Physics 116, Winter 2007

OE Vilches

Solutions to some selected problems from text, Chapter 13

$\boxed{13-5}$ 74 heartbeats/minute

$$f = 74\,\frac{\text{beats}}{\text{min}} \times \frac{1}{60\ \text{sec/min}} = 1.23\ \text{Hz}$$

$$T = \frac{1}{f} = 0.81\ \text{seconds}$$

$\boxed{13-11}$ $x = A\cos\omega t \;=\; x = A\cos 2\pi f\, t$

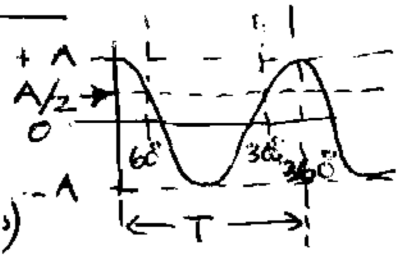
(a) $x = 3.50\cos 2\pi \times 2\times10^{14}\,\text{Hz} \times t =$

$= 3.50\cos(1.26\times10^{15})\,t$

(b) If at $t=0$ $x=0$ then $\boxed{x = A\sin\omega t}$

$[\text{or } x = A\cos(\omega t - \pi/2)]$

$\boxed{13-15}$ $x = A\cos\frac{2\pi}{T}t$



The $\cos\frac{2\pi}{T}t = 0.500$

when the angle $\frac{2\pi t}{T}$ is $60°$ (in radians)

or $300°$ (in radians)

So total for $x > \frac{A}{2}$ is time between 0 and $60°$ $(\frac{1}{3}\pi)$

and between $300°$ $(1\frac{2}{3}\pi)$ and $360°$ (2π). Total is $\boxed{\frac{1}{3}T}$

(or $\frac{1}{3}$ of 2π in angle).

$\boxed{13-19}$ Written in terms of the maximum values

$$\boxed{\begin{aligned} x &= A\cos\omega t \\ v &= -v_{max}\sin\omega t = -A\omega\sin\omega t \\ a &= -a_{max}\cos\omega t = -A\omega^2\cos\omega t \end{aligned}}$$

(a) $v_{max} = A\omega = A\,\frac{2\pi}{T}$ (1)

$a_{max} = A\omega^2 = A\,\frac{(2\pi)^2}{T^2}$ (2)

If we take the ratio

$$\frac{(2)}{(1)} = \frac{a_{max}}{v_{max}} = \frac{A\left(\frac{2\pi}{T}\right)^2}{A\left(\frac{2\pi}{T}\right)} = \frac{4\pi}{T} \Rightarrow T = \frac{2\pi\, v_{max}}{a_{max}} = \frac{2\pi \times 4.3\,\text{m/s}}{0.65\,\text{m/s}^2} = 41.6\,\text{s}$$

(b) $A = \dfrac{v_{max} T}{2\pi} = \dfrac{4.3\,m/s \times 41.6\,s}{2\pi} = 28.5\ m$

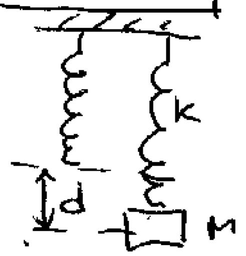
13-27 (25) The horse and rider have a mass M. At some amplitude the force of the spring down will equal the force of gravity on the horse plus rider. At that instant the rider feels no normal force (is weightless).

$$kA = Mg \quad \text{(both point down)}$$

$$A = \frac{M}{k} g = \omega^2 g$$

$$= \frac{T^2}{4\pi^2} g = \frac{(0.74\,s)^2}{4\pi^2} \times 9.8\,m/s^2 = 0.14\ m.$$

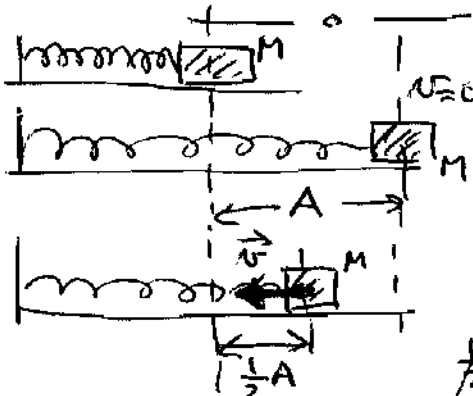
13-35 (33)



$$kd = Mg \Rightarrow d = \frac{M}{k} g = \frac{T^2}{4\pi^2} g = \left(\frac{56.7\,s}{102\,osc}\right)^2 \frac{9.8\,m/s^2}{4\pi^2}$$

$$d = 0.0767\ m$$

13-42 (40)



(a) Conservation of energy

$$E = \underbrace{\tfrac{1}{2} k A^2}_{\text{initial}} = \tfrac{1}{2} k x^2 + \tfrac{1}{2} M v^2$$

So at $x = \frac{1}{2} A$.

$$\cancel{\tfrac{1}{2}} kA^2 = \cancel{\tfrac{1}{2}} k\left(\frac{A}{2}\right)^2 + \cancel{\tfrac{1}{2}} M v^2$$

$$\frac{3}{4} kA^2 = Mv^2 \Rightarrow v_{1/2} = \left(\frac{3}{4}\frac{k}{M}\right)^{1/2} A$$

$$v_{1/2} = \left[\frac{3}{4} \times \frac{26\,N/m}{0.40\,kg}\right]^{1/2} 0.032\,m = 0.22\,m/s$$

13-43 (41) v_{max} occurs at $x = 0 \Rightarrow \cancel{\tfrac{1}{2}} kA^2 = \cancel{\tfrac{1}{2}} M v_{max}^2$

(a) $v_{max} = \sqrt{\dfrac{k}{M}}\, A = \sqrt{\dfrac{26\,N/m}{0.40\,kg}}\ 0.032\,m = 0.258\,m/s$

(b) $\cancel{\tfrac{1}{2}} kA^2 = \cancel{\tfrac{1}{2}} kx^2 + \cancel{\tfrac{1}{2}} M \left(\dfrac{0.258\,m/s}{2}\right)^2$

$$x^2 = A^2 - \frac{M}{k}\left(\frac{0.258\,m/s}{2}\right)^2 \Rightarrow x = \left[(0.032\,m)^2 - \frac{0.40\,kg}{26\,\frac{N}{m}}\left(\frac{0.258\,m/s}{2}\right)^2\right]^{1/2}$$

$$= 0.028\ m$$

13-50 (48)

This is $\frac{1}{4}T \Rightarrow t = \frac{1}{4} \times 2\pi\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{10\,m}{9.81\,\frac{m}{s^2}}} = 1.59\,s$

13-65 (61)

$$a = -(0.302\ m/s^2)\cos[2.41t]$$

(a) $\omega = 2\pi f = 2.41\frac{rad}{s} \Rightarrow f = \frac{2.41\ rad/s}{2\pi\ rad} = 0.384\ 1/s$

(b) $a_{max} = \omega^2 A \Rightarrow A = \frac{a_{max}}{\omega^2} = \frac{0.302\ m/s^2}{(2.41\ rad/s)^2} = 0.052\ m$

(b) $v_{max} = \omega A \Rightarrow v_{max} = (2.41\ rad/s)(0.052\ m) = 0.125\ m/s$

13-74 (72) (a) Referring to Fig. 13-28 (3rd edition), T = 4 seconds and $\frac{1}{4}T$ = 1 second. The maximum speed (slope) is at t = 0, 2 (negative velocity), 4 s etc.

In $\frac{1}{4}T$, $\Delta x = 0.50\ m$. The slope is higher than this, if we draw a tangent to the graph at t = 0 s. So $v \simeq \frac{0.5}{0.7\,s} > 0.5\ m/s$.

(b) $v_{max} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{4s} \times 0.5\,m = 0.79\ m/s$

(c) $E = \frac{1}{2}M v_{max}^2 = \frac{1}{2} \times 3.8\,kg \times (0.79\ m/s)^2 = 1.19$ Joules

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13-75 (73) (a) Maximum force is when spring is stretched the most (and the acceleration is the largest)

So t = 1 s, 3 s, 5 s

(b) For a spring-mass system, $T = 2\pi\sqrt{\frac{M}{k}} \Rightarrow T^2 = 4\pi^2\frac{M}{k}$

So $k = \frac{4\pi^2 M}{T^2}$

and $F_{max} = kA = \frac{4\pi^2 MA}{T^2} = \frac{4\pi^2 \times 3.8\,kg \times 0.50\,m}{(4s)^2} = 4.69\,N$

(c) 0, 2, 4, 6 seconds (the spring is not stretched)

(d) $F = kx = kA\sin(\frac{1}{4}\pi) = 4.69\,N\ \sin\frac{\pi}{4} = 3.32\,N$

$\underset{45^\circ}{\nearrow}$
