Phys 121A EXAM 03/~~13~~/06

I

Solution to multiple choice:

$x = 10 - 5t + 3t^2$ (1)

$y = 5 + 5t$ (2)

Question 1 - The equation (1) is the equation for motion with constant acceleration ($x = x_0 + v_0 t + \frac{1}{2}at^2$) and equation (2) is the one for motion with constant velocity, $y = y_0 + v_0 t$) — (B)

Question 2 -

This is like a projectile motion, except that the acceleration is positive and in the x direction - Trajectory is still a parabola, which at $t=0$ goes through $x = 10\,m$, $y = 5\,m$ - y is always increasing so answer has to be A or B, but since a_x is constant(+) answer is (A) -

Question 3

$$a = a_x \text{ , and } \frac{1}{2}a_x = 3\,m/s^2 \Rightarrow a_x = 6\,m/s^2$$

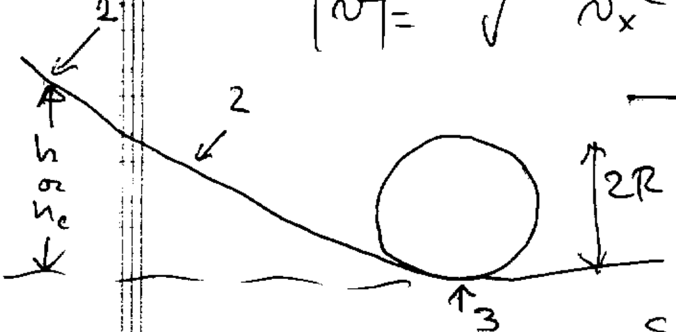
(C)

Question 4 Here we have to calculate.

$$v_x = \frac{dx}{dt} = -5 + 6t \Rightarrow \text{at } t = 3s \Rightarrow v_x = +13\,m/s$$

$$v_y = \frac{dy}{dt} = 5\,m/s \text{ (constant)}.$$

$$|v| = \sqrt{v_x^2 + v_y^2} = +13.9\,m/s \quad (A)$$



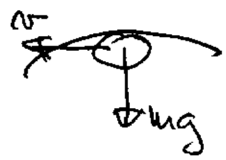
Question 5 - The forces on the hoop are $\vec{F}_W$, $\vec{F}_N$ and $\vec{F}_S$, this last one since the hoop does not slip (or slide) (D)

Question 6, The objects rotate about their Center of Mass (or an axis through their center of mass) in the same direction, clockwise according to picture at position 2. So all vector quantities point in same direction and according to convention into plane of top figure or back of picture in question. (B)

Question 7 The one with the smallest moment of inertia arrives first. So Sphere, Cylinder, Hoop is the order. (S, C, H)

(D)

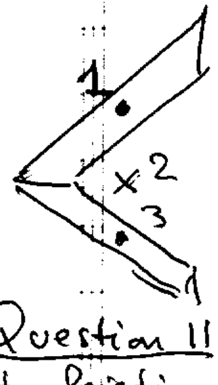
Question 8, "Falling" from the top of the loop depends on the translational velocity and the radius of the loop.



$$mg = m\frac{v^2}{R_{loop}} \Rightarrow v = \sqrt{R_{loop}\, g} \quad \text{and}$$

is the same for all objects. Since the potential energy is also the same for all objects the one that can move the fastest on question 7 can afford to start lowest. $h_S < h_C < h_H$ (B)

Question 9 If we think of this piece as two parts, the top one has C of M at 1 and the lower one at 3, so the C of M of both halves is at 2 (B)



Question 10

The explosion is an internal force, so the $\vec{P}_{system}$ is conserved and $\vec{V}_{cm} = \text{constant}$. (A)

Question 11 Linear Momentum decreases during explosion due to friction, an external force, by $\int F_{friction} \Delta t$. (D)

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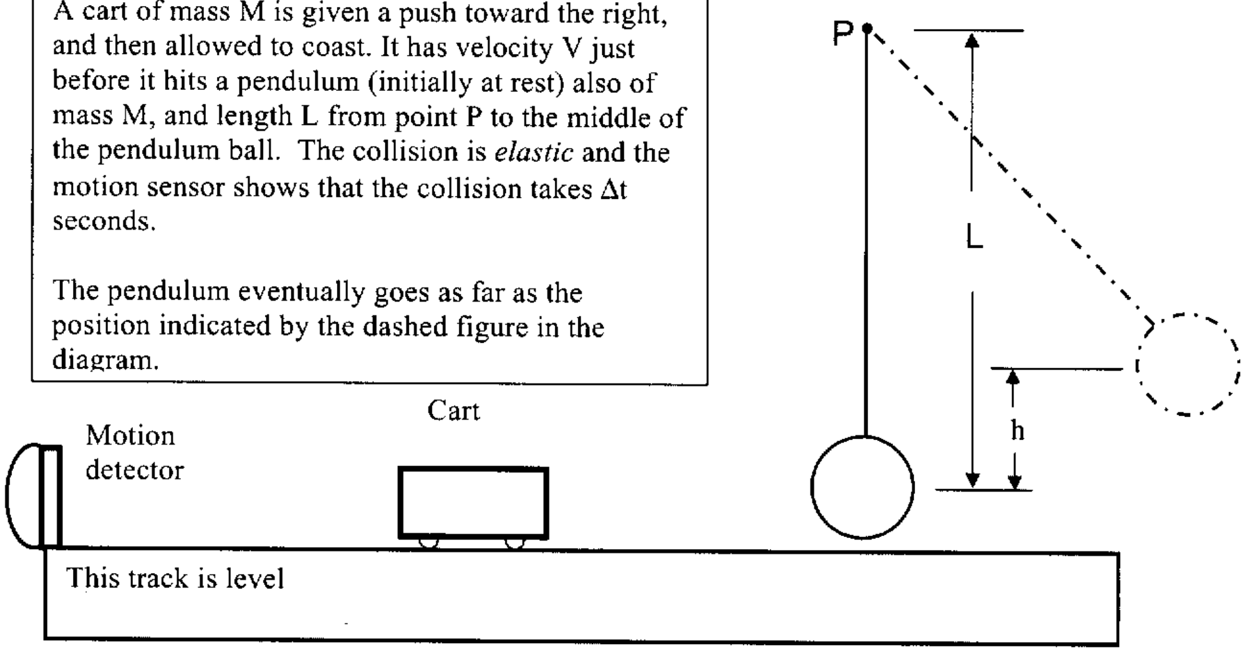
last first

II. (25 pts)

A cart of mass M is given a push toward the right, and then allowed to coast. It has velocity V just before it hits a pendulum (initially at rest) also of mass M, and length L from point P to the middle of the pendulum ball. The collision is *elastic* and the motion sensor shows that the collision takes Δt seconds.

The pendulum eventually goes as far as the position indicated by the dashed figure in the diagram.



Values: $M = 0.500$ kg, $V = 0.400$ m/s, $L = 0.500$ m, and $\Delta t = 0.050$ sec. Imagine the uncertainties are very small.

12. The speed of the cart after the elastic collision is
 The mass of cart and pendulum are equal. Therefore in an elastic collision all the momentum and kinetic energy is transferred from the cart to the pendulum, leaving the cart with V=0. Answer B.

13. The average force exerted by the cart on the ball is
 $F=\Delta P/\Delta t = MV/\Delta t = 0.500 \times 0.400/0.050 = 4.0$ N -- only 2 significant figures, since Δt is given to only two significant figures. Answer B.

14. The pendulum swings until the height h is
 Energy is conserved, with initial kinetic energy converting into a change in potential energy. Thus $(1/2)MV^2 = Mgh$ with V the initial velocity, and h the final height.
 Thus $h = V^2/(2g) = 0.400^2/(2 \times 9.80) = 0.00816$ m $= 0.816$ cm Answer B.

15. Now imagine the uncertainty in V is 0.008 m/s and that in Δt is 0.015 sec. The uncertainties in M, L and g are still very small. What is the *fractional* uncertainty in the calculated value of h?
 h is proportional to V^2 so the fractional uncertainty in h is $.2 \times (\delta V/V)$
 Δt does not enter into the computation of V so its uncertainty doesn't contribute.

Note: Everyone got credit here – Answer should have been $\frac{2 \times 0.008 \text{ m/s}}{0.400} = 0.040$, or 4% but there was no such a choice

16. When is the tension in the string of the pendulum the greatest?
 The tension in the string includes the component of the weight along the string, plus the centripetal force. Both of these have maximum values immediately after the collision. Answer B

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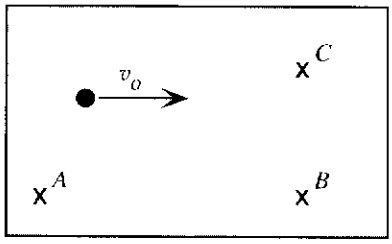
III. [20 points] A small ball moves in a weightless, frictionless environment. The ball moves with constant velocity v_o, to the right (without spinning).

Three locations (all in the plane of the page) are marked: points *A*, *B*, and *C*. One dashed line shows the straight-line path of the ball; other dashed lines are drawn to help show the locations of points *A*, *B*, and *C*.



Let "$\vec{L}_{\text{ball,A}}$" represent the angular momentum of the ball with respect to point *A*, "$\vec{L}_{\text{ball,B}}$" represent the angular momentum of the ball with respect to point *B*, *etc.*

For the instant shown in the figure:

17. [5 pts] Is the magnitude of $\vec{L}_{\text{ball,A}}$ *greater than*, *less than*, or *equal to* the magnitude of $\vec{L}_{\text{ball,B}}$?

C. Equal to, and both are **not** zero

Using $\vec{L} = \vec{r} \times \vec{p}$, the component of r (the vector pointing from the reference point to the ball) that is perpendicular to the linear momentum p is the same for points A and B.

18. [5 pts] Is the magnitude of $\vec{L}_{\text{ball,B}}$ *greater than*, *less than*, or *equal to* the magnitude of $\vec{L}_{\text{ball,C}}$?

A. Greater than

Using reasoning similar to that in question 17, the component of r perpendicular to p is greater for point B than for point C.

19. [5 pts] Which of the following best describes the directions of $\vec{L}_{\text{ball,A}}$, $\vec{L}_{\text{ball,B}}$, and $\vec{L}_{\text{ball,C}}$?

D. Two of these vectors point into the page, and one of them points out of the page.

The component of r that is perpendicular to p is directed up the page for points A and B, and down the page for point C. Using the right-hand rule, (up)X(right)=(into the page), and (down)X(right)=(out of the page).

20. [5 pts] Which of the following options describes how the magnitudes of $\vec{L}_{\text{ball,A}}$ and $\vec{L}_{\text{ball,B}}$ are changing in time?

A. They are both constant.

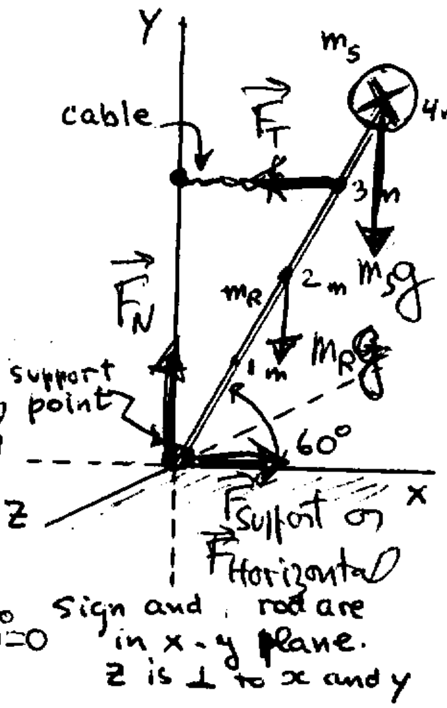
The magnitude of $\vec{L}$ is the product of $|\vec{p}|$ and the component of $\vec{r}$ that is perpendicular to $\vec{p}$. Neither of these quantities changes in magnitude or direction.

Physics 121A, Final Exam, Winter 2006 Name: OEV Solution

5a

Problem IV (25 points).

1. A long rigid rod of length L = 4 m and mass m_R = 5 kg is held by a support against the ground and by a horizontal cable attached to the rod 3 m away from the ground. A sign of mass m_S = 20 kg rigidly is attached to the rod. The rod makes an angle of 60 degrees with the ground (the horizontal). For later reference, I_R (center of mass) = (1/12) $m_R L^2$, I_R (end) = (1/3) $m_R L^2$.



a. (5 points) Draw on the picture at right all the forces acting on the rod and label them.

[It is possible also to have a single force on the rod from support point ground instead of F_N and F_H

b. (6 points) Write down all the equations necessary to calculate all the forces acting on the rod given the information above (static equilibrium, do not solve the equations)

$$\sum F_x = 0 \quad -F_T + F_H = 0 \quad (\text{or } F_R \cos 60° = F_H$$

$$\sum F_y = 0 \quad F_N - m_S g - m_R g = 0$$

$$\sum \tau = 0 \quad F_T \times 3m \sin 60° - m_S g\, 4m \sin 30° - m_R g\, 2m \sin 30° = 0$$

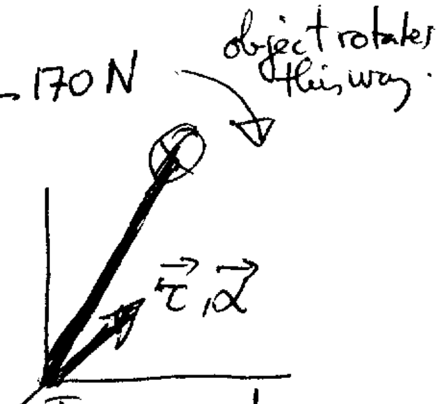
← Torque about point on ground.

c. (5 points) Calculate the magnitude of the tension force on the horizontal cable. Show your work.

Use the torque equation.

$$F_T = \frac{(m_S \times 4m + m_R \times 2m) g \sin\theta}{3m \sin 60°} = \frac{(20kg \times 4m + 5kg \times 2m)\, 9.8\, m/s^2 \sin 30°}{3m \sin 60°} = 169 \text{ or } 170 N$$

2. The horizontal cable snaps!



a. (2 points) The axis of rotation of the rod plus sign is at the end on the ground. At that point, draw in the figure above a vector showing the angular acceleration vector **α** and write down its direction.

Z

Torque and angular acceleration point in −Z direction, or into page (clockwise acceleration)

b. (4 points) Calculate the moment of inertia of the rod plus sign about the axis of rotation (show your work).

$$I_R + I_S = \frac{1}{3} m_R L^2 + m_S L^2 = \left(\frac{1}{3} m_R + m_S\right) L^2 = \left(\frac{1}{3} 5kg + 20kg\right)(4m)^2 =$$

c. (3 points) Calculate the total initial torque that makes the rod and sign falls.

$$\tau_{initial} = -2m\,(m_R g) \sin 30° - 4m\,(m_S g) \sin 30°.$$

$$= -2 \times 5 \times 9.8 \times \sin 30° - 4 \times 20 \times 9.8 \times \sin 30° = \boxed{-441 \text{ Nm}}$$

or 441 N clockwise rotation.

6a

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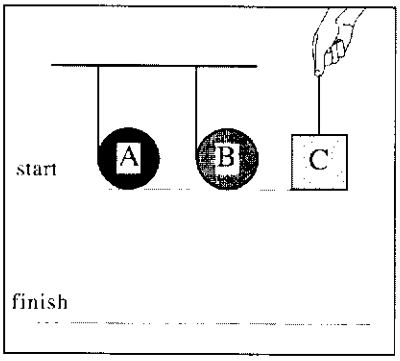
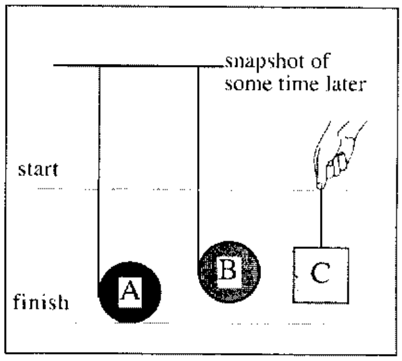
last *first*

V. [25 points] Three objects, A, B, and C, of <u>equal mass</u>, start moving downward from rest at the same time from the same height. Objects A and B have the same radius. However, the mass inside them is distributed differently, so they have different moments of inertia ($I_A \neq I_B$).

Side view



Side view



A light thread is attached to each object. The threads attached to objects A and B are wrapped many times around them and fixed to the ceiling. The hand pulls on the thread on object C so that object C and object B have the *same* vertical position *at every instant.*

Object A reaches the finish line first; B and C reach the finish line at the <u>same time</u>, after object A.

A. [6 pts] Rank, from greatest to least, the magnitudes of the acceleration of the center of mass of each object. If any of these are equal, state so explicitly. Explain.

A>B=C. All three objects have constant acceleration (because the forces on them after they start moving don't change with time), start from rest, and travel the same distance (d). Therefore, the motion of each object is described by $(1/2)a_{cm}t^2 = d$. The time (t) is least for object A, so A's acceleration is greatest. B and C take equal time, so they have the same acceleration.

B. [6 pts] Rank, from greatest to least, the magnitude of the tension force exerted on each object by the thread. If any of these are equal, state so explicitly. Explain.

B=C>A. Newton's second law says that the net force on an object is equal to its mass times the acceleration of its center of mass. The objects all have the same weight force (mg, downward), so the different accelerations must be due to different tension forces. Since the tension is opposing the motion in each case, greater acceleration of the center of mass means less tension force.

C. [6 pts] Is the angular speed of object A when it reaches the finish line *greater than, less than* or *equal to* the angular speed of object B when it reaches the finish line? Explain.

Greater than. The (vertical part of the) strings attached to objects A and B don't move, so both objects A and B move in the same manner as rolling-without-slipping, which means that $v_{cm} = \omega r$ for each object at every instant. The velocity of the center of mass (v_{cm}) is greater for object A when it reaches the bottom (because it travels the same distance with greater acceleration: $(v_{cm,f})^2 = 2a_{cm}d$). Objects A and B have the same radius, so $\omega_f = v_{cm,f}/r$ is greater for object A.

D. [7 pts] Is the rotational kinetic energy of object A when it reaches the finish line *greater than, less than,* or *equal to* the rotational kinetic energy of object B when it reaches the finish line? Explain.

Less than. Object A has greater v_{cm} at the finish line. Therefore, A has greater translational kinetic energy at the bottom than B. The total kinetic energy of A at the bottom is equal to the total kinetic energy of B at the bottom (since the work done by the gravitational force is the same, and the tension force did zero work, the change in total energy of the two spools must be the same.) Therefore A must have less rotational kinetic energy than B.