Ekman layers

So far, we have looked at friction within the ocean. But there is also friction between the ocean water and the surroundings. In fact, an important driver of ocean currents is friction from winds blowing over the surface of the ocean. This friction imparts momentum of the wind onto the surface layer of the ocean – effectively, the wind drags the sea surface – and creates a current.

Intriguingly, over coastal to oceanic scales of motion, Earth’s rotation has a decisive impact on this wind-driven current. For one, it deflects the current from the downwind direction; for another, it limits the vertical extent of this wind-driven flow to within the top 10-50m of the ocean. The layer over which this rotation-influenced current occurs is called the Ekman layer. In this section we consider the dynamics of the wind-driven Ekman layer.

Ekman transport

In the Ekman layer, there is a balance between friction and the Coriolis force. Since the Coriolis force always acts perpendicular to the current, the balancing force would also be perpendicular to the current. From this follows the rather amazing result that the current within the Ekman layer is, on the whole, at right angles to the wind rather than along the wind! From this balance we can deduce the magnitude of the current and its direction. In the northern hemisphere, the vertically averaged transport in the Ekman layer is at 90 degrees to the right of the wind, as depicted.

We can calculate the magnitude of this transport from the force balance between Coriolis force and turbulent shear stress:

\[
(7.1a) \quad -fv = \frac{1}{\rho} \frac{\partial \tau^y}{\partial z} \\
(7.1b) \quad fu = \frac{1}{\rho} \frac{\partial \tau^x}{\partial z}
\]

We integrate (7.1) vertically from the depth where the turbulent stress dies out \(z = -D_E\), which we define to be the base of the Ekman layer.

\[
\int_{-D_E}^{0} \rho v \, dz = -\frac{1}{f} \int_{z=-D_E}^{0} \frac{\partial \tau^y}{\partial z} \, dz \quad \int_{-D_E}^{0} \rho u \, dz = \frac{1}{f} \int_{z=-D_E}^{0} \frac{\partial \tau^x}{\partial z} \, dz
\]
where $\tau_w = (\tau_w^x, \tau_w^y)$ is the wind stress at the sea surface, $z = 0$. The symbol $M$ will represent total mass transport. $M_{xE}$ is the total Ekman transport in the $y$-direction. The above expression simplifies to:

$$M_{xE} = -\frac{1}{f} (\tau_w^x) \quad M_{xE} = \frac{1}{f} (\tau_w^y)$$

(7.2)

So the net transport is $90^\circ$ from the direction of the wind (to the right in the northern hemisphere and to the left in the southern hemisphere).

To know the wind stress at the surface of the ocean, we can use a good empirical relationship between the wind stress and the wind speed ten meters above the sea surface (where most ships have anemometers) $W_{10\text{meters}}$ that gives

$$\tau_{\text{wind}} = \rho_A C_D W_{10\text{meters}}^2$$

in the direction of the wind

where

$C_D \equiv 2 \times 10^{-3}$ (drag coefficient)

$\rho_A = 1.3 \text{kg} / \text{m}^3$ (density of air)

Note: these are transports per meter of horizontal distance. Units are kg m$^{-1}$ s$^{-1}$. To convert these mass transports to volume transports (per unit horizontal distance, i.e., m$^2$/s) divide by the mean ocean density.

$$V = \frac{1}{\rho f} \tau_w^x \quad U = \frac{1}{\rho f} \tau_w^y$$

To clarify, this is the transport for a cross-sectional area of height equal to the depth of the Ekman layer and width equal to one meter perpendicular to the transport direction.

**Ekman pumping**

The transport in the Ekman layer is proportional to the wind stress. What happens when the wind stress varies spatially? In the diagram the southward stress increases toward the west, so the transport increases. What is the vertical flow required to conserve mass? Divergences and convergences occur in the Ekman layer in response to spatially varying wind stress. The divergences cause upward motion ("suction") and the convergences cause downward motion ("pumping") at the bottom of the surface Ekman layer. In this way, there can be an exchange of water between the Ekman layer and the ocean underneath. This exchange is
very important for the transport of materials such as nutrients into and out of the surface ocean, as well as for the driving of the current below the Ekman layer.

**Vertical structure of the Ekman layer.**

The above argument about the Ekman transport applies to the Ekman layer as a whole. Within the layer, the structure of the current is rather intricate. If we assume the stress in the interior to be given by an eddy viscosity times the vertical shear, equations (7.1a,b) become

\[-fv = A_z \frac{\partial^2 u}{\partial z^2}\]

\[fu = A_z \frac{\partial^2 v}{\partial z^2}\]

This pair of equations can be solved, with the stress matched to the wind stress at the surface and assumed to vanish at depth. We will not examine the mathematical details of the solution, but rather just comment on some important aspects of the solution.

First we can draw a “hodograph”, a plot of $u$ versus $v$.

Features of the solution include:
1) Surface flow is 45 degrees to the right of the wind and rotates to the right in the northern hemisphere, and to the left in the southern hemisphere.
2) Flow decays exponentially with depth over a vertical scale of

\[D_e = \sqrt{\frac{2A_z}{f}}\]

Observations show that the relationship between the transport in the Ekman layer and the wind stress is very robust. However, the details of the vertical structure of the velocity field are not as well explained by the simple theory. However, the simple theory does predict the turning. See the two figures below taken from Chereskin (1995).
The next figure is from Tomczak and Godfrey (1994), Regional Oceanography: An Introduction and show the annually averaged wind.

What is the direction of the Ekman transport at 50N 180W? At 30W 50S based on the wind figure given above?