Dynamic height (at pressure p relative to pressure p_0.)

First, we recall the hydrostatic relationship

$$\frac{\partial p}{\partial z} = -g\rho$$

From this equation, we can find an expression for z as a function of pressure

$$D = -\frac{1}{g}\int_{p_0}^{p} \frac{1}{\rho} dp$$

where p_0 is a reference level and D is in units of meters, m. The dynamic height is usually measured as the difference or the anomaly from a reference value. Typically it has a range of a couple of meters.

*Interpretation of dynamic height***.**

Roughly, $D \sim 1/\rho *$height above reference level.
High ρ (cold, salty) $\Leftrightarrow$low D
Low ρ (warm, fresh) $\Leftrightarrow$ high D

A flow field in *geostrophic* balance will have high pressure on the right facing the direction of flow (in the northern hemisphere of course), so this is the origin of the maxim "light on the right".

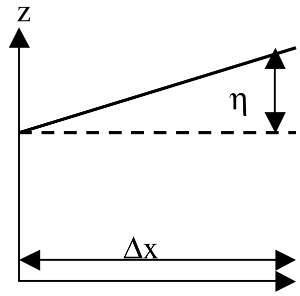
gradient in D = (gradient in pressure at p) – (gradient in pressure at p_0)

Substituting the gradient of dynamic height for the pressure gradient in the geostrophic equations (4.1), we obtain new equations to diagnose geostrophic velocity in the ocean.

$$\begin{aligned} v(p) - v(p_0) &= \frac{g}{f}\frac{\partial D}{\partial x} \\ u(p) - u(p_0) &= -\frac{g}{f}\frac{\partial D}{\partial y} \end{aligned} \qquad (5.1)$$

The right hand side is now something we can compute from our observations of temperature T, salinity S and pressure p. The velocity determined in this way is called the **relative (geostrophic) velocity**. It is the velocity at p relative to the velocity at the **reference level**, p_0. If we have some way of independently specifying the velocity or the pressure gradient at any given level (say by current meters, floats, or altimetry), we can combine these to obtain the **absolute velocity,** u(p) and v(p). Another way of understanding this "reference level" problem is to observe that our measurements of horizontal gradients in density (from which we compute the horizontal gradients in dynamic height) are not related directly to velocity itself, but only to a vertical difference in velocity.

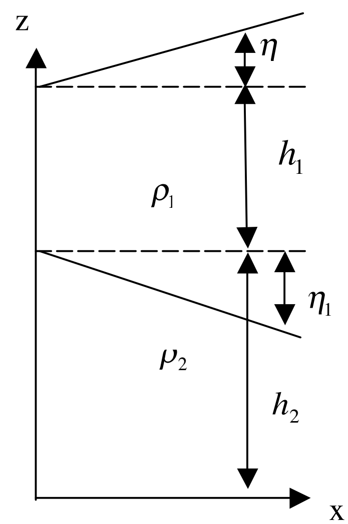
Margules's equation



We can quantify this in a simpler situation through *Margules's equation*. The first example is a one-layer (constant density) ocean. The geostrophic relationship gives us

$$fv = \frac{1}{\rho_o}\frac{\partial p}{\partial x} = g\frac{\eta}{\Delta x} = gi_x$$

where the dynamic height is just η and i_1 is the slope of the free surface.



In a *two layer* ocean, things get a little more complicated. For layer 1, as with the one-layer ocean, we have

$$fv_1 = \frac{1}{\rho_o}\frac{\partial p_1}{\partial x} = g\frac{\eta}{\Delta x} = gi_1$$

In layer 2, we have

$$fv_2 = \frac{1}{\rho_o}\frac{\partial p_2}{\partial x} = \frac{g}{\rho_o}\frac{\partial}{\partial x}\left[\rho_1(\eta-\eta_1)+\rho_2(\eta_1)\right]$$
$$= \frac{g}{\rho_o}\left[\rho_1\frac{\partial \eta}{\partial x} + (\rho_2-\rho_1)\frac{\partial \eta_1}{\partial x}\right] = gi_1 + g\frac{(\rho_2-\rho_1)}{\rho_o}i_2$$

Where i_2 is the slope of the interface between layer 1 and layer 2 and we recall the definition of **reduced gravity**
$g' = \frac{(\rho_2-\rho_1)}{\rho_0}g$.
We can then write the velocity shear between layer 1 and layer 2 as
$f(v_1 - v_2) = -g' i_2$

We will next derive a more general relationship between density and velocity differences.

Thermal wind

Underlying the "reference level" problem is the fact that our measurements of horizontal gradients in density (from which we compute the horizontal gradients in dynamic height) are not related directly to velocity itself, but only to a vertical difference in velocity. This relationship is quantified in the **thermal wind equation**.
Starting with *geostrophy*

$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$, we take the vertical derivative and reverse the order of the differentiation

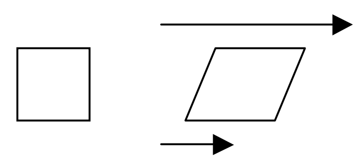
$$f\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial y}\right) = -\frac{1}{\rho_0}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right)$$

We next substitute for the pressure derivative using the *hydrostatic equation* $\frac{\partial p}{\partial z} = -\rho g$ to obtain the **thermal wind equations** (and doing the same thing for the x-component)

$$\boxed{\frac{\partial u}{\partial z} = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y} \qquad \frac{\partial v}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x} \qquad (5.2)}$$

The thermal wind equations relate horizontal gradients of density to the vertical derivative of velocity. The dynamic height relationship to geostrophic velocity (5.1) is simply the vertically integrated version of this thermal wind equation. They are called the thermal wind equations because the relationship was originally derived for the atmosphere using air temperature in place of density, so the wind gradient can be derived from the air temperature.

This vertical gradient of the geostrophic velocity is called the **geostrophic shear**. (The gradient of velocity in general is called the shear due to tendency of velocity gradients to shear a material element.)



How do we get absolute velocity? Recall that the reason that we derived the thermal wind equations and Margules's relation was that we have difficulty measuring the absolute pressure field. There are generally three solutions to this problem: 1) using a measured velocity (at any depth) to get the absolute velocity profile, 2) using a measurement of pressure (or the sea surface height) to get the absolute dynamic height (and then the velocity), 3) assuming a velocity (usually zero) at some depth. This latter method has historically been the most common and that depth is known as the "level of no motion." Of course, an error occurs where there is substantial motion at that depth.

Using the altimeter for a reference level.

An important new tool for oceanographers is the radar altimeter, which is mounted on a satellite. The altimeter measures the distance between the satellite and the sea surface. To get the height of the sea surface η(x,t) that corresponds to the pressure gradient from geostrophic motion, there are several other quantities that need to be measured or estimated. First, we have to know the height of the satellite h_o relative to some fixed reference ellipsoid, which is done by measuring the satellite's altitude h (this is currently done using a laser as shown, as well as with GPS, the global positioning system). This gives us the height of the ocean surface above the ellipsoid, h_s. Then we have to know the shape of the earth's geoid g(x), which is a surface of constant gravitational potential, because

$$h_s(x,t) = g(x) + \eta(x,t)$$

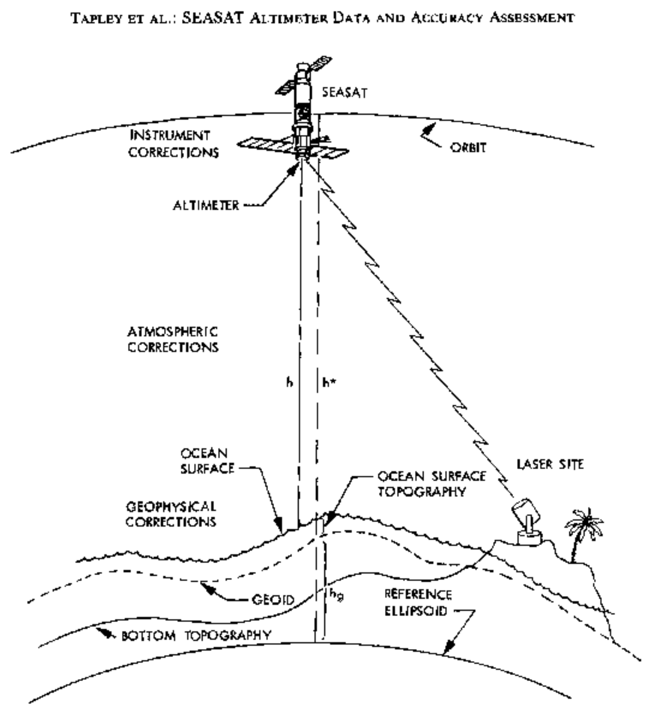


Fig. 1. Schematic of SEASAT data collection, modeling, and tracking system.

Spatial variations in the density of the solid earth give slight spatial variations in gravity, and correspond to variations in the height of the geoid of as much as 100 m. The geoid is currently not sufficiently well known to allow us to resolve typical currents scales of 100 km, but there are two gravity missions in progress designed to improve the geoid estimate.

There are several alternatives to an accurate geoid estimate. One is to just look at the time-varying sea level η', the anomalies relative to the mean. Because the geoid is relatively constant with time, averaging gives us

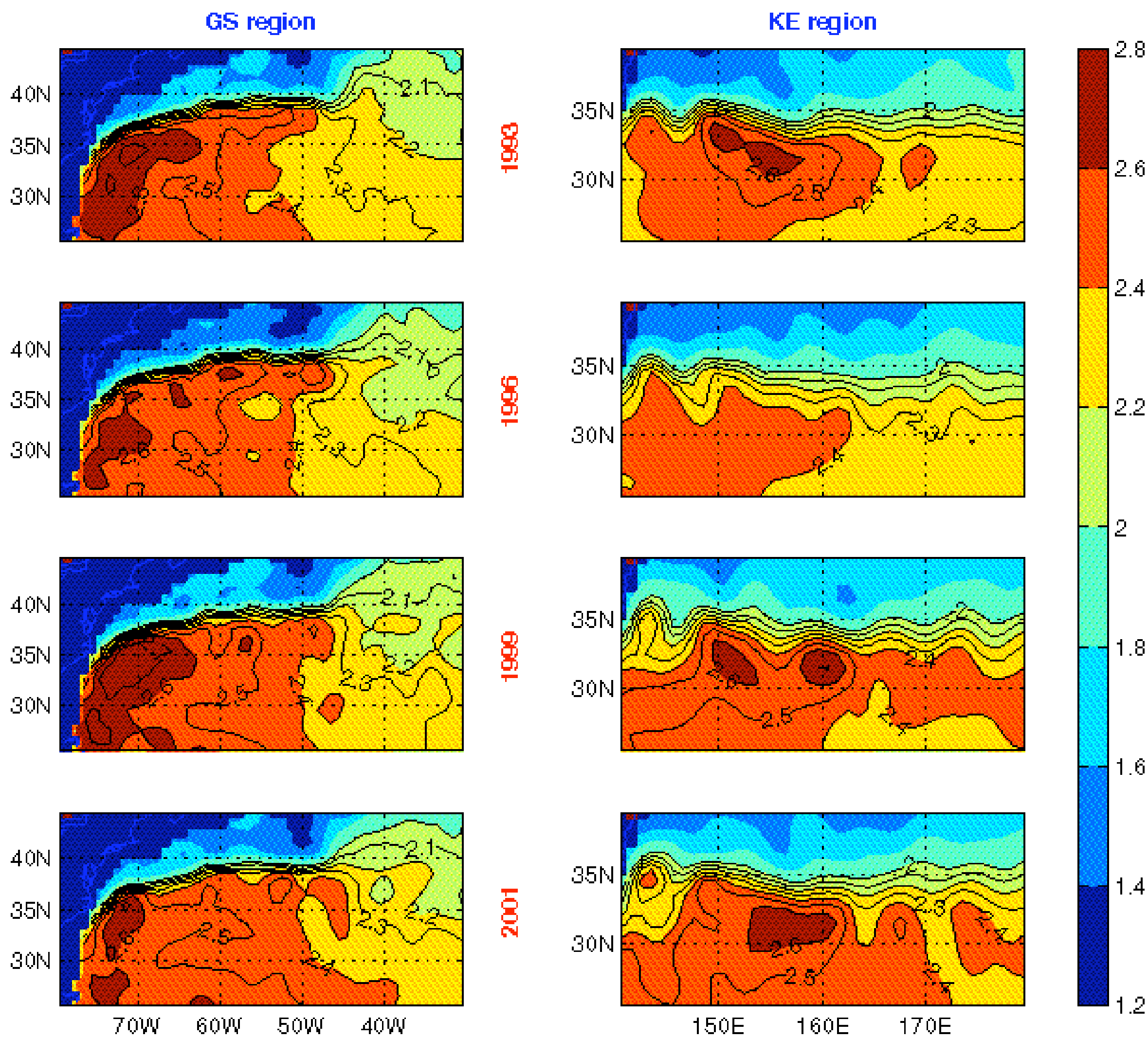
$$\overline{h_s(x,t)} = g(x) + \overline{\eta(x,t)}$$

so removing the mean of the sea level data removes the geoid along with $\overline{\eta}$

$$\eta'(x,t) = h_s(x,t) - \overline{h_s}(x)$$

Other solutions include creating an estimate of $\overline{\eta}$ with some sort of modeling or using other observations,

Sea level maps from the TOPEX/Poseidon altimeter. The maps show contours of sea level that include a mean sea surface. Each map is an average over 3 months in the *winter for the Gulf Stream (left) and for the Kuroshio Extension (right) for the year shown.*



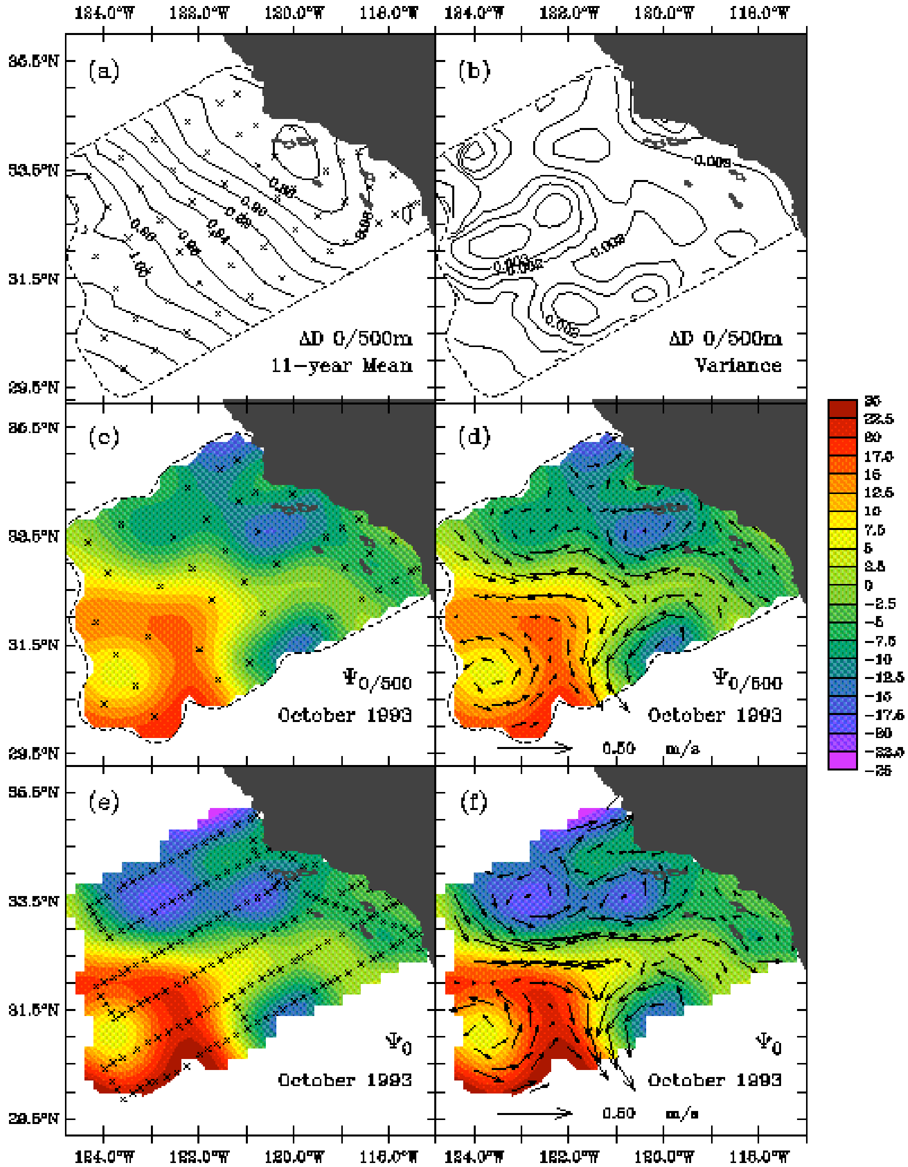


Figure 1. (a) Dynamic height anomaly 0/500 m, 11-yr mean. Note that the mean field bears little resemblance to any single realization of the flow, such as the 10/93 cruise shown here, due to averaging over the spatially varying locations of the California Current and eddies. (b) Variance about the 11-year mean. The high variance zone corresponds to the (meandering) path of the California Current. (c) Dynamic height anomaly 0/500, October 1993. Locations of CalCOFI hydrographic stations are shown. (d) as in (c), with geostrophic velocity vectors superposed. (e) Dynamic height corresponding to the absolute flow field. An ADCP velocity reference at 200 m. (f) as in (e), with velocity vectors superposed. Differences between (d) and (f) result from the ADCP reference. The maximum surface flow in the California Current deduced from (f) is about 35 cm/s, about 50% larger than estimated assuming zero flow at 500 m as in (e). The other striking difference between the maps is the increased gradients that are resolved by including the ADCP vector reference. (From Chereskin and Trunnell, 1996.