1b

Phys 121B Exam

I – Multiple choice

$$x = 100 + 20t - 2t^2$$

This is the equation of uniform accelerated motion (a=constant)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Question 1 (A)

The acceleration is independent of time.

$$\frac{1}{2} a = -2 \text{ m/s}^2 \rightarrow a = -4 \text{ m/s}^2$$

Question 2 (C)

$$v = \frac{dx}{dt} = 20 - 4t \xRightarrow{5s} 20 - 20 = 0 \text{ m/s}$$

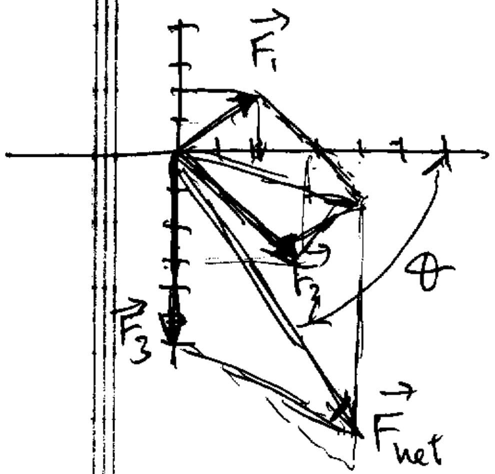
Question 3 (B)

Car starts at 100 m at $t=0$, moving with positive velocity of 20 m/s – So motion is like "free fall" of an object thrown straight up – At $t=5$ s the velocity is zero –

Question 4 (B)

Inside the car the ball was moving with 0 velocity – Ball is thrown straight up, but ball comes down when car has negative acceleration, so ball keeps moving forward with +acceleration (without any real force pushing!)

$\vec{F}_1 = 2\hat{i} + 2\hat{j}$ $\quad \vec{F}_2 = 3\hat{i} - 3\hat{j}$ $\quad \vec{F}_3 = -6\hat{j}$



Question 5 $\quad F_{net}(x) = 2 + 3 = 5\,N$

$F_{net}(y) = +2 - 3 - 6 = -7N$

$|\vec{F}_{net}| = \sqrt{(5)^2 + (7)^2} = 8.6\,N$

(E)

$\tan\theta = \dfrac{-7N}{5N} \Rightarrow 54.5°$ below x-axis.

Question 6 $\quad F_{net} = 8.6N$ (essentially given in Question 5), and is constant.

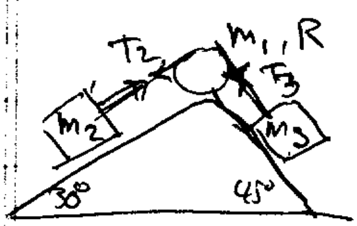
(D) $\quad$ So $a = \dfrac{F}{m} = \dfrac{8.6N}{2\,kg} = 4.3\,m/s^2$

In 2 seconds the displacement is

$r = \frac{1}{2}at^2 = \frac{1}{2} \times 4.3\,m/s^2 \times (2s)^2 = 8.6m.$

Force and displacement are parallel

So $W = Fd = 8.6N \times 8.6m \cong +74J$



Question 7 $\quad$ (D)

$m_3 g \sin 45° - m_2 g \sin 30° = (m_2 + m_3)\,a$

$a = \dfrac{m_3 g \sin 45° - m_2 g \sin 30°}{m_2 + m_3} = \dfrac{(3kg \sin 45° - 2kg \sin 30°)g}{5kg}$

$= 2.19\,m/s^2 \simeq 2.2\,m/s^2$

for mass m_2 this is uphill -

(4b)

Question 11 (A)

$$I_{\text{rotating masses}} = I(\text{rod}) + I(\text{disc})$$

$$= \frac{1}{12} m_{rod} l_{rod}^2 + \frac{1}{2} m_{disc} R_{disc}^2$$

$$= \frac{1}{12} \times 4\,kg \times (2m)^2 + \frac{1}{2}\, 3\,kg\, (0.2m)^2$$

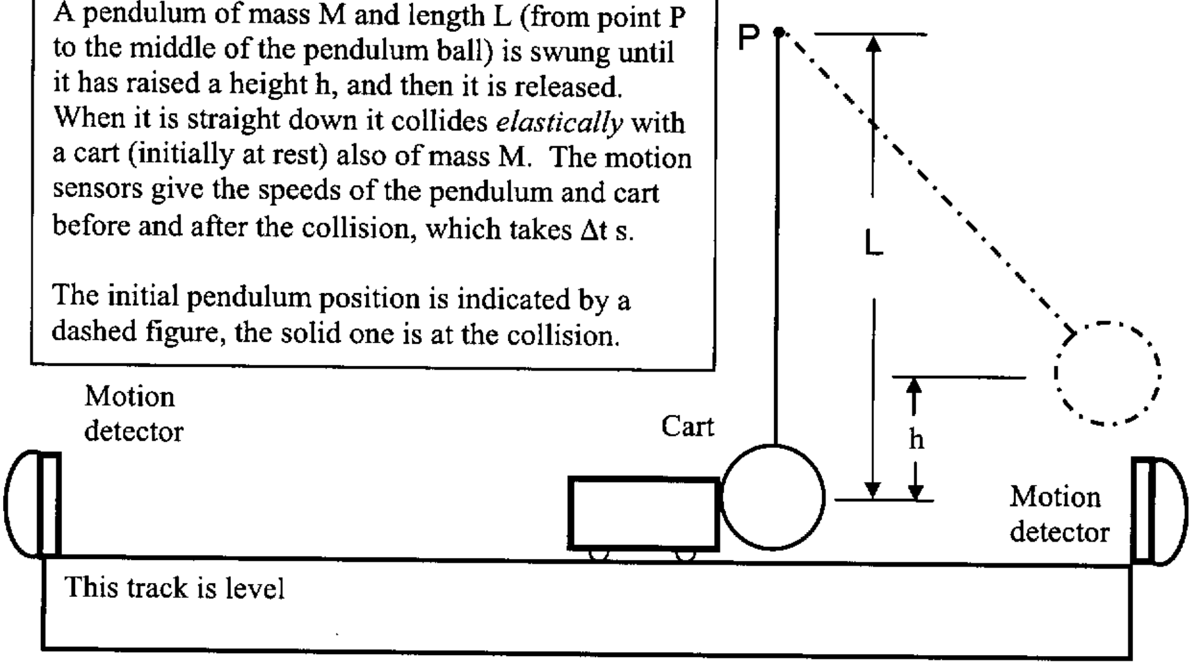
$$= 1.39\ kg\,m^2$$

Name______________________________Student ID__________Score________
last first

II. (25 pts)

A pendulum of mass M and length L (from point P to the middle of the pendulum ball) is swung until it has raised a height h, and then it is released. When it is straight down it collides *elastically* with a cart (initially at rest) also of mass M. The motion sensors give the speeds of the pendulum and cart before and after the collision, which takes Δt s.

The initial pendulum position is indicated by a dashed figure, the solid one is at the collision.



Values: $M = 0.500$ kg, $h = 0.0080$ m, $L = 0.500$ m, and $\Delta t = 0.050$ sec. Imagine the uncertainties are very small.

12. The speed of the pendulum just before the elastic collision is (positive is toward the left)
Energy is conserved, with the change in potential energy converting into kinetic energy. Thus $(1/2)MV^2 = Mgh$ with V the final velocity, and h the initial height.
Thus $V^2 = 2gh = 2 \times 9.80 \times 0.0080$, so $V = 0.40$ m/s Answer E.

13. The speed of the pendulum just after the elastic collision is
The mass of cart and pendulum are equal. Therefore in an elastic collision all the momentum and kinetic energy is transferred from the pendulum to the cart, leaving the pendulum with V=0. Answer B.

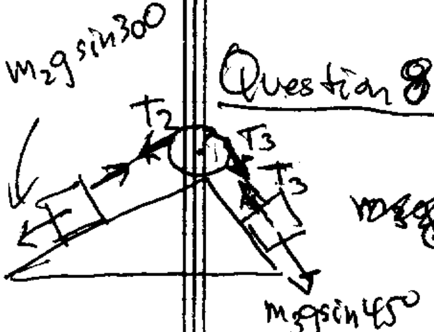
14. The magnitude of the average torque about **P** exerted by the cart on the pendulum during the collision is

Torque, τ, about **P** is LF_x so $\langle\tau\rangle = L\langle F_x\rangle = L\,\Delta P_z/\Delta t = L\,MV/\Delta t = 0.500\ 0.500\ 0.40 / 0.05 = 2.0$ N-m Answer B. $\Delta P = MV$ because in an elastic collision of equal masses, all the momentum and energy are transferred from one mass to the other. The value for V comes from question 12.

15. Now imagine the uncertainty in Δt is 0.015 sec. The uncertainties in M, L, h and g are still very small. What is the *fractional* uncertainty in the calculated value of the average torque?
The average torque is inversely proportional to Δt, so the fractional uncertainties are equal.
$\delta\Delta t/\Delta t = 0.015/0.050 = 0.30$ Answer C

A.) 0.020 B.) 0.028 C.) 0.30 D.) 0.35 E.) much smaller than those

16. When is the tension in the string of the pendulum the greatest?
The tension in the string includes the component of the weight along the string, plus the centripetal force. Both of these have maximum values immediately before the collision. Answer B



Question 8 – The two tensions are not equal – Now Using positive along track.

$$\sin 45^\circ m_3 g - T_3 = m_3 a \qquad (1)$$

$$T_2 - m_2 g \sin 30^\circ = m_2 a \qquad (2)$$

$$\cancel{R} T_3 - \cancel{R} T_2 = \frac{1}{2} m_1 R^{\cancel{2}} \frac{a}{\cancel{R}} \qquad (3)$$

Using (1) and (2) to replace in (3)

$$m_3 g \sin 45^\circ - m_3 a - m_2 a - m_2 g \sin 30^\circ = \frac{1}{2} m_1 a$$

$$\frac{g(m_3 \sin 45^\circ - m_2 \sin 30^\circ)}{m_3 + m_2 + \frac{1}{2} m_1} = a$$

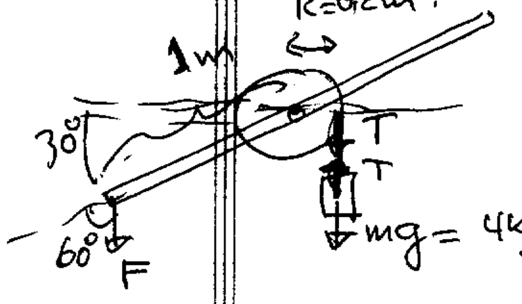
$$a = \frac{9.8\,(3 \sin 45^\circ - 2 \sin 30^\circ)}{3 + 2 + 0.5} = 1.998\,m/s^2$$

$$\simeq 2\,m/s^2$$

m_2 still goes <u>up</u>   A

Question 9   B

Total mechanical energy = KE + PE
So this is conserved – The mass m_3 will go down while mass m_2 (smaller) will go up. – Loss of Potential Energy will turn into a gain of Kinetic energy –



Question 10   D

In equilibrium,

$$F \times 1m \sin 60^\circ = mgR$$

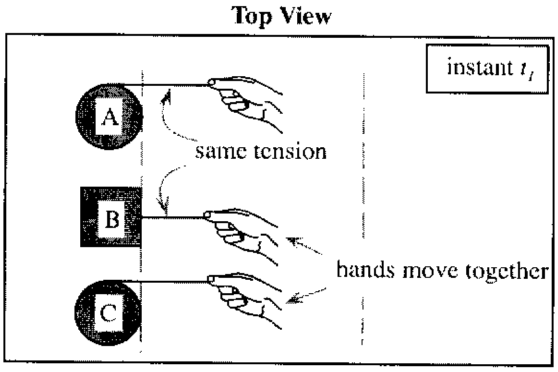
$$F = \frac{mgR}{1 \sin 60^\circ} = \frac{0.2kg \times 9.8 m/s^2 \times 0.2m}{1m \sin 60^\circ}$$

$$= 0.45N$$

Name ______________________________ **Student ID** ____________ **Score**______
last · *first*

III. [20 pts total] Three equal mass objects, Spool A, Block B, and Spool C, are each pulled across a frictionless table by a string. The strings pulling the spools are wrapped many times around the spools and may unwind as they are pulled. The strings pulling Spool A and Block B are pulled so that they have the same tension; the strings pulling Block B and Spool C are pulled so that the hands move together.

All objects start from rest. All strings start pulling at instant t_1. At instant t_2, Block B crosses the finish line.



17. [5 pts] Which of these options describes the motion of Spool A?

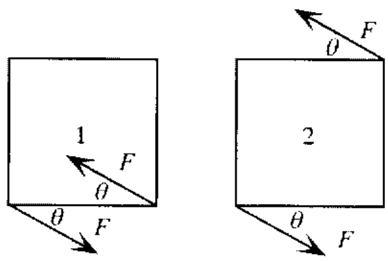
C. A rotates; A crosses the finish line at instant t_2.
The tension applied to spool A creates a torque on the spool about its center of mass, because the radius vector from the CM to the point where the force is applied is not parallel to the force itself. Spool A crosses the finish line at the same instant as block B because the net force on them is the same and they have the same mass, so they have the same center-of-mass acceleration.

18. [5 pts] Which of the following is a correct ranking of the work done on each object by its string between instants t_1 and t_2? (Let W_A stand for the work done on Spool A by String A, *etc.*)

C. $W_A > W_B > W_C$
The work done on A is greater than the work done on B, because the amount by which the energy of A increases is greater than the amount by which the energy of B increases. At the finish line, they have the same translational KE, while A will also have rotational KE. Alternatively, hand A does more work than hand B, since it travels a longer distance than hand B while exerting the same force. (Hand A travels a longer distance than spool B because some of the thread is let out.) The work done by each hand on the string is the same amount as the work done by each string on the object (A,B, or C) since the string cannot store energy.

The work done on B is greater than the work done on C. Hand B and hand C move the same distance, but the force exerted by hand C must be less than the force exerted by hand B. This is because, as spool C unwinds, it must lag behind block B. If it accelerates less quickly, the net force on it must be less.

Two identical square blocks of uniform mass density are on a horizontal, frictionless surface. The top-view diagram at right shows the horizontal forces exerted on each block.



All forces have the same magnitude F. All angles marked have the same value θ.

<u>At the instant shown:</u>

19. [5 pts] Is the magnitude of the angular acceleration of block 1 (α_1) about its center of mass *greater than, less than,* or *equal to* the magnitude of the angular acceleration of block 2 (α_2) about its center of mass?

B. Less than: $\alpha_1 < \alpha_2$
The net torque on block 1 about its center of mass is less than that on block 2. This is because the force that is up-and-to-the-left on block 1 is more closely directed at the center of mass than the up-and-to-

the-left force that is on block 2 (the perpendicular moment arm for that force is less on block 1 than on block 2). Since the blocks are identical, the block with the greater net torque has the greater angular acceleration.

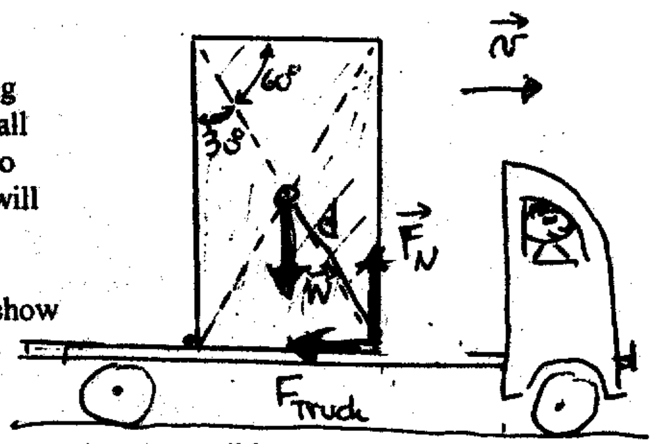
20. [5 pts] Is the magnitude of the acceleration of the center of mass of block 1 ($a_{cm,1}$) *greater than, less than,* or *equal to* the magnitude of the acceleration of the center of mass of block 2 ($a_{cm,2}$)?

    C. Equal to: $a_{cm,1} = a_{cm,2}$

The applied forces on each block are equal in magnitude and opposite in direction, so the net force on each block is zero. Since the net force on each block is zero, the center-of-mass accelerations are both zero. The place where the force is exerted makes no difference to Newton's 2nd law.

Physics 121B, Final Exam, Winter 2006 Name.............................

IV. (25 points).



1. A truck moving at constant velocity (20 m/s) is carrying a box of mass m = 200 kg. The box is secured by two small stops, A and B, so it can not slide. The car suddenly has to put the brakes to stop at the maximum possible rate that will not tip the box.

i . (3 points). Draw vectors in the picture at right which show all the forces acting on the box when the brakes are on as described above, drawn at the point they act.

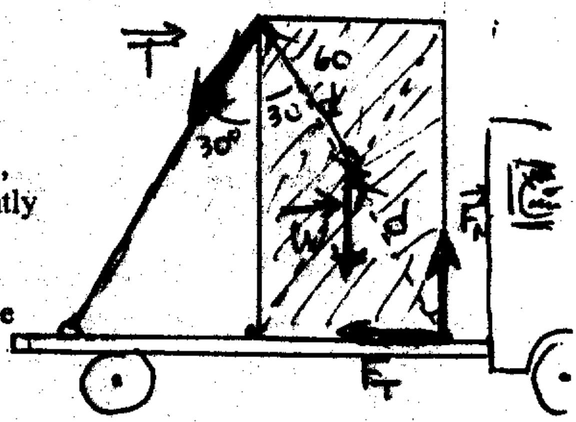
SEE FIGURE

ii. (8 points). Calculate the magnitude and direction of the maximum possible acceleration that will not tip the box. Positive x-direction is the direction of travel of the truck. The box will have to "rotate" to tip.

About the center of mass $F_N d \sin 30° = F_{truck} d \sin 60°$. (about C of M)

$F_N = W = mg$, $F_{truck} = ma$.

So $a = \frac{F_{truck}}{m} = \frac{\cancel{m}g \sin 30°}{\sin 60° \cancel{m}} = 5.66\ m/s^2$ in the $\underline{-x \text{ direction}}$. $\rightarrow \vec{v}$



2. After the difficult experience above, a cable is added to prevent the box from tilting forward. The cable does not provide any tension when the truck is moving at constant velocity. The driver now suddenly has to apply the brakes again, this time resulting in an acceleration a = - 12 m/s^2, larger than the one allowed in part (ii). The crate lifts very slightly above the rear stop (b).

iii. (4 points) Draw vectors in the picture at right showing all the forces acting on the box, at the point in which they act, under this extreme braking condition. Label or name the forces.

iv. (6 points) Write down all the necessary force and torque equations needed to calculate all the unknown forces acting on the crate (normal, truck, tension). The only things known are m = 200 kg, and the acceleration of the truck. Do not solve for the forces here!

$$\Sigma F_x = -T \sin 30° - F_T = ma$$
$$\Sigma F_y = -T \cos 30° - mg + F_N = 0$$
$$\Sigma \tau = 0 \text{ (about C of Mass)} = T d \sin 60° + F_N d \sin 30° - F_T d \sin 60° = 0$$

v. (4 points) Is the normal force on the crate under these braking conditions larger, equal, or smaller than the one experienced by the box on part 1(ii) above? Write a short sentence justifying your answer.

F_N is larger because it compensates for $(T \cos 30°)$ and $\vec{W}$)

~~the~~ W has not changed.

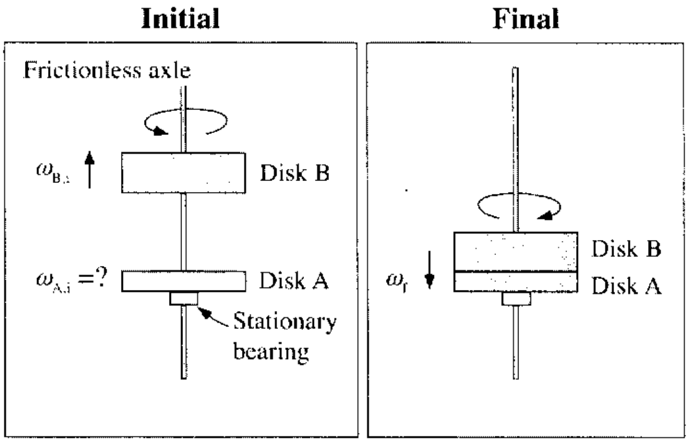
Name (please print): ______________________________Student ID:___________Total Points:_____

(last) *(first)*

V. [25 points] Disk A is spinning on a vertical frictionless axle with an *unknown* initial angular velocity $\omega_{A,i}$. The moment of inertia of disk A about this axis is $I_A = 1\ \text{kg}\cdot\text{m}^2$.

A second disk, B, rotates about the same axis. Disk B has moment of inertia $I_B = 2\ \text{kg}\cdot\text{m}^2$ and initial angular velocity $\omega_{B,i} = 2$ rad/s, directed upward (counter-clockwise, when viewed from above).

Disk B is dropped onto disk A. Disk A is prevented from sliding down the axle by a frictionless bearing. After the collision, the disks rotate together with the same final angular velocity $\omega_f = 2$ rad/s, directed downward (clockwise, when viewed from above).



Side view

For the following questions, all torque and angular momentum vectors are taken ***with respect to the axle.***

1. [6 pts] Determine the magnitude and direction of the angular momentum of the system of both disks <u>after</u> the collision. Show your work.

*$L_{system,f} = I_{total}\omega_f$ = (1 kg*m^2 + 2 kg*m^2)*(2 rad/s, down) = 6 kg*m^2/s, down*

2. [6 pts] What is the direction of the torque on <u>disk A</u> by <u>disk B</u> during the collision? If this torque is zero, state so explicitly. Explain.

The direction of the torque on A by B is opposite the direction of the torque on B by A. The direction of the torque on B by A is the same direction as its change in angular momentum. Since the initial angular momentum of B is up and the final angular momentum of B is down, the change in its angular momentum vector must point down. Therefore, the torque on B by A is down, and the torque on A by B is up.

3. [7 pts] Determine the magnitude and direction of the initial angular velocity of disk A ($\omega_{A,i}$). Explain.

Because there is no external torque on the system of both disks, the total angular momentum of the system must stay the same throughout the collision:

*$L_{system,f}$ = 6 kg*m^2/s, down = $L_{system,i}$*
*$L_{system,i} = L_{A,i} + L_{B,i} = I_A\omega_{A,i} + I_B\omega_{B,i}$ = (1 kg*m^2)*$\omega_{A,i}$ + (2 kg*m^2)*(2 rad/s, up)*
$\Rightarrow \omega_{A,i}$ = 10 rad/s, down

4. [6 pts] Of the following options, circle the one that best describes how the rotational kinetic energy of the system of both disks is changing during the collision. Explain your choice.

ii. It is decreasing.

The total energy of the system of both disks must stay the same throughout the collision, because there are no external forces doing work on the system. There is kinetic friction between the disks at every instant during the collision. Thus, thermal energy must be increasing during the whole process, so the rotational kinetic energy must be decreasing.