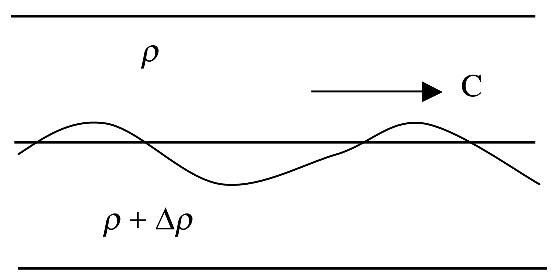
I. Two-layer internal waves

The simplest internal waves are like those in the clear acrylic boxes you can buy at the store. You can make for yourself with a bottle half full of water and half full of oil. There are two immiscible fluids, usually of different colors and you can set up an internal wave along the interface by tilting the box back and forth. The ocean analogy is a wave much like the surface gravity waves we have studied, but which acts on the *thermocline*. In reality, the physics is identical -- the wind waves we have studied can be viewed as internal waves on the interface between two largely immiscible fluids (air and water) with a very large density contrast.



Imagine raising a parcel of water across the interface between the layers. The restoring force that will cause the parcel to return to its equilibrium position (actually, overshoot its equilibrium position) is equal to gravity times the difference in density between the water parcel and its surroundings. This quantity,

$$B = -g\rho' = -g\Delta\rho$$

is called the *buoyancy force*. The buoyancy force is positive, $B > 0$, if ρ' (the density of a water parcel minus the density of its surroundings) is negative, resulting in an upward force. In general, the *frequency of a wave increases with strength of the restoring force*:

$$\omega \propto \Delta\rho$$

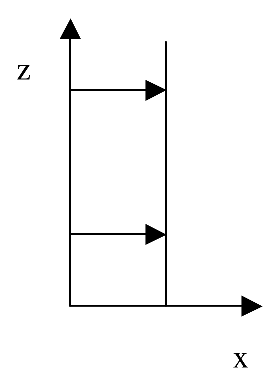
Thus, surface waves on the thermocline will have longer characteristic periods (typically hours) than surface waves on the air-sea interface. For gravity waves, the wave speed, as well as the frequency, is proportional to the buoyant restoring force.

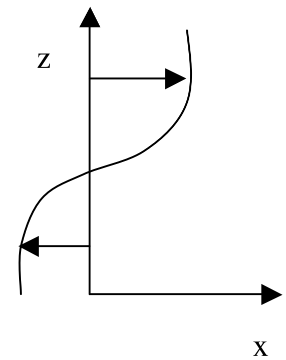
Thus, in a two-layer fluid there are two modes of variability.

a) the *barotropic* (or external) mode which is just like the surface gravity wave mode that we have been discussing. In the shallow water limit, the phase speed is

$$C = \sqrt{gH}$$

The surface displacement is comparable to the interface displacment $\frac{\eta}{h} \approx 1$, and the velocities are the same in both of the layers $\frac{u_1}{u_2} \approx 1$.





b) The *baroclinic* (or internal) mode. In this case, the phase velocity is given by

$$C = \sqrt{g' \frac{H_1 H_2}{H_1 + H_2}} \quad (16.1)$$

where $g' = g \Delta\rho / \rho$ is the reduced gravity. In this case, the surface displacement is much smaller than the interface displacement where

$$\frac{\eta}{h} \approx -\frac{g'}{g}$$

The fluid parcel velocity is opposite in the two layers

$$\frac{u_1}{u_2} \approx -\frac{H_2}{H_1}$$

1. Let's think of the coastal ocean as a two layer fluid with upper layer thickness 100 m and lower layer thickness 500m. The density in the surface layer is 1024 kg/m3 and in the lower layer is 1026 kg/m3. How fast would a barotropic internal gravity wave propagate? How fast would a baroclinic internal gravity wave propagate?

The barotropic wave would travel at $C = \sqrt{gH} = \sqrt{9.8 * 600} = 77 m/s$

The baroclinic wave would travel $C = \sqrt{g' \frac{H_1 H_2}{H_1 + H_2}} = \sqrt{\frac{9.8 * 2}{1025} \frac{100 * 500}{600}} = 1.3 m/s$

2. For the situation in (1) if the maximum flow in the upper layer 20 cm/s, what would the maximum flow be in the lower layer, associated with the wave?

$$u_2 = \frac{u_1 H_1}{H_2} = 0.03 m/s$$

Waves on the thermocline. The thermocline usually separates a relatively shallow upper layer from a deep lower layer, $\frac{H_1}{H_2} << 1$, which for the internal mode (16.1) gives the approximation

$$C = \sqrt{g' H_1}$$

where H_1 is the depth of the thermocline. Across the ocean thermocline, typically, $\Delta\rho/\rho_0$ = 1% at most, so $g' \sim 1\%$ of g. Therefore, the phase speed of an internal wave on the thermocline is at least 100 times slower than the speed of a wave on the sea surface. Likewise, the displacement at the interface is much larger than the sea surface displacement.

Waves on the ocean surface are also waves on an interface, that between the ocean and the air. In this case

$$g' = \frac{\rho_{OCEAN} - \rho_{ATM}}{\rho_{OCEAN}} g \cong g$$
$$C = \sqrt{gH}$$

because the ocean density is so much larger than the air density.

6.2 The Case of Two Superposed Fluids of Different Density **125**

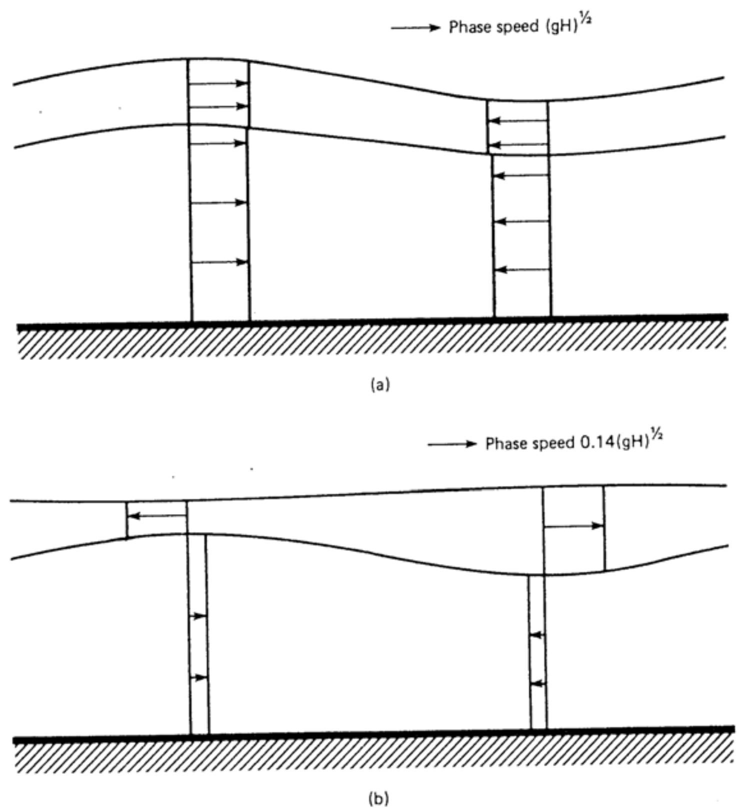


Fig. 6.3. Layer configuration in a two-layer system for a barotropic wave (a) and a baroclinic wave (b) propagating from left to right. For the case shown, the lower layer is three times deeper than the upper layer and has density 10% greater. Also shown are the directions of flow at troughs and crests, and the relative velocities of the two layers at these points.