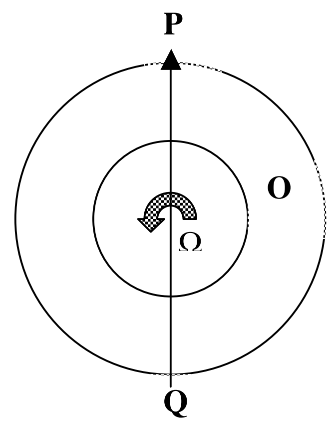
Rotation

An important concept in understanding ocean circulation is the effect of the earth's rotation on the motion of water parcels. Rotation is simple when viewed from a stationary (non-moving or Newtonian) frame of reference. For example, imagine the motion of a ball thrown from point Q to point P – the trajectory is a straight line. Now imagine that same trajectory viewed by an observer at point O on a merry-go-round rotating counterclockwise.

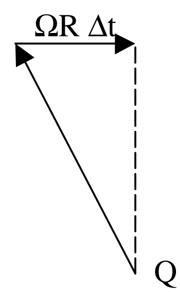


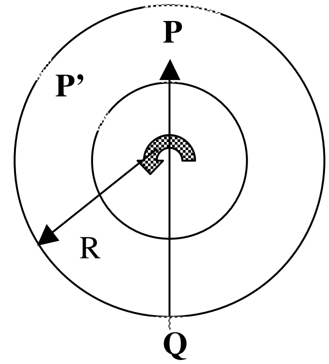
Let's examine the effect of rotation in three cases.

Case I. A ball is thrown across the merry-go-round from a person on the ground at point Q to another person on the ground at point P. What is the effect of rotation?

The ball clearly moves in a straight line to an observer on the ground, and the ball reaches the person at point P, so there is no effect of rotation here. (However, the trajectory appears quite complex as viewed by a person on the merry-go-round (point O).

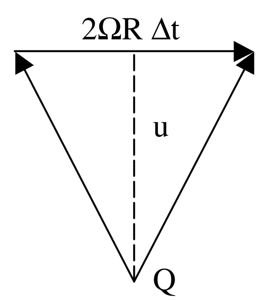
Case II. The person at point P gets on the merry-go-round. The person at Q throws the ball toward P, but by the time the ball gets there, the receiver at P is at P'. What is the effect of rotation?

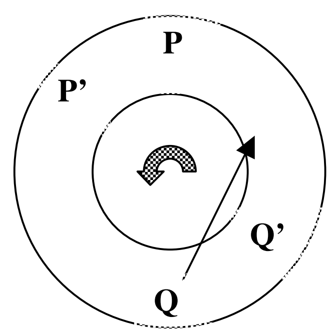




The ball misses its target. From the point-of-view of the receiver, the ball moved sideways by the amount $\Omega R \Delta t$, where Δt is the time for the ball to cross the merry-go-round.

Case III. The person at point Q also gets on the merry-go-round. The person at Q throws the ball toward P, but the ball moves toward the right. What is going on here? Because the ball-thrower is moving toward the right, and was holding the ball, the ball had a component of velocity toward the right, ΩR. When it was thrown toward P, it continued to move right. (The person at Q moves to Q'.)





What is the effect of rotation?

The ball misses its target by twice as much as before. From the point-of-view of the receiver, the ball moved twice as in Case II. From the point-of-view of an observer on the ground, the ball moves to the right.

How can we explain this sideways motion (we live in the rotating frame) in terms of Newton's law, $F = ma$, or, as we usually write it in oceanography, $F/m = a$?

The distance d the ball moved is proportional to the rotation rate of the merry-go-round, $2\Omega R\Delta t$, where Δt is $u/(2R)$, the time for the ball to cross the merry-go-round at the speed at which the ball is thrown, u. Now we can obtain the acceleration a from a basic physics formula,

$$d = \frac{1}{2} a \Delta t^2$$

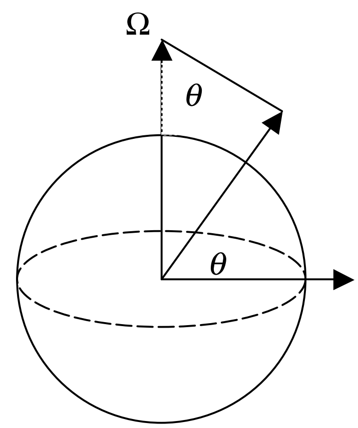
so

$$a = \frac{2d}{\Delta t^2} = \frac{4\Omega R \Delta t}{\Delta t^2} = \frac{4\Omega R}{2R/u} = 2\Omega u$$

This is the "force" that deflects the ball sideways. The rotation rate dependence seems straightforward (the faster the rotation, the larger the deflection of the ball), but why does it depend on the speed u at which the ball is thrown (the water parcel motion)? Because the faster the ball is thrown, the shorter the time required to cross the merry-go-round, and the larger the acceleration needed to account for the sideways deflection.

Conversely, if one wanted to throw the ball the P at the new location P', the thrower at Q would have to apply an force (acceleration) of $2\Omega u$.

The earth is like the merry-go-round, rotating at a rate of one revolution per day. A water parcel moving on the earth with speed u experiences an apparent force that deflects it (to the right in the northern hemisphere) called the *Coriolis force*. To make a water parcel move in straight line an opposing force proportional to $2\Omega u$ must be applied. You can think of the factor of 2 as arising from both ends points of a parcel trajectory being on the rotating earth, as in Case III.



There is another factor in the Coriolis force, the *latitude*. Why is that? For horizontal motions on the surface of the earth, it is the local vertical component of rotation that is important. (Vertical motions are relatively small in the ocean.) The magnitude of the local vertical component of rotation is

$$\boxed{f = 2\,\Omega \sin\theta}$$

where θ is the latitude and we refer to f as the *Coriolis parameter*.
Note that f is a maximum at the North Pole, that f is zero at the equator, and that f changes sign (becomes negative) in the southern hemisphere.

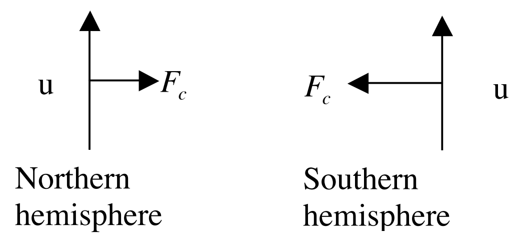
The magnitude of the Coriolis force is given by

$$F_c = 2\,\Omega \sin\theta\, u = f\, u$$

(Note that mass does not appear here. As we discussed before, because we do not have point masses in the ocean, but rather continuous water parcels, we use pressure, density, etc.) The use of the non-Newtonian reference frame, the rotating earth, gives rise to a vector cross product term for the Coriolis force. Actually,

$$\underline{F_c} = -2\,\underline{\Omega} \times \underline{u}$$

If the right-hand-rule does not come readily to mind, then remember that the Coriolis force is directed to the *right* of the current in the *northern* hemisphere and to the *left* of the current in the *southern* hemisphere.



There are two senses of rotation from the perspective of the earth as a frame of reference: in the same direction as the earth rotates (*cyclonic*) or opposite to the earth's rotation direction (*anticyclonic*).

We use cyclonic/anticyclonic rather than counterclockwise/clockwise because the direction of rotation appears to change sign between hemispheres. Cyclonic rotation in the northern hemisphere is counterclockwise, but cyclonic rotation in the southern hemisphere is clockwise.