Geostrophy

What other forces act on water parcels in the ocean?

We just reviewed the Coriolis force – the extra term that arises from our rotating reference frame. This additional term in the momentum equations leads to the possibility of new balances, in particular, a balance between the Coriolis force and the pressure gradient force (geostrophic balance). Recall that in order for a water parcel to move in a nearly straight line, there must be a force to balance the Coriolis force as in the diagram where the pressure gradient is directed from the high in the center (H) outward to regions of lower pressure.

To understand how the ocean attains this balance, imagine that initially there is a pressure gradient, as shown in the diagram. What happens? There is an acceleration from high to low pressure, as would happen without any influence of rotation.

acceleration = pressure gradient force

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}
\]

Over time, as the velocity \( u \) increases, the Coriolis force becomes important. If the pressure gradient persists over a time longer than the inertial period, \( T = \frac{2\pi}{f} \), the water parcel will be deflected (to the right in the N hemisphere), until the Coriolis force exactly balances the pressure gradient force. The parcel will then experience no further accelerations (at least in our rotating reference frame). Remember that \( f = 2\Omega \sin \theta \) where \( \Omega = \frac{2\pi}{24 \text{ hours}} = 7.292 \times 10^{-5} \text{ s}^{-1} \) and \( \theta \) is latitude. Then we have

geostrophy: \( fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \quad \left( F_c = -F_p \right) \quad (4.1a) \)

Geostrophy also holds for the other velocity component, \( v \),

\[
f v = \frac{1}{\rho_0} \frac{\partial p}{\partial x} \quad (4.1b)
\]
The signs for the component equations (4.1) are such that high pressure is on the right when looking downstream in the northern hemisphere (and on the left in the southern hemisphere, where $f$ is negative).

If we know the pressure, then we can determine the geostrophic flow. In the ocean, the pressure field is dominated by the vertical increase of pressure with depth. To first order, pressure and depth are just scaled versions of the same vertical coordinate. Below the ocean surface, it is impractical to separate out the small horizontal pressure gradients from direct measurements of pressure, but we can measure horizontal gradients of density. How do we get from density to geostrophic velocity? We need a way to infer pressure gradients from density – we will use the quantity called dynamic height.

*Note: the ocean surface is a pressure surface and it can be measured using a radar altimeter to get the surface pressure gradient. More about that later.*

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