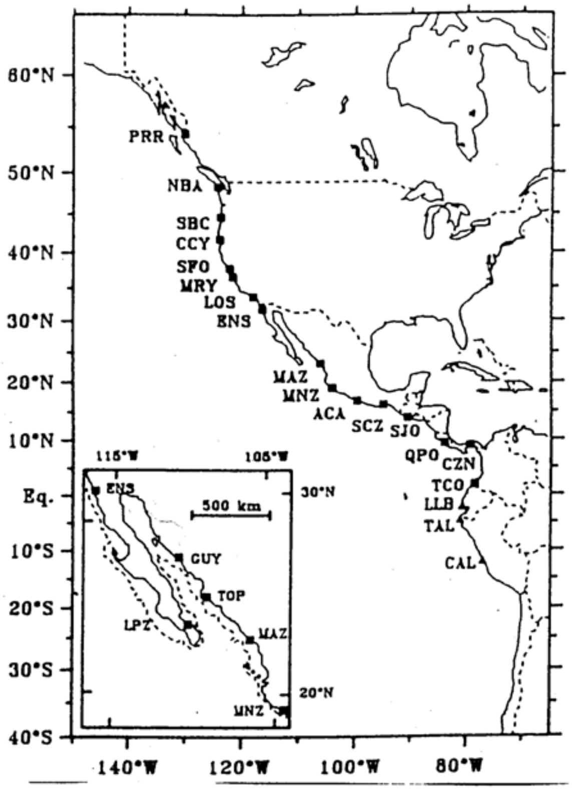
The *dispersion relation* is

$$\omega = k\sqrt{gH}$$

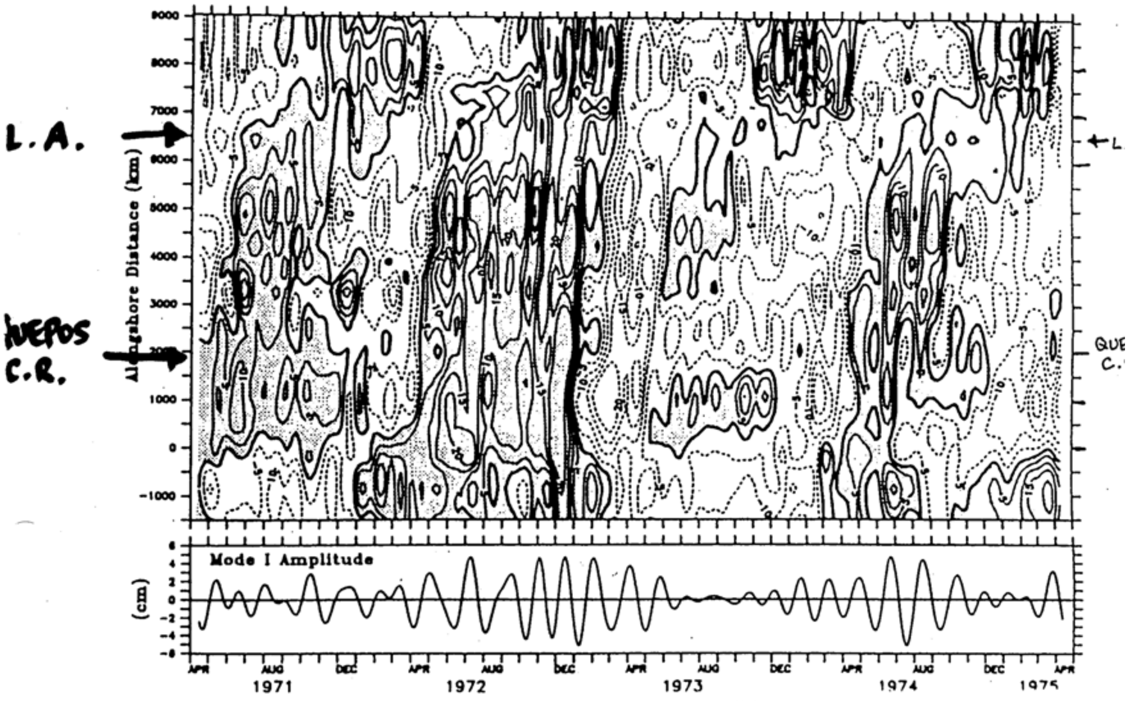
the same as for the shallow water wave. The wave is *non-dispersive* and travels at the shallow water surface gravity wave speed. There are also *internal* Kelvin waves that travel at the internal gravity wave speed and whose decay scale offshore is the *internal* deformation radius.

For examples of Kelvin waves, look at the animation for the M2 tide on *http://www.oce.orst.edu/po/research/tide/*. You can see propagation of the tide around each basin in the Kelvin wave sense.

For a coastal example, consider long period motions along the coast (50 days or more). Sea level pressure shows the propagation of an internal Kelvin wave up the coast. To follow the Kelvin wave, shown is a longitude/time plot. Notice the negative anomaly that starts towards the end of 1972 and propagates up the coast over the next month or so. The internal gravity wave speed is about 1m/s. Then the wave will travel about 5000km in about 2 months.



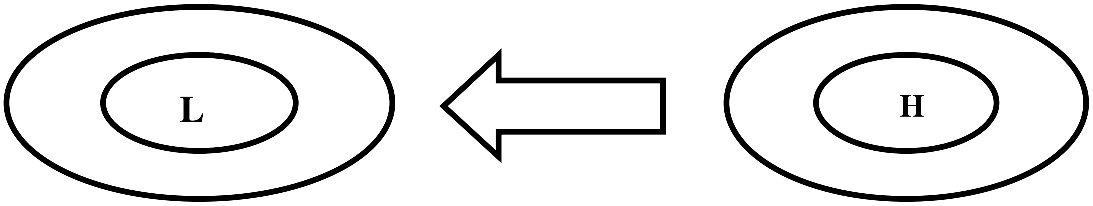
Spillane et al (1987) J. Phys. Ocean.



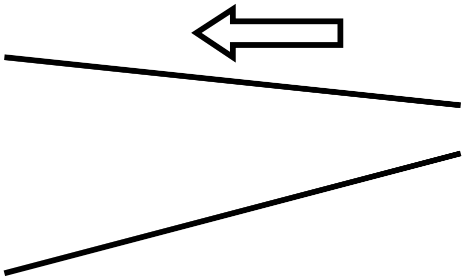
El Niño and Kelvin waves

The El Niño/Southern Oscillation (ENSO) is a coupled ocean/atmosphere phenomenon that causes global deviations of the climate on 2 to 5 year time scales. The El Niño part refers to variations in the ocean circulation, originally observed off the coast of South America in wintertime. The Southern Oscillation Index is the sea level pressure in Tahiti (in the Central Pacific) minus the sea level pressure in Darwin (in Northern Australia), a measure of large-scale atmospheric changes in the Southern Hemisphere. The mean atmospheric circulation has a low-pressure center in the west, and a high in the east, giving a positive mean difference. This implies that there are winds from the east to the west (*easterlies*). If this pressure difference weakens the SOI index will be negative. Likewise, if the SOI is positive, then the pressure difference increases.

"Normal conditions":

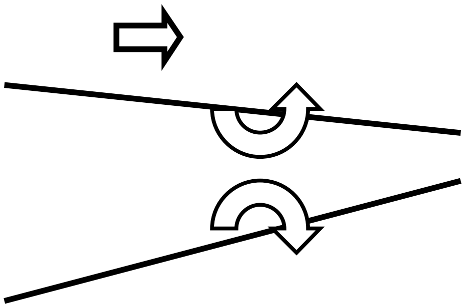


The winds pile up water in the western Pacific (at the equator the Coriolis force is small and water and air move down the pressure gradient). The thermocline mirrors the sea surface giving a deep upper layer in the west and a shallow layer in the east, with upwelling of cold water. For easterlies that straddle the equator, northward Ekman transport on the northern side and southward transport on the southern side cause a divergence and a strong equatorial upwelling circulation.



"El Niño": the warm phase of ENSO

If the east-west pressure gradient driving the winds weakens, the winds on the equator also weaken. In addition, the region of convection moves to the east, resulting in relative droughts over Indonesia, and more rain in the east. The tropical upwelling in the eastern Pacific shuts down as the winds are not as strongly westward, which results in a dramatic drop in productivity in the eastern Pacific with drastic consequences for the fisheries off of Peru. In addition, the SST in the eastern Pacific rises dramatically, by as much as 4 to 8 degrees C.



What happens to the thermocline and the sea surface? With weak easterlies or westerlies, the sea surface and the thermocline tend to *flatten.* This happens as a Kelvin wave, rather than as a gradual process. These equatorially trapped Kelvin waves, following a period of westerly winds, often signal the start of an El Niño. The westerly winds cause anomalous *downwelling* along the equator that then propagates to the east. The source of the westerlies can be tropical cyclones, something called the Madden-Julian Oscillation, or outbreaks from the subtropics. The ***Madden-Julian Oscillation*** is probably the most common source of westerly wind events. It is a cyclical weather pattern with period of 30-60 days and is characterized by a weak low pressure center that develops in the east Indian Ocean and then propagates eastward along the equator. These oscillations reach their peak amplitude during the transition from winter to summer and summer to winter.

Now back to Kelvin waves. As we described them earlier in class, they are trapped to a boundary (in this case the "boundary" between positive and negative Coriolis parameter, the equator) and propagate to the east to flatten the thermocline. This results in a deeper thermocline and warmer surface temperatures in the east. As the sea surface temperatures increase in the central Pacific, the region of rising motion associated with deep convection also moves to the east, consistent with the negative anomaly in the SOI and the shifting of the Walker circulation to the east. As more westerly wind events occur, the thermocline flattens further still.

The Kelvin wave has a Gaussian shape, symmetric about the equator

$$\eta(x,t) = \exp(-y^2 / L_D^2)\cos(kx - \omega t)$$

It propagates eastward with the shallow water gravity wave speed. The decay length scale is the ***internal equatorial deformation radius***

$$L_R{}^2 = \frac{\sqrt{g' H_1}}{2\beta}$$

where

$$\beta = \frac{2\Omega\cos\theta}{R_e} = \frac{df}{dy}$$

and *Re* is the radius of the Earth. So at the equator, we find that $\beta = 2.3 \times 10^{-11} / m / s$. If g'=0.02, and $H_1 = 200m$, then the internal equatorial deformation radius will be about 200 km. The internal Kelvin was would travel at speed 2 m/s.