Name $\qquad$

There are 85 points possible on this exam. Each portion of a question is worth 5 points. You can do your work directly on the exam pages and turn to additional sheets of paper as necessary. Show your work for quantitative answers and your reasoning for qualitative answers.

1. We are interested in the biological activity in a region of the North Pacific.
a. We have measured the oxygen concentration in the water on a float that moves with the fluid at 100 m depth. We observe oxygen concentration decreases over the course of 10 days and that the float moves a distance of 50 km . We conclude that there has been active respiration (biological activity) in the fluid column. Explain how we can make this conclusion.

Since the measurements are taken on the moving float, and if diffusion is small, advection will not change the concentration, so any changes must come from respiration. This is a Lagrangian measurement.
b. Next we measure oxygen at a fixed mooring that has a sensor at 100 m depth. This time, we see the oxygen levels decrease over 10 days. Can we say anything about the biological activity in this case? Why or why not?

Since we are measuring in one place, advection could have brought in oxygen of lower concentration, so we can't tell if the changes are coming from advection or biological activity. This is an Eulerian measurement.

2. Shown above is a two layer density section at $180 \mathrm{~W} . \rho_{1}=1025 \mathrm{~kg} / \mathrm{m}^{3}$ $\rho_{2}=1027 \mathrm{~kg} / \mathrm{m}^{3}$ whose center is at 30 N
a) What is the reduced gravity?

$$
g^{\prime}=g \frac{\Delta \rho}{\rho_{1}}=9.8 \mathrm{~m} / \mathrm{s}^{2} 2 \mathrm{~kg} / \mathrm{m}^{3} \frac{9.8 \mathrm{~m} / \mathrm{s}^{2} 2 \mathrm{~kg} / \mathrm{m}^{3}}{1025 \mathrm{~kg} / \mathrm{m}^{3}}=0.019 \mathrm{~m} / \mathrm{s}^{2}
$$

b) What is the value of the Coriolis parameter?

$$
f=2 \Omega \sin \theta=2 * 7.29 \times 10^{-5} s^{-1} \sin 30=7.29 \times 10^{-5} s^{-1}
$$

c) If there is no flow in the lower layer, what is the sea surface height difference across the box? The distance across the box is 222 km . Sketch the SSH on the figure.

$$
\begin{aligned}
& \frac{\Delta \eta}{\Delta y}=i_{0}=-\frac{g^{\prime}}{g} i_{1}=\frac{0.019 m / s^{2}}{9.8 m / s^{2}} \frac{100 m}{\Delta y} \\
& \Delta \eta=0.194 m
\end{aligned}
$$

d) If there is no flow in the lower layer, what is the velocity in the upper layer? Give magnitude and direction.

$$
\begin{aligned}
& f u=-g \frac{d \eta}{d y} \\
& u=-\frac{g}{f} \frac{0.194}{222 \mathrm{~km}}=-0.12 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Since the SSH is sloping up to the north, and the high pressure is on the right of the flow, the flow is into the page, to the west and is negative.
e) If the flow is $5 \mathrm{~cm} / \mathrm{s}$ in the lower layer towards the west (negative) in the bottom layer, what is the flow in the upper layer? Give magnitude and direction.

The flow will be to the west, and will be the sum of the answer in (d) plus the $5 \mathrm{~cm} / \mathrm{s}$. $u=-0.12 m / s-0.05 m / 2=-0.17 \mathrm{~m} / \mathrm{s}$
f) Sketch the SSH for case (e) on the figure.
3. You are going on a research cruise to study upwelling off the coast of Chile in the Southern Hemisphere. You've memorized the upwelling patterns for the US West Coast, but summer is winter and winter is summer in South America! Is everything backwards in Chile?
a) Show the direction of the Ekman transport in the figure at right. Will the wind cause upwelling or downwelling? Why?

The Ekman flow at the surface is on shore because it Is to the left of the wind. This causes downwelling.

b) Sketch the sea surface height on the figure below.

c) Show the direction of the alongshore geostrophic flow in both figures.

Give your reasoning for the direction of the flow.
The flow is to the south, negative, out of the page. In the southern hemisphere, the high pressure (and SSH) is on the left of the geostrophic flow.

4. Shown above is a density section at 30 N in the Atlantic down to 1200 m . Consider the geostrophic flow implied by the density section
a) If there is a level of no motion at about 250 m , what direction (north or south or east or west) is the flow near the surface at 77 W ? Why?

In all of the parts, use the Margules relation
$f\left(v_{1}-v_{2}\right)=-g^{\prime} i$
In this part, with a level of no motion at 250 m , we set $\mathrm{v}_{2}=0$, and note that the slope is positive above 250 m , so that the flow must be negative, so the south.
b). What would the direction of the flow be at 800 m at 77 W if the level of no motion were at 1200 m ? Why?

If the level of no motion were at 1200 m , then we once again set $\mathrm{v}_{2}=0$ and note that since the slopes are negative between 800 m and 1200 , the flow must be positive, to the north.
c). What would the direction of the flow be at 800 m at 77 W if the level of no motion were at 250 m ? Why?

In this case, we set $\mathrm{v}_{1}=0$, and because the slope between 800 m and 250 m , is negative, we must have $v_{2}$ negative, that is the flow at 800 m will be negative, to the south.

5. Shown is wind stress as a function of latitude. The density of the ocean is $1025 \mathrm{~kg} / \mathrm{m}^{3 /}$.
a. Give the magnitude and direction of the Ekman volume transport at 35 N .

Since the zonal stress is positive, the Ekman volume transport must be negative, to the south.
At 35 N ,
$f=2 \Omega \sin \theta=8.3 \times 10^{-5} s^{-1}$
$V=-\frac{\tau}{\rho f}=\frac{0.05 \mathrm{~N} / \mathrm{m}^{2}}{1025 \mathrm{~kg} / \mathrm{m}^{3} 8.3 \times 10^{-5} \mathrm{~s}^{-1}}=0.59 \mathrm{~m}^{2} / \mathrm{s}$
b. The magnitude of the Ekman transport at 10 N is (greater or less ) than the magnitude at 20 N because:

Since the stress is about the same at 10 N and 20N, but the Coriolis parameter is smaller at 10 N , we would expect that the Ekman transport would be bigger at 10 N since it depends on the inverse of the Coriolis parameter
c. The sign of the Ekman pumping (the vertical velocity at the base of the mixed layer at 25 N ) is (positive or negative) because:

The Ekman pumping is negative. You can see this because north of 25 N the stress is larger than south of 25 N . That means that more mass is coming to the south at say 26 N than going to the south at 24 N . That means that to conserve mass, there must be transport out of the Ekman layer into the thermocline, and thus the vertical velocity must be negative.

## Variables and units in Ocean 420

$u$ zonal velocity (east-west) $\mathrm{m} / \mathrm{s}$
$v$ meridional velocity (north-south) $\mathrm{m} / \mathrm{s}$
$w$ vertical velocity (up-down) $\mathrm{m} / \mathrm{s}$
$t$ time $s$
$x$ east-west location $m$
$y$ north-south location $m$
$z$ vertical location (positve up) m
$p$ pressure ${\mathrm{N} / \mathrm{m}^{2} \text { or } \mathrm{kg} / \mathrm{s}^{2} / \mathrm{m}}_{\tau \text { stress } \mathrm{N} / \mathrm{m}^{2} \text { or } \mathrm{kg} / \mathrm{s}^{2} / \mathrm{m}}^{\rho \text { density } \mathrm{kg} / \mathrm{m}^{3}}$
$h$ thickness $m$
$H$ depth $m$
$v$ kinematic viscosity $\mathrm{m}^{2} / \mathrm{s}$
$\mu$ dynamic viscosity $\mathrm{kg} / \mathrm{m} / \mathrm{s}$
$\kappa$ diffusivity $\mathrm{m}^{2} / \mathrm{s}$
$C_{D}$ drag coefficient unitless
T temperature ${ }^{\circ} \mathrm{C}$
S salinity parts $/$ thousand
$\eta$ sea surface height $m$
g gravitational constant $\mathrm{m} / \mathrm{s}^{2}$
$f$ Coriolis parameter $1 / \mathrm{s}$
$\theta$ latitude radians
$D$ dynamic height $m$
$i$ slope of isopycnals unitless

## Some constants

$\Omega$ Rotation rate of the Earth $=7.292 \times 10^{-5} s^{-1}$
$R_{e}$ Radius of the Earth=6370 km
g graviational acceleration $9.8 \mathrm{~m} / \mathrm{s} 2$
$\rho_{A}$ Density of air $1.3 \mathrm{~kg} / \mathrm{m}^{3}$
$\rho_{0}$ Density of ocean $1025 \mathrm{~kg} / \mathrm{m}^{3}$
$C_{D}$ Typical drag coefficient $2 \times 10^{-3}$ (no dimensions)
Distance in one degree of latitude 111 km

## Equations in Ocean 420

$f=2 \Omega \sin \theta$
$\frac{\partial p}{\partial z}=-g \rho$
$f v=\frac{1}{\rho_{0}} \frac{\partial p}{\partial x}$
$f u=-\frac{1}{\rho_{0}} \frac{\partial p}{\partial y}$
$v(p)-v\left(p_{0}\right)=\frac{g}{f} \frac{\partial D}{\partial x}$
$u(p)-u\left(p_{0}\right)=-\frac{g}{f} \frac{\partial D}{\partial y}$
$f\left(v_{1}-v_{2}\right)=-g^{\prime} i_{1}$
$f\left(u_{1}-u_{2}\right)=g^{\prime} i_{1}$
$g^{\prime}=g \frac{\Delta \rho}{\rho}=g \frac{\rho_{2}-\rho_{1}}{\rho_{1}}$
$f v_{1}=g i=g \frac{\partial \eta}{\partial x}$
$f u_{1}=-g i=-g \frac{\partial \eta}{\partial y}$
$f=2 \Omega \sin \Theta$
$\frac{\partial u}{\partial t}-f v=0$
$\frac{\partial u}{\partial t}=-\frac{1}{\rho} \frac{\partial p}{\partial x}$
$\frac{\partial u}{\partial t}=$ friction
$0=-\frac{1}{\rho} \frac{\partial p}{\partial x}+$ friction
$-f v=$ friction
$\tau=\mu \frac{\partial u}{\partial z}$
$F_{\text {friction }}=\frac{\partial \tau_{x z}}{\partial z}$

Coriolis Parameter
Hydrostatic balance

Geostrophy

Velocity relative to reference level from dynamic height

Margules relation for vertical shear

Reduced gravity

Relationship from geostrophic flow at the surface (top
layer) given the sea surface height
Coriolis parameter
Acceleration balancing Coriolis
Acceleration balancing pressure gradient

Acceleration balancing friction

Friction balancing pressure gradient
Coriolis balancing friction
Stress
Frictional force
$\tau=-\rho \overline{u^{\prime} w^{\prime}} \approx \rho A_{v} \frac{d \bar{u}}{d z}$ Turbulent stress
$\tau_{\text {wind }}=\rho_{A} C_{D} W_{10 \text { meetrs }}\left|W_{10 \text { meters }}\right| \quad$ Wind stress (in the direction of the wind)
$M_{y E}=-\frac{1}{f}\left(\tau_{W}^{x}-0\right)$
Mass transport in Ekman layer
$M_{x E}=\frac{1}{f}\left(\tau_{W}^{y}-0\right)$
$V_{y E}=-\frac{\tau_{W}^{x}}{f \rho}$
$U_{x E}=\frac{\tau_{W}^{x}}{f \rho}$
$-f v=A_{z} \frac{\partial^{2} u}{\partial z^{2}}$
Volume transport in the Ekman layer
$f u=A_{z} \frac{\partial^{2} v}{\partial z^{2}}$
Ekman layer balance
$D_{e}=\sqrt{\frac{2 A_{z}}{f}} \quad$ Ekman layer depth
$\underline{\tau}=\rho C_{D}|u| \underline{u} \quad$ Stress at the top of log layer
$\tau_{\text {bottom }}=\rho_{0} C_{D} U_{1}^{2} \quad$ Bottom stress
$\frac{D C}{D t}=\frac{\partial C}{\partial t}+u \frac{\partial C}{\partial x}=0 \quad$ One-dimensional tracer advection
$\frac{D C}{D t}=\frac{\partial C}{\partial t}+u \frac{\partial C}{\partial x}+v \frac{\partial C}{\partial y}+w \frac{\partial C}{\partial z}=0 \quad$ Tracer advection
$\frac{\partial C}{\partial t}=A_{h}\left(\frac{\partial^{2} C}{\partial x^{2}}\right) \quad$ One dimensional tracer diffusion
$\frac{\partial C}{\partial t}=A_{h}\left(\frac{\partial^{2} C}{\partial x^{2}}+\frac{\partial^{2} C}{\partial y^{2}}\right)+A_{z} \frac{\partial^{2} C}{\partial z^{2}}$

