

Question 1 - (C)

$\Delta p = \int F dt \equiv$  area under force vs time graph -

Ball 1, area  $\frac{1}{2} (0.3 - 0.1) s \times 100 N = 10 \text{ kg m/s}$

Ball 2, area  $25 N \times 0.4 s = 10 \text{ kg m/s}$

$\Delta p_1 = \Delta p_2$

Question 2 (E)

Past 0.4 s ball 2 moves with constant velocity ( $F_{net} = 0$ )

$\frac{\Delta p}{m} = v = \frac{10 \text{ kg m/s}}{0.05 \text{ kg}} = 200 \text{ m/s}$

Question 3 Since ball started at  $v = 0 \text{ m/s}$ , if  $v_{100s} = 100 \text{ m/s}$

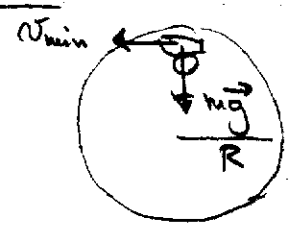
(E)

Work done by force  $F_1 =$  change in Kinetic Energy.

$= \frac{1}{2} m v_1^2 = \frac{1}{2} \times 0.05 \text{ kg} \times (100 \frac{\text{m}}{\text{s}})^2 = 250 \text{ J}$

Question 4

(D)



The minimum speed is given by  $mg = \frac{mv^2}{R}$   $v_{min} = \sqrt{Rg}$   $v_{min}^2 = Rg$

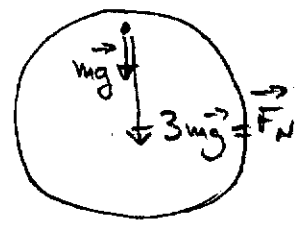
if it goes at twice the minimum speed,

$\frac{(2v_{min})^2 m}{R} = mg + F_N$

$4 \frac{v_{min}^2 R}{R} = 4 \frac{Rg m}{R} = mg + F_N$

$F_N = 4mg - mg = 3mg$

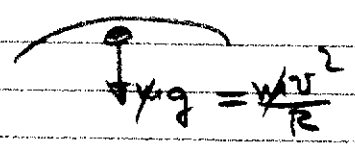
and it points down -



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Question 5

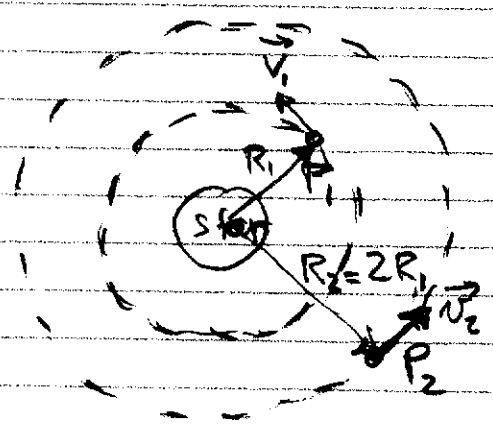
(B)  $mgH_{min} = \frac{1}{2}mv^2 + mg2R =$   
 $= \frac{1}{2}gR + 2gR = \frac{5}{2}R.$



Question 6

(A)  $mgH = \frac{1}{2}k\Delta x^2$  All (PE)<sub>initial</sub> goes into all (PE)<sub>final</sub>, it doesn't matter what happens in between if there is no friction.

Question 7 (B)



$$\frac{GM_s M_1}{R_1^2} = \frac{M_1 v_1^2}{R_1}$$

$$\frac{GM_s M_2}{R_2^2} = \frac{GM_s M_2}{2^2 R_1^2} = \frac{M_2 v_2^2}{2R_1}$$

Take ratios

$$\frac{1}{R_1^2} = \frac{v_1^2}{R_1}$$

$$\frac{1}{2R_1^2} = \frac{v_2^2}{2R_1}$$

$$v_1^2 = 2v_2^2 \Rightarrow v_1 = \sqrt{2} v_2$$

Question 8 Use Kepler's 3<sup>rd</sup> law

(C)  $\frac{R_1^3}{T_1^2} = \frac{R_2^3}{T_2^2}$   
 so  $T_1^2 = T_2^2 \frac{R_1^3}{R_2^3} = T_2^2 \frac{R_1^3}{(2R_1)^3} = \frac{1}{8} T_2^2$   
 $T_1 = (8)^{-\frac{1}{2}} T_2$  or  $T_2 = \sqrt{8} T_1$

Question 9 (B)

$$\left. \begin{aligned} G \frac{m M_P}{R_E^2} &= W_P \\ G \frac{m M_E}{R_E^2} &= W_E \end{aligned} \right\} \text{Take ratios}$$

$$\frac{M_P}{M_E} = \frac{W_P}{W_E} \Rightarrow M_P = \frac{W_P}{W_E} M_E$$

$$M_P = \frac{1000 \text{ N}}{800 \text{ N}} M_E \Rightarrow \boxed{M_P = 1.25 M_E}$$

Question 10 (C)

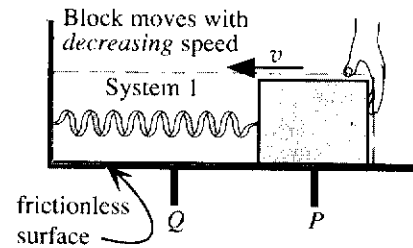
Since planet makes one rotation every 24 hours and the radius of the planet is the same as the radius of the Earth, then

$$F_{\text{net equator}} = m_{\text{person}} \left( \frac{2\pi}{T_{\text{Earth}}} \right)^2 R_{\text{Earth}}^2$$

The apparent weight (normal force) will decrease the same as on Earth, but percentage wise actually by about 0.24% -

It can not increase or decrease by a larger percentage than on Earth.

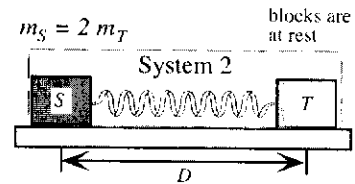
III. A. [7 pts] A block on a frictionless table is connected to an ideal spring. At time  $t_1$ , the block is at point  $P$ , moving with speed  $v$  to the left. A hand pushes the block until it reaches point  $Q$  at time  $t_2$ , compressing the spring. The block moves from point  $P$  to point  $Q$  with *decreasing* speed.



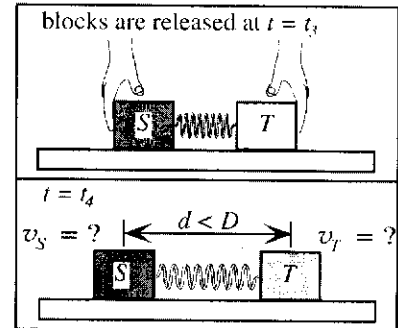
Between  $t_1$  and  $t_2$ , does the energy of the block-spring system **increase**, **decrease**, or **stay the same**?

The only external force doing work on the system is the force of the hand. (The wall does no work on the system because the point where the normal force by the wall is applied does not move, and all other forces on the system are perpendicular to the displacement of the block.) The hand does positive work on the block, because it exerts a force in the same direction that the block is moving. Since the net work on the system is positive, the energy of the system must increase.

B. System 2 consists of two blocks, S and T, and a spring. The mass of block S is twice that of block T. They are on a frictionless table. Initially, the blocks are at rest a distance  $D$  apart.



The blocks are now pushed together, compressing the spring. They are then released from rest at time  $t_3$ . At time  $t_4$ , the blocks are a distance  $d$  apart ( $d < D$ ) and moving away from each other with speeds  $v_S$  and  $v_T$ , respectively.



i. [7 pts] Calculate the ratio  $v_S / v_T$ . Show your work.

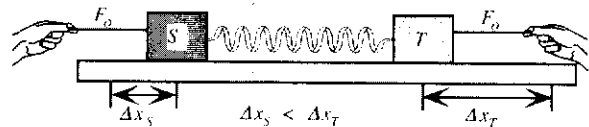
Since there is no net force on system A, the momentum of the system must remain constant.

$$\begin{aligned} \vec{p}_{2,i} &= \vec{p}_{2,f} \\ 0 &= \vec{p}_{S,f} + \vec{p}_{T,f} \\ 0 &= m_S \vec{v}_{S,f} + m_T \vec{v}_{T,f} \\ m_T / m_S &= v_S / v_T \\ 1/2 &= v_S / v_T \end{aligned}$$

ii. [5 pts] Between  $t_3$  and  $t_4$ , is the absolute value of the change in kinetic energy of block S ( $\Delta K_S$ ) **greater than**, **less than**, or **equal to** the absolute value of the change in potential energy of system 2 ( $\Delta U$ )? Explain.

Since no external forces do work on system A, the total energy of the system must remain constant. At time  $t = t_3$ , the energy of the system is entirely potential energy. Between times  $t_1$  and  $t_2$ , some of the energy changes from potential to kinetic:  $\Delta E = \Delta U + \Delta K_S + \Delta K_T \Rightarrow -\Delta U = \Delta K_S + \Delta K_T$ . Since both block S and block T have increasing kinetic energy (i.e.,  $\Delta K$  is positive for both blocks), the change in kinetic energy of either block must be **less than** the change in potential energy of the system.

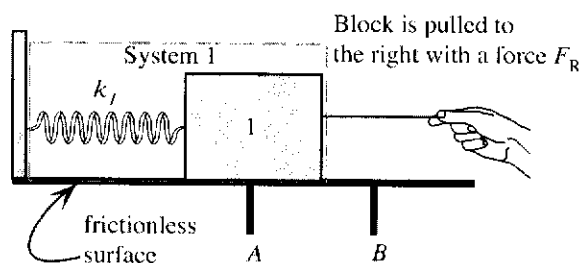
C. [6 pts] Later, the blocks are again at rest, a distance  $D$  apart. At time  $t_5$ , two strings begin pulling the blocks apart as shown. At time  $t_6$ , block S has moved to the left a distance  $\Delta x_S$ , and block T has moved to the right a distance  $\Delta x_T$ .



Between  $t_5$  and  $t_6$ , is the absolute value of the net work on system 2 **greater than**, **less than**, or **equal to** the quantity  $|F_0 \Delta x_T|$ ? If the net work on system 2 is zero, state so explicitly. Explain.

The total work on system A is  $W_{net,A} = W_{S,string} + W_{T,string} = |F_0 \Delta x_S| + |F_0 \Delta x_T|$ . Both of these terms are positive, so  $W_{net,A}$  must be **greater than**  $|F_0 \Delta x_T|$ .

III. A. Block 1 sits on a frictionless table and is connected to spring 1, an ideal spring of negligible mass and constant  $k_1$ . A rope exerts a constant force  $F_R$  on the block, as shown. At time  $t_1$ , the block is at rest at point A and the spring is at its equilibrium length. The block comes to rest momentarily at point B at time  $t_2$ .



Consider system 1, consisting of block 1 and spring 1.

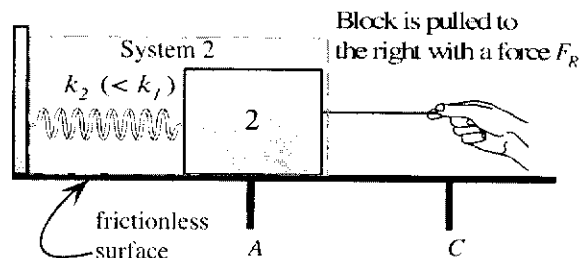
- i. [6 pts] For an instant between  $t_1$  and  $t_2$ , list all forces acting on system 1. For each force, indicate the object exerting that force and the type of force.

There are four external forces acting on system 1: The gravitational force on system 1 by the Earth, the normal force on system 1 by the surface, the tension force on system 1 by the rope, and the tension force on system 1 by the wall.

- ii. [7 pts] For each force you listed in part A, state whether the work done on system 1 by that force is *positive*, *negative*, or *zero* as the block moves from A to B. Explain.

The gravitational force by the earth and the normal force by the surface each do zero work because they act perpendicular to the displacement of the block. The tension force by the wall also does zero work, because the point at which it is applied does not move. The tension force by the rope does positive work because it is a force to the right, and the point at which it is applied moves right.

B. System 2 consists of block 2 (which is identical to block 1) and spring 2 (also of negligible mass), which has spring constant  $k_2$ .  $k_2$  is less than  $k_1$ . Block 2 is also pulled to the right by a rope exerting the same constant force  $F_R$ . Block 2 begins at rest at point A at time  $t_3$ , and comes to rest momentarily at point C at time  $t_4$ . The distance from A to C is larger than the distance from A to B.



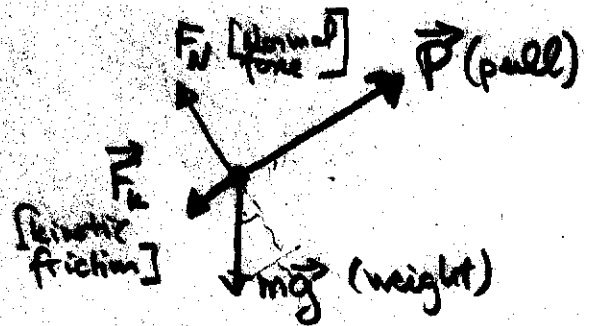
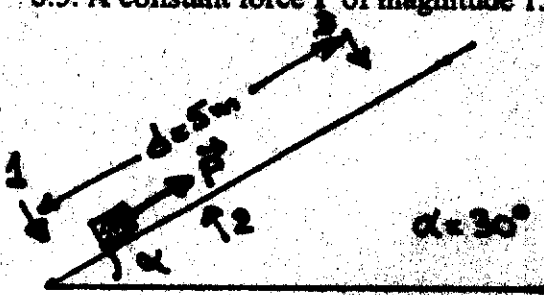
- i. [6 pts] Is the change in total energy of system 2 between  $t_3$  and  $t_4$  **greater than**, **less than**, or **equal to** the change in total energy of system 1 between  $t_1$  and  $t_2$ ? Explain.

The change in total energy of system 2 is **greater than** system 1. Both systems experience the same force from the rope, but it acts over a larger distance on system 2. This means that the rope does more work on system 2. Since work is equal to the change in total energy of the system, system 2 experiences a larger change in total energy.

- ii. [6 pts] When block 2 is at point C, is the magnitude of the force exerted on block 2 by the spring **greater than**, **less than**, or **equal to**  $F_R$ ? Explain.

The magnitude of the force on block 2 by the spring is **greater than**  $F_R$ . The block had to slow down to come to a stop at point C. Since it was moving to the left, this means that it was accelerating to the right just before it reached point C, and therefore that the net force on the block was to the right. In order for the net force on the block to be to the right, the force of the spring would have to be larger than the force from the rope. Since the spring force is larger just before reaching point C, it must also be larger at point C, since as the spring stretches, that force continues to grow larger.

**Problem 3 (25 Points).** The picture below shows a fixed inclined plane and a solid box of mass  $m = 1 \text{ kg}$ . The coefficient of friction between the box and the inclined plane is  $\mu_k = 0.5$ . A constant force  $P$  of magnitude  $15 \text{ N}$  pulls the box uphill.



a. (5 points) The dot at right represents the box when it is at position "2". At the dot, draw a free body diagram of all the forces acting on the box and label them.

b. (8 points) Calculate the work done by each one of the forces acting on the box between positions "1" (initial) and "3" (final). Show your work, not just a number with units.

Forces are all constant.

$$W_P = P \times d = 15 \text{ N} \times 5 \text{ m} = 75 \text{ Joules}$$

$$W_{F_N} = 0 \quad (\cos 90^\circ = 0) = F_N d \cos 90^\circ$$

$$W_{F_k} = |F_k| |d| \cos 180^\circ = -\mu_k mg \cos 30^\circ d = -0.5 \times 1 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times \cos 30^\circ \times 5 \text{ m} = -21.2 \text{ J}$$

$$W_{mg} = mg d \cos 120^\circ = 1 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 5 \text{ m} \cos 120^\circ = -24.5 \text{ J}$$

c. (4 points) State in your own words the content of the work-energy theorem.

The work done by the net force acting on a mass  $m$  goes into a change in the kinetic energy of that mass.

d. (8 points) Calculate the change in the kinetic energy of the box between positions "1" and "3" using the work-energy theorem. Show your work and write down any assumptions you make.

$$\Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W_{F_{\text{net}}}$$

Using the x-axis along the inclined plane, then  $F_{\text{net}}(y) = 0$

$$F_{\text{net}}(x) = P - F_k - mg \sin \alpha = 15 \text{ N} - 0.5 \times (1 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2}) \cos 30^\circ - mg \sin 30^\circ = 5.85 \text{ N}$$

in the +x direction

$$\Delta KE = F_{\text{net}} d = 5.85 \text{ N} \times 5 \text{ m} = 29.3 \text{ J}$$

This is the same result as if we added all the work in part (b)

$$75 \text{ J} - 21.2 \text{ J} - 24.5 \text{ J} = 29.3 \text{ J}$$

