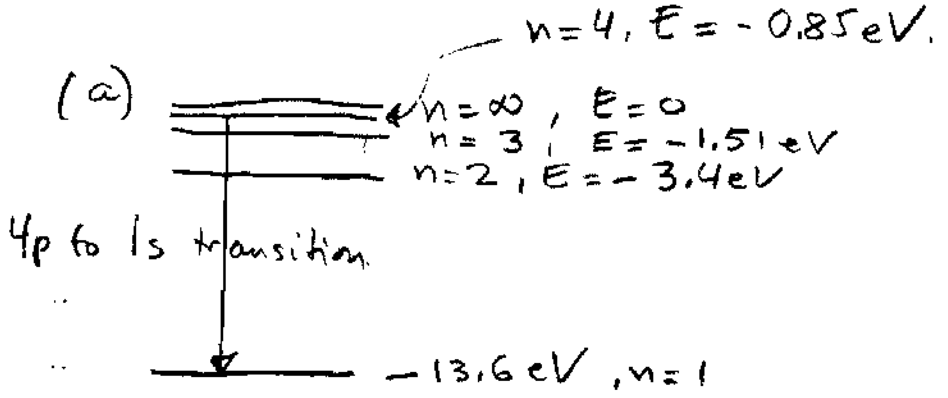
Answers to "Sample Final" from Summer 1992

OE Vildas

PROBLEM 1

(a)



(b) Atomic levels written in terms of n, ℓ quantum numbers (not done this year), $4p \rightarrow n=4, \ell=2$

$1s \rightarrow n=1, \ell=0$

(c) Wavelength:

$$\frac{hc}{\lambda} = 13.6\,eV\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = 12.75\,eV.$$

$$\lambda = \frac{hc}{12.75\,eV} = \frac{1.24\times10^{3}\,eV\,nm}{12.75\,eV} = 97.6\,nm.$$

This transition is in the ultraviolet.

(d) No - The difference between Hydrogen and Deuterium is in the nucleus - ^1_1H has one proton, ^2_1H has one proton and one neutron - In Bohr atom's derivation the nuclear mass was assumed infinite.

In reality though, there is a small difference since both the nucleus and the electron are an isolated system, so they "spin" (or orbit) about their center of mass [(M) o$_m$] - There is a difference in the

↑ C of Mass

Wavelengths of visible light emitted of about 0.5 nm. (2)
This is how ^2_1H was discovered.

Problem 2

(a)

	n	l	m	s
1	1	0	0	$-\frac{1}{2}$
2	1	0	0	$+\frac{1}{2}$
3	2	0	0	$-\frac{1}{2}$
4	2	0	0	$+\frac{1}{2}$
5	2	1	−1	$-\frac{1}{2}$
6	2	1	−1	$+\frac{1}{2}$
7	2	1	0	$-\frac{1}{2}$
8	2	1	0	$+\frac{1}{2}$
9	2	1	+1	$-\frac{1}{2}$
10	2	1	+1	$+\frac{1}{2}$

$l < n$

$-l \le m \le +l$

$s = \pm \frac{1}{2}$

No two electrons can have the same four quantum numbers: Dirac's exclusion principle

(b) See exam note.

$$E = -\frac{13.6\,eV\,Z^2}{n^2} \underset{\substack{n=1\\ l=0}}{=} -13.6\,eV(10)^2$$

Problem 3

(a) Electrons going through D have 100 eV energy or 1.6×10^{-17} Joules.

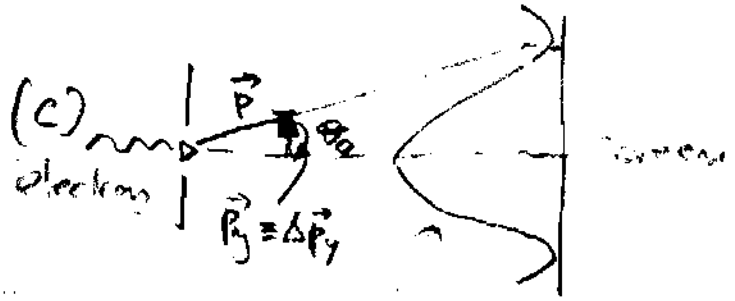
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}} = \frac{6.63\times10^{-34}\,Js}{\sqrt{2\times9.11\times10^{-31}kg\times1.6\times10^{-17}J}}$$
$$= 1.23\times10^{-10}\,m$$

(b) 1^{st} diffraction zero → $D\sin\theta = \lambda \Rightarrow D = \frac{\lambda}{\sin\theta}$.

$$\tan\theta = \frac{Y}{L} = \frac{0.05\,m}{0.3\,m} = 0.167 \Rightarrow \theta = 9.46°$$

$$D = 1.23\times10^{-10}\,m / \sin 9.46° = 7.48\times10^{-10}\,m.$$

(c)



For these electrons, at the slit, $\pm\Delta y = \frac{D}{2}$
First diffraction zero is at $D\sin\theta = \lambda = \frac{h}{p}$.

If we use $\pm\Delta p_y$ (uncertainty in p_y) as the p_y of an electron hitting the first zero of diffraction zero

$$\Delta p_y \cong p\sin\theta \Rightarrow \boxed{\Delta y\,\Delta p_y \cong \frac{D}{2} \times p\sin\theta = \cong \frac{h}{2p\sin\theta} \times p\sin\theta \simeq \frac{h}{2}}$$

The actual uncertainty can be little bit smaller

$$\Delta y\,\Delta p_y \geq \frac{h}{2\pi}$$

Problem 4

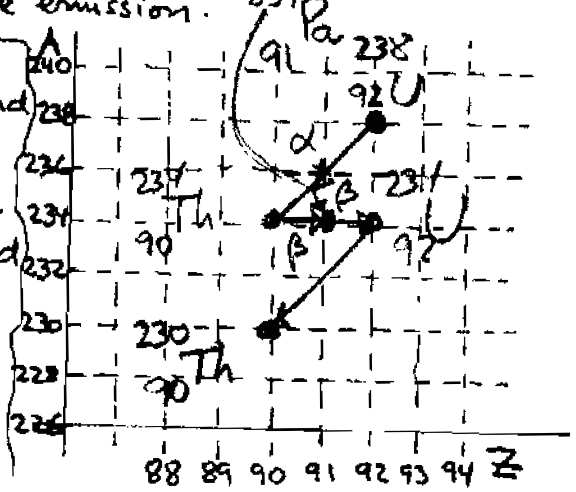
See next page

④

PROBLEM 4 (26 Points)

The $^{238}_{92}U$ decay series has this isotope decaying to $^{A'}_{Z'}Th$ by α-particle emission. This Th isotope goes to $^{A''}_{Z''}Pa$ by β emission, followed by a transformation to $^{A'''}_{Z'''}U$ by β-particle emission, and to $^{A''''}_{Z''''}Th$ by α-particle emission.

(10) (a) Find $A', A'', A''', A'''', Z', Z'', Z''', Z''''$, and write below each one of the nuclear reactions in this chain until you reach $^{A''''}_{Z''''}Th$ - Map the path followed in the A vs Z grid at right.



$^{238}_{92}U \rightarrow {}^{4}_{2}\alpha + {}^{234}_{90}Th$

$^{234}_{90}Th \rightarrow \beta^- + {}^{234}_{91}Pa$

$^{234}_{91}Pa \rightarrow \beta^- + {}^{234}_{92}U$

$^{234}_{92}U \rightarrow \alpha + {}^{230}_{90}Th$ (230 ← A'''', 90 ← Z'''') etc

(8) (b) If the half life of $^{230}_{90}Th$ was 8×10^4 years and you had half a mole of it (about 3×10^{23} atoms) how many particles do you expect to decay per second? (Calculate the number)

$$\frac{\Delta N}{\Delta t} = \frac{0.693}{T_{1/2}} N = \frac{0.693}{2.52\times10^{12}s} \times 3\times10^{23} \text{atoms} = 8.25\times10^{10}/\text{second}$$

$$1 \text{ year} = 3600\frac{s}{h} \times 24\frac{hr}{day} \times 365\frac{days}{year} = 3.15\times10^7 \text{ seconds}; \quad T_{1/2} \equiv 8\times10^4 yr \times 3.15\times10^7 \frac{s}{year} = 2.52\times10^{12} s$$

(8) (c) If the α-particles emitted in part (b) are emitted in all directions of space, calculate how many counts/second a circular detector of 1 cm diameter will record if it is located 20 cm away from the source.

This is like an "Intensity" question.

$$\frac{N}{\text{Second}} = 2.52\times10^{12}\frac{\text{alphas}}{\text{Second}} \times \frac{\pi\times(0.5\,cm)^2}{4\pi\times(20\,cm)^2} = 3.9\times10^8 \frac{\text{alphas}}{\text{Second}}$$

SCORE
PROB. 1 (27)
PROB. 2 (20)
PROB. 3 (27)
PROB. 4 (26)
Total (100)