STEADY-STATE FORCE BALANCES

(1) Pressure gradient and Coriolis forces

Geostrophic Flow

$$u = -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}$$

Thermal Wind Equation

$$\frac{\partial u}{\partial z} = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y}$$

(2) Coriolis force and wind stress

Ekman transport

$$M_{xE} = \frac{1}{f}\tau^{y}_{Wind}$$

(3) Geostrophy (1) + continuity

Potential vorticity conservation

Lagrangian

$$\frac{(f+\zeta)}{H} = \text{const}$$

Eulerian

$$\beta v = f\frac{\partial w}{\partial z}$$

(4) Ekman transport (2) + continuity

Ekman pumping

$$w_E = \frac{1}{\rho_0 f}\left(\frac{\partial \tau^{y}_{W}}{\partial x} - \frac{\partial \tau^{x}_{W}}{\partial y}\right) + \frac{\beta}{\rho_0 f^2}\tau^{x}_{W}$$

(5) Ekman pumping (4) + potential vorticity conservation (3) + continuity

Wind-driven circulation

$$M_{yG} + M_{yE} = \frac{1}{\beta}\left(\frac{\partial \tau^{y}_{W}}{\partial x} - \frac{\partial \tau^{x}_{W}}{\partial y}\right)$$

(6) Potential vorticity conservation (3) + continuity

Dynamics of deep circulation (even if we don't have a model yet!)

PERIODIC TIME-DEPENDENT BALANCES (WAVES) **Restoring Force**

(1) Acceleration + pressure gradient force (+ Coriolis force) *GRAVITY*

Wind waves

Shallow Water *Deep Water*

$C = \sqrt{gH} = C_g$ $C = \sqrt{\frac{g}{\kappa}} = 2C_g$

Internal waves

2-layer *Continuously stratified*

$C = \sqrt{g'H} = C_g$ $\tan^2\theta = \frac{k^2}{m^2} = \frac{\omega^2 - f^2}{N^2 - \omega^2}$

When does rotation matter? Horizontal scale $L_R = \frac{\sqrt{gH}}{f}$ or $L_R = \frac{\sqrt{g'H}}{f}$;

Wave period ~ a day

TIDES

Poincare waves, Inertia-gravity waves

$\omega = \sqrt{f^2 + gHk^2}$

Kelvin waves (boundary waves; veloc ⊥ coast = 0)

$C = \sqrt{gH}$ or $C = \sqrt{g'H}$

(2) Acceleration + Coriolis force *NO FORCES*

Inertial oscillations

Limit of internal waves as $\omega \to f$

(3) Acceleration + pressure gradient force + Coriolis force *POTENTIAL VORTICITY*

Rossby waves