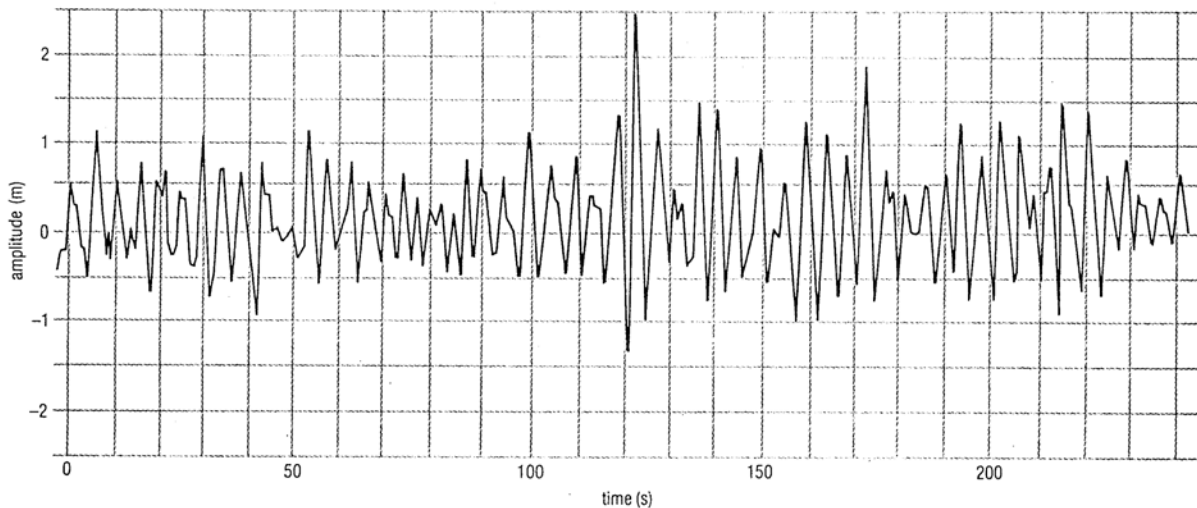


Surface Gravity Wave Generation

A typical wave record is shown below

Figure 1.5 A typical wave record, i.e. a record of variation in water level with time at one position.

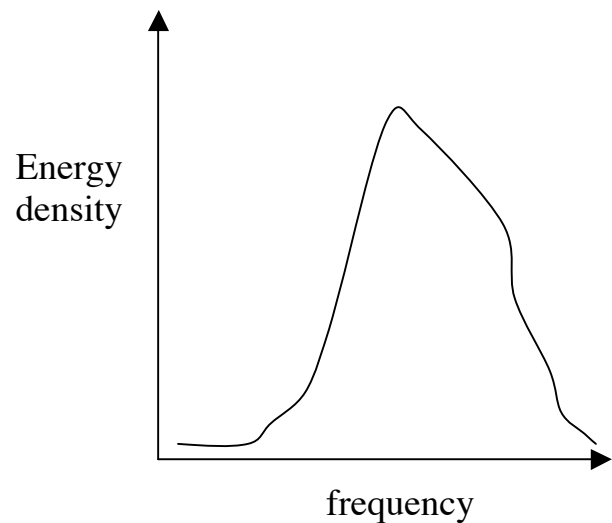


(from Waves, Tides and Shallow Water Processes).

From the record, we can see that the wave cannot be simply described as one sinusoid. Instead, it is characterized by many waves with different periods and phases. The spectrum of energy is usually plotted as energy density, (unit of energy/unit frequency interval, Hz). The energy density is given by the amount of energy in a particular frequency interval

$$\frac{1}{2} \rho g a^2 / Hz$$

To quantify the sea state, we make a plot of the energy density as a function of frequency.



Another way we can describe the waves, or the *sea state*, is with the *significant wave height*. The significant wave height $H_{1/3}$ is the average height of the highest 1/3 of all waves observed in a given period of time. This is the average height given by an experienced observer. The Beaufort scale gives a relationship between the wave height and the wind speed, so given the observed significant wave height, one can infer the wind speed.

Table 1.1 A selection of information from the Beaufort Wind Scale.

Beaufort No.	Name	Wind speed		State of the sea-surface	Wave height* (m)
		knots	ms ⁻¹		
0	Calm	<1	0.0-0.2	Sea like a mirror.	0
1	Light air	1-3	0.3-1.5	Ripples with appearance of scales; no foam crests.	0.1-0.2
2	Light breeze	4-6	1.6-3.3	Small wavelets; crests have glassy appearance but do not break.	0.3-0.5
3	Gentle breeze	7-10	3.4-5.4	Large wavelets; crests begin to break; scattered white horses.	0.6-1.0
4	Moderate breeze	11-16	5.5-7.9	Small waves, becoming longer; fairly frequent white horses.	1.5
5	Fresh breeze	17-21	8.0-10.7	Moderate waves taking longer form; many white horses and chance of some spray.	2.0
6	Strong breeze	22-27	10.8-13.8	Large waves forming; white foam crests extensive everywhere and spray probable.	3.5
7	Moderate gale	28-33	13.9-17.1	Sea heaps up and white foam from breaking waves begins to be blown in streaks; spindrift begins to be seen.	5.0
8	Fresh gale	34-40	17.2-20.7	Moderately high waves of greater length; edges of crests break into spindrift; foam is blown in well-marked streaks.	7.5
9	Strong gale	41-47	20.8-24.4	High waves; dense streaks of foam; sea begins to roll; spray may affect visibility.	9.5
10	Whole gale	48-55	24.5-28.4	Very high waves with overhanging crests; sea-surface takes on white appearance as foam in great patches is blown in very dense streaks; rolling of sea is heavy and visibility reduced.	12.0
11	Storm	56-64	28.5-32.7	Exceptionally high waves; sea covered with long white patches of foam; small and medium-sized ships might be lost to view behind waves for long times; visibility further reduced.	15.0
12	Hurricane	>64	>32.7	Air filled with foam and spray; sea completely white with driving spray; visibility greatly reduced.	>15

* $H_{1/3}$, i.e. the significant wave height.

While it is fairly intuitive how strong winds can generate larger waves, it is not as clear how the strength of the wind influences the dominant frequency of the waves, and thus the wave spectrum. To understand this, it is important to think about how waves grow. Initial disturbances are due to turbulent pressure fluctuations on small capillary waves.

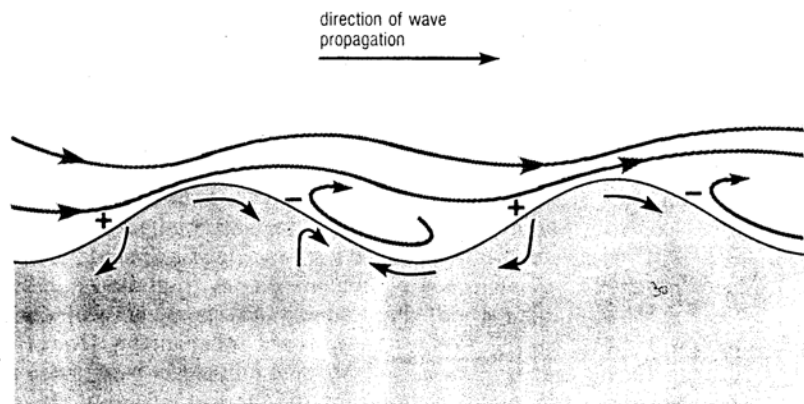
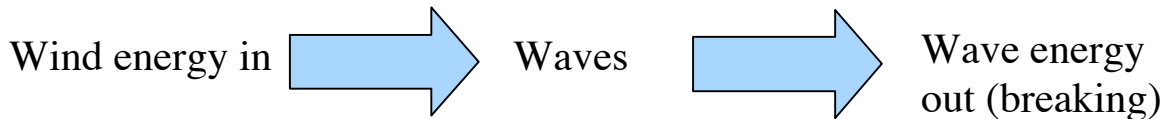


Figure 1.3 Jeffreys' 'sheltering' model of wave generation. Curved lines indicate air flow; short, straight arrows show water movement, which will be explained more fully in Section 1.2.1. The rear face of the wave against which the wind blows experiences a higher pressure than the front face, which is sheltered from the force of the wind. Air eddies are formed in front of each wave, leading to differences in air pressure. The excesses and deficiencies of pressure are shown by plus and minus signs respectively. The pressure difference pushes the wave along.

As in flow over topography, the wave causes air pressure fluctuations and eddies to form on the downwind side of the wave. Higher pressure on the upwind side of the wave creates a pressure gradient force on the wave and causes them to grow. As the waves grow, they steepen and non-linear effects become increasingly important and the waves break. In *deep water*, wave breaking tends to happen when $a/\Lambda = 1/12$.

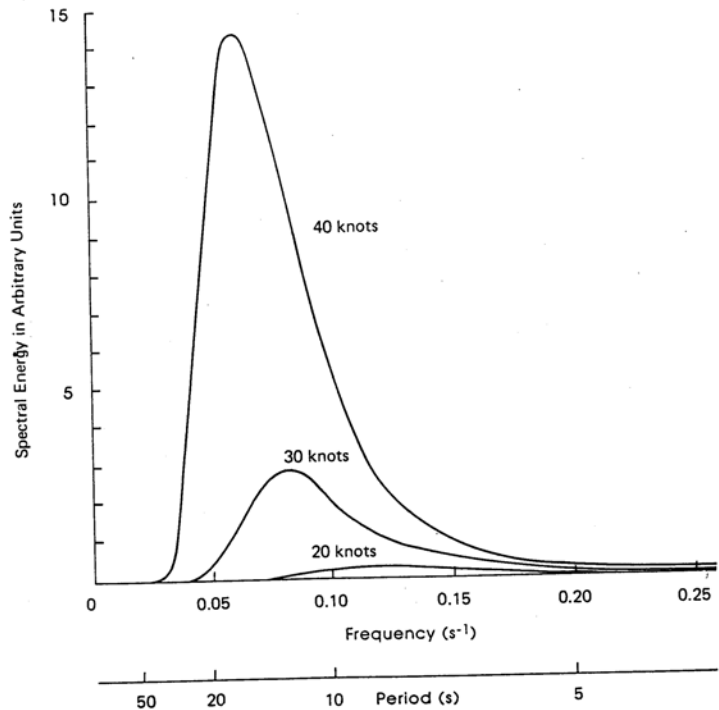
Now how does the speed of the wind effect the wave spectrum? First we need a definition for a *fully developed sea*. A fully developed sea occurs when the input by the wind is balanced by breaking of the waves.



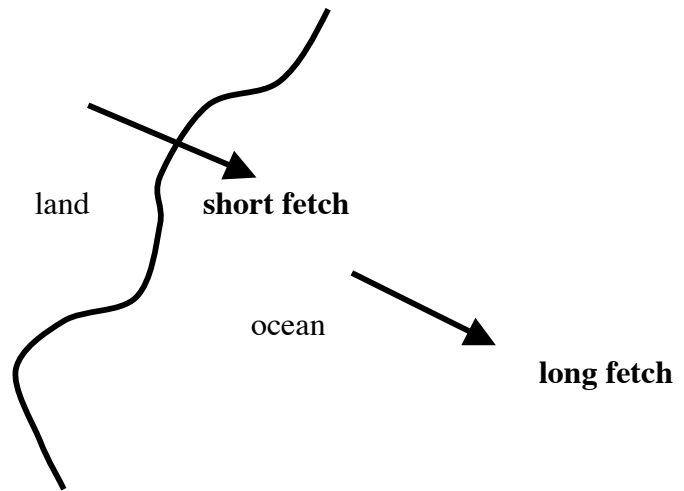
The wind must blow as fast, if not faster, than the phase speed C of the wave to impart energy to it, where

$$C = \frac{\omega}{k} = \frac{\omega}{\omega^2/g} = \frac{g}{\omega} = \frac{gT}{2\pi}$$

Therefore, for a fully developed sea, the greater the wind speed, the longer the period of the most energetic waves (the peak in the spectrum). Also, since $\omega^2 = gk$, the longer the period T , the lower the frequency, and the longer the wavelength.



However, a fully developed sea does not always occur. Whether it does depends on the *fetch* (the distance over which the wind blows) and the *duration* (the length of time the wind blows) of the storm. *Young seas*, those with limited duration or fetch will tend to be peaked at shorter wavelengths. Waves tend to grow exponentially so that the seas quickly come into equilibrium with the winds, which is why the Beaufort scale is useful. See figure 9.12 in Knauss that shows the minimum fetch and duration required for a fully developed sea as a function of wind speed. In general, the *stronger the wind, the larger fetch and duration is needed for a fully developed sea to form*. Figure 9.13 shows the spectrum for a fully developed sea and for ones with more limited fetches.



Aside. The speed with which the ocean waves come into equilibrium with near-surface winds makes possible the measurement of winds using satellite-mounted radars called “scatterometers.” The radar signal is scattered back from the ocean surface by waves with wavelengths of a few centimeters – their resonant backscatter, called “Bragg scatter”, is used to estimate the speed of the wind that produces the short waves. By varying the look angle of the radar, wind direction can also be obtained.

Example: A storm is generated offshore of Washington. We measure the energy spectrum of surface gravity waves at Astoria on February 1 at noon and then again on February 3 at noon.

- a. The peak energy occurs at a period of 12s on February 1, and 6 s on February 3. What is the wavelength for each of those waves?

We know that

$$\omega^2 = gk$$

Rearranging we have

$$\omega^2 = \frac{g2\pi}{\Lambda}$$

$$\Lambda = \frac{g2\pi}{\omega^2} = \frac{gT^2}{2\pi}$$

So plugging in for T we have 224 m for the 12 s wave and 56 m for the 6 s wave

- b. What is the phase velocity and group velocity for each of the waves found in b?

$$C = \sqrt{g/k} = \sqrt{g\Lambda/2\pi}$$

18.7 m/s for the 12 s wave

9.35 m/s for the 10 s wave

The group velocity will be half that: 9.35 m/s and 4.68 m/s respectively.

c. Estimate *when* the storm occurred. Let d be the offshore distance (unknown, but the same for both cases)

$$d = C_1 t_1$$

$$d = C_2 t_2$$

and

$$t_2 = t_1 + 48 \text{hours}$$

where C_1 and C_2 are the group velocity for the 12s and 6 s waves, respectively. Then

$$C_1 t_1 = C_2 t_2 = C_2 (t_1 + 48 \text{hours})$$

$$(C_1 - C_2) t_1 = C_2 48 \text{hours}$$

$$t_1 = \frac{C_2}{(C_1 - C_2)} 48 \text{hours} = 48 \text{hours}$$

d. Estimate *where* the storm occurred.

$$d = C_1 t_1 = 9.35 * 48 * 60 * 60$$

$$d = 1616 \text{ km}$$