

Answers to 1st Exam

$x = 20 \cos(\pi t + 0.25\pi)$, x in cm, t in seconds.

Q1 $\frac{2\pi}{T} = \pi \Rightarrow T = 2s$ (A)

Q2 At $t=0$ $x = 20 \cos(0.25\pi) \approx +14$ cm

Moving to right, left, or stopped? Amplitude of motion is 20 cm, so mass is moving. As time increases above zero, angle goes towards $\frac{\pi}{2}$ (90°) where $\cos 90^\circ = 0$. So mass is moving towards $x=0$ (to the left in picture) - (D)

Q3 At $t=1.25s$, $x = (20 \text{ cm}) \cos(\underbrace{1.5\pi}_{270^\circ}) = 0$ (origin)

So mass has all kinetic energy (C)

Q4 Same as Tycho problem, except with 4 springs instead of 2.

Each $\frac{1}{4}$ segment stretches as if it had a $k_{\text{effective}} = 4k$ whole spring

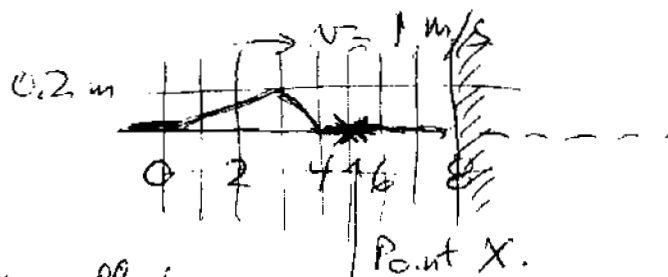
Since there are four springs with $(4k)$ spring constant, total stretch is $\frac{1}{16}$ of original $\Rightarrow \frac{1}{16} \times 0.24 \text{ m} = 0.015 \text{ m}$ (E)

Q5 For pendulum $T = 2\pi \sqrt{\frac{L}{g}}$ (independent of M)

$T_{\text{new}} = T_{\text{old}}$ (A)

Q6 $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$ $\frac{f_k}{f_E} = \sqrt{\frac{g_k}{g_E}} = \sqrt{\frac{2g_E}{g_E}} = \sqrt{2}$

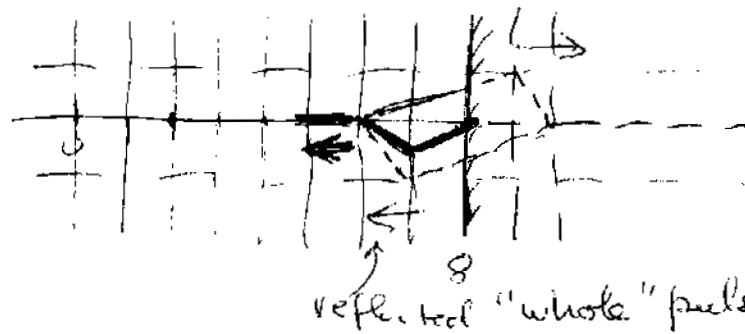
So $f_k = f_E \cdot \sqrt{2}$ (C)



(2)

Q7] Maximum speed will be when point X is going up $\Rightarrow \Delta y = 0.2 \text{ m}$, $\Delta t = 1 \text{ s}$ (from $v = 1 \text{ m/s}$)
 $v_y = \frac{\Delta y}{\Delta t} = \frac{0.2 \text{ m}}{1 \text{ s}} = 0.2 \text{ m/s}$ (A)

Q8] for $t = 6 \text{ seconds}$, take of pulse will be at $x = 6 \text{ m}$.



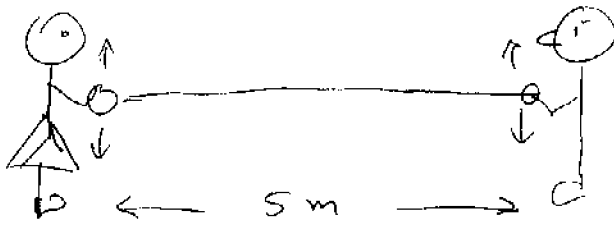
Sum of pulses going right and coming left leaving wall point fixed, in heavy lines (E)

$$\begin{aligned}
 Y_a &= 10 \cos(3x - 4t) &= 10 \cos\left[3\left(x - \frac{4}{3}t\right)\right] \\
 Y_b &= 10 \cos(5x + 4t) &= 10 \cos\left[5\left(x + \frac{4}{5}t\right)\right] \\
 Y_c &= 20 \cos(10x + 60t) &= 20 \cos\left[10\left(x + \frac{60}{10}t\right)\right] \\
 Y_d &= 20 \cos(4x - 20t) &= 20 \cos\left[4\left(x - \frac{20}{4}t\right)\right]
 \end{aligned}$$

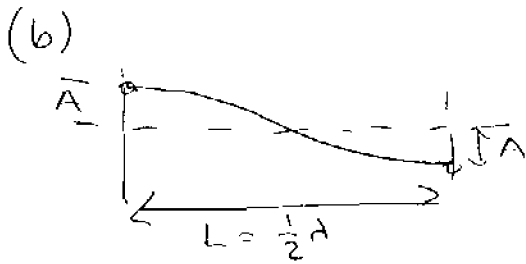
Q9] To travel in the $+x$ direction the sign in the time term has to be negative. The time coefficient is the speed of the wave. Y_a and Y_d are waves travelling in positive x -direction [another way of looking at it is that as time advances, waves that were in the negative side of $x=0$ will appear at $x=0$] (C)

Q10] $|v_{\text{wave}}| = \frac{60}{10} = 6 \frac{\text{m}}{\text{s}}$ (B)

Problem 1. $M = 0.5 \text{ kg}$ $T = 10 \text{ N}$

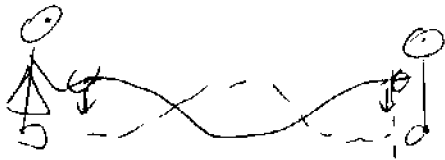


(a) $v = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{10 \text{ N}}{0.5 \text{ kg}/5 \text{ m}}} = \sqrt{\frac{10 \text{ N}}{0.1 \frac{\text{kg}}{\text{m}}}} = \sqrt{\frac{100 \text{ m}^2}{\text{s}^2}} = 10 \text{ m/s}$



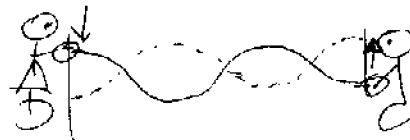
$f = \frac{v}{2L} = \frac{10 \text{ m/s}}{2 \times 5 \text{ m}} = \frac{10 \text{ m/s}}{10 \text{ m}} = 1 \text{ Hz}$

(c) 2nd harmonic
(1st overtone)



moving in the same direction.

3rd harmonic
(2nd overtone)



moving in opposite directions.

Problem 2

(a) Wavelength is longer - Usually one works with frequencies
When approaching each other the frequency heard by cyclist is

$$f_{\text{app}} = f_0 \left(\frac{1}{1 - \frac{v_p}{v_s}} \right) \left(1 + \frac{v_{\text{cyc}}}{v_s} \right) = \frac{v_s}{v_{\text{app}}}$$

When moving away from each other

$$f_{\text{away}} = f_0 \left(\frac{1}{1 + \frac{v_p}{v_s}} \right) \left(1 - \frac{v_{\text{cyc}}}{v_s} \right) < f_{\text{approach}}$$

(b) $I = \frac{400 \text{ Watts}}{4\pi R^2} = \frac{400 \text{ W}}{4\pi (20 \text{ m})^2} = 7.96 \times 10^{-2} \frac{\text{Watts}}{\text{m}^2}$

(c) $\text{db} = 10 \log_{10} \frac{7.96 \times 10^{-2} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = \underline{\underline{109 \text{ db}}}$