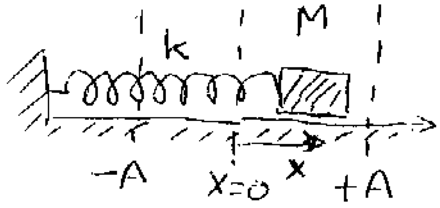
OE Vilches

The ideal spring-mass system and the ideal capacitor - inductor circuit



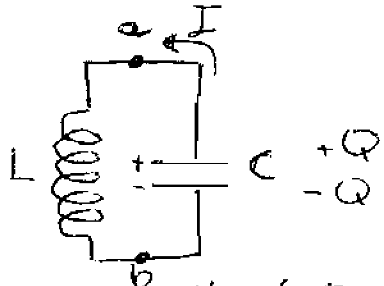
Stretching spring to +A and then releasing the mass M leads to simple harmonic oscillations.

Starting from Newton's 2nd Law

$$F = ma$$

$$\boxed{-kx = ma = m\frac{\Delta v}{\Delta t}} \quad (1)$$

with $a = \frac{\Delta v}{\Delta t}$ and $v = \frac{\Delta x}{\Delta t}$



Charging capacitor ($+Q_0$ and $-Q_0$) and then connecting to the inductor (or "coil") a current will start to flow to discharge the capacitor. A potential difference can be measured between points "a" and "b", which is the same if we travel along the circuit through L or through C. Thus one starts by

$$V_{ab}(\text{Inductor}) = V_{ab}(\text{capacitor})$$

$$-L\frac{\Delta I}{\Delta t} = \frac{Q}{C}, \text{ or}$$

writing it like (1)

$$\boxed{-\frac{Q}{C} = L\frac{\Delta I}{\Delta t}} \quad (2)$$

and $I = \frac{\Delta Q}{\Delta t}$

Equations (1) and (2), while representing different phenomena, have the same mathematical form if we use the equivalence

$$x \longrightarrow Q$$

$$k \longrightarrow \frac{1}{C}$$

$$M \longrightarrow L$$

$$a = \frac{\Delta v}{\Delta t} \longrightarrow \frac{\Delta I}{\Delta t}$$

$$v = \frac{\Delta x}{\Delta t} \longrightarrow I = \frac{\Delta Q}{\Delta t}$$

It is not surprising then that both (1) and (2) lead to mathematically identical solutions (let's neglect phase issues)

Spring - Mass

$x = A\cos(\omega t)$

$v = -A\omega\sin(\omega t) = -v_{max}\sin\omega t$

$a = \frac{\Delta v}{\Delta t} = -A\omega^2\cos(\omega t)$

$= -v_{max}\,\omega\cos(\omega t)$

$= -a_{max}\cos(\omega t)$

We found out that in equation (1)

$$\boxed{\frac{\Delta v}{\Delta t} = a = -\frac{k}{m}x = -\omega^2 x}$$

with $\omega = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$

Capacitor - Inductor

$Q = Q_0\cos(\omega t)$

$I = -Q_0\omega\sin(\omega t) = -I_{max}\sin\omega t$

$\frac{\Delta I}{\Delta t} = -Q_0\omega^2\cos(\omega t)$

$= -I_{max}\,\omega\cos(\omega t)$

→ Similarly, from equation (2)

$$\boxed{\frac{\Delta I}{\Delta t} = -\frac{1}{LC}Q = -\omega^2 x}$$

so **now** $\omega = \sqrt{\frac{1}{LC}} = 2\pi f = \frac{2\pi}{T}$

A fundamental characteristic of the spring-mass system is that one can store energy in <u>TWO</u> places: (a) Potential Energy $\frac{1}{2}kx^2$, and (b) Kinetic Energy, $\frac{1}{2}Mv^2$. The same is true for the capacitor-inductor system. Energy can be stored in the Capacitor (electric field) and the Inductor (magnetic field)

⇒ Potential Energy $\frac{1}{2}kx^2$

⇒ Kinetic Energy $\frac{1}{2}Mv^2$

$$\varepsilon_0 = 8.85\times10^{-12}\ \frac{\text{Coulomb}^2}{\text{Newt m}^2}$$

$$\mu_0 = 4\pi\times10^{-7}\ \frac{\text{Newt s}^2}{\text{Coulomb}^2}$$

$$\sqrt{\frac{1}{\varepsilon_0\mu_0}} = 3\times10^8\ \text{m/s} = c$$

⇒ Energy in capacitor: $\begin{cases} \frac{1}{2}\frac{Q^2}{C} \\ \frac{1}{2}\varepsilon_0 E^2 \times \text{Volume of Capacitor} \end{cases}$

⇒ Energy in inductor: $\frac{1}{2}\frac{B^2}{\mu_0}\times$ Volume of Inductor.

$\frac{1}{2}LI^2$

$u_E = \frac{1}{2}\varepsilon_0 E^2$ is the energy density or energy per unit volume in any electric field.

$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$ is the energy density, or energy per unit volume, in any magnetic field.

In simple harmonic oscillation the energy goes back and forth from one pot to the other.