Equations in Ocean 420

$$\frac{\partial p}{\partial z} = -g\rho$$ Hydrostatic balance

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = g\frac{\partial \eta}{\partial x}$$ Pressure gradient in a fluid with constant density

$$\frac{1}{\rho}\frac{\partial p_2}{\partial x} = g\frac{\partial \eta}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$ Pressure gradient in the lower layer of a two layer fluid

$g\eta = g'\eta_1$ The relationship between the sea surface height and interface (thermocline displacement) when the lower layer is thick and motionless

$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

Geostrophy

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$$

$$v(p) - v(p_0) = \frac{g}{f}\frac{\partial D}{\partial x}$$

Velocity relative to reference level from dynamic height

$$u(p) - u(p_0) = -\frac{g}{f}\frac{\partial D}{\partial y}$$

$$f(v_1 - v_2) = -g' i_2$$

Margules relation for vertical shear

$$f(u_1 - u_2) = g' i_2$$

$f = 2\Omega\sin\Theta$ Coriolis parameter

$$\frac{\partial u}{\partial z} = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y}$$

Thermal wind

$$\frac{\partial v}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}$$

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} - Ju$$ Simple force balance

$$\frac{\partial u}{\partial t} - fv = 0$$ Acceleration balancing Coriolis

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$ Acceleration balancing pressure gradient

$$\frac{\partial u}{\partial t} = -Ju$$ Acceleration balancing friction

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} - Ju$$ Friction balancing pressure gradient

$-fv = -Ju$ Coriolis balancing friction

$$\tau = \mu\frac{\partial u}{\partial z}$$ Stress

$F_{friction} = \frac{\partial \tau_{xz}}{\partial z}$	Frictional force
$\tau = -\rho \overline{u'w'} \approx \rho A_v \frac{d\overline{u}}{dz} = \rho u_*^2$	Turbulent stress
$\tau_{wind} = \rho_A C_D W_{10meters}^2$	Wind stress
$M_{yE} = -\frac{1}{f}\left(\tau_W^x - 0\right)$ $M_{xE} = \frac{1}{f}\left(\tau_W^y - 0\right)$	Mass transport in Ekman layer
$-fv = A_z \frac{\partial^2 u}{\partial z^2}$ $fu = A_z \frac{\partial^2 v}{\partial z^2}$	Ekman layer balance
$D_e = \sqrt{\frac{2A_z}{f}}$	Ekman layer depth
$A_V = ku_* z$	Eddy viscosity in log layer
$u_*^2 = \tau / \rho$	Relationship between the friction velocity and stress
$u_*^2 = -\overline{u'w'} = ku_* z \frac{du}{dz}$	Friction velocity in the log layer
$u(z) = \frac{u_*}{k} \ln \frac{z}{z_o}$	Velocity profile in log layer
$C_D = \left(\frac{k}{\ln\left(z / z_0\right)}\right)^2$	Drag coefficient in log layer
$\underline{\tau} = \rho C_D \lvert u \rvert \underline{u}$	Stress at the top of log layer
$\tau_{bottom} = \rho_0 C_D U_1^2$	Bottom stress
$-g \frac{\partial \eta}{\partial x} H = C_D u^2$	Balance between pressure gradient and bottom stress
$h \frac{\partial T}{\partial t} = \frac{Q}{\rho C_p}$ $h \frac{\partial T}{\partial t} = \frac{Q}{\rho C_p} - \frac{\partial h}{\partial t}(T_s - T_h)$	Temperature balance in mixed layer
$\frac{1}{2}\alpha g h \Delta T \frac{\partial h}{\partial t} = m_0 u_*^3 - \frac{\alpha g h}{2\rho C} Q$	Energy balance in mixed layer
$h = 2\frac{C_p \rho m_0 u_*^3}{g \alpha Q}$	Monin-Obukhov depth

$$u = -\frac{\partial \psi}{\partial x}$$
$$v = \frac{\partial \psi}{\partial y}$$
Stream function

$$\frac{DC}{Dt} = \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = 0$$ Tracer advection

$$\frac{\partial C}{\partial t} = A_h\left(\frac{\partial^2 C}{\partial x^2}\frac{\partial^2 C}{\partial y^2}\right) + A_z\frac{\partial^2 C}{\partial z^2}$$ Tracer diffusion

$\eta(x,t) = a\cos(kx - \omega t)$ Sea surface height in a surface gravity wave

$C_g = \frac{\partial \omega}{\partial k}$ Group velocity

$C = \frac{\omega}{k}$ Phase velocity

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$
Force balance in surface gravity wave

$\omega^2 = gk \tanh kH$ Dispersion relation for surface gravity waves

$\omega = \sqrt{gH}k$ Dispersion relation for long (shallow water) gravity waves

$C = \sqrt{gH}$ Phase speed for shallow water gravity waves

$\omega^2 = gk$ Dispersion relation for deep water gravity waves

$C = \frac{\omega}{\kappa} = \sqrt{\frac{g}{\kappa}}$ Phase speed for deep water gravity waves

$C_g = \frac{\partial \omega}{\partial \kappa} = \frac{1}{2}\frac{g}{\omega} = \frac{1}{2}\sqrt{\frac{g}{\kappa}} = \frac{1}{2}C$ Group velocity for deep water gravity waves

$u = \frac{gak}{\omega}\cos(kx - \omega t)$ Horizontal velocity in deep water gravity waves

$w = \frac{gak^2}{\omega}\sin(kx - \omega t)(z + H)$ Vertical velocity in deep water gravity waves

$x = \frac{gak}{\omega^2}\sin\omega t$ Particle Excursion

$\Lambda > 20H$ Shallow water gravity waves

$\Lambda < 4H$ Deep water gravity waves

$E = \frac{1}{2}\rho g a^2$ Energy per unit wave crest in surface gravity waves

$\frac{E_2}{E_1} = \sqrt{\frac{H_1}{H_2}}$ Energy for shoaling waves

$g' = g\frac{\rho_2 - \rho_1}{\rho}$ Reduced gravity

$\eta = a\cos(\omega t - \varphi)$ Tidal amplitude

$\Delta g = \frac{GM}{R^2} 2 \frac{R_E}{R}$	Gravitational force from the moon to a unit mass on earth
$T = \frac{4L}{n\sqrt{gH}}$	Quarter wave oscillator period
$L_R = \frac{\sqrt{gH}}{f}$	Deformation radius (external,barotropic)
$L_R = \frac{\sqrt{g'H_1}}{f}$	Deformation radius (2 layer, internal, baroclinic)
$\omega^2 = N^2 \cos^2 \varphi$	Dispersion relation for internal gravity waves
$N^2 = -\frac{g}{\rho}\frac{\partial \rho_\theta}{\partial z}$	Buoyancy frequency
$\omega = f$	frequency of inertial oscillations