CHAPTER 5: 
FINITE ELEMENT ANALYSIS OF 
REINFORCED CONCRETE ELEMENTS

5.1 Introduction

The individual material models may be assembled to generate a global finite element model; however, the assembly and global solution process as well as calibration of the global model must be considered before the model is applied to the problem of analysis of beam-column connections. Introduction of the bond element requires implementation of non-standard meshing algorithms and provides an opportunity to investigate non-standard modeling techniques to represent bond zone response. Introduction of material softening requires development of a non-standard solution algorithm to ensure that the solution can be advanced in the event that multiple solution paths develop. Identification of appropriate model parameters and modeling techniques for representing the response of general reinforced concrete connections follows from comparison of observed and computed response for increasingly complex systems. A finalized model is used to investigate the response of reinforced concrete beam-column connections subjected to simulated earthquake loading.

5.2 Development of the Global Finite Element Model

The proposed finite element model incorporates non-standard element formulations, solution algorithms and material models. As a result, some modification of the standard finite element computer code is necessary to accommodate the model and to enhance representation of three dimensional reinforced concrete structural elements. The two aspects of the proposed model that require the most significant modification of the base finite element code include meshing of the model to accommodate the bond element for appropriate representation of bond-zone behavior and introduction of a solution algorithm that is appropriate for systems that include material models with degrading strength and stiffness. The proposed methods for addressing these issues are discussed in the following sections.
5.2.1 Including Bond Elements in the Global Finite Element Mesh

Introduction of the bond model into the global finite element model requires consideration of the behavior of the bond zone under multi-dimensional loading as well as the issues associated with implementation of the element into the global finite element model. As discussed in Chapter 4, the bond element is intended to represent the microscopic behavior of the concrete and reinforcing steel in the vicinity of the interface, averaged over a volume that is of the scale of the reinforcing bar. Consideration of global model development as well as prior research suggests that this representation is accomplished best through the introduction of a bond element that has zero width and a finite length. In order to represent radial forces developed in association with tangential bond forces, the model includes definition of a radial bond response mode. From this it follows that one bond element is required on either side of a reinforcing bar. Figure 5.1 shows an idealization of the node and element layout within the vicinity of the reinforcing bar. The introduction of two nodes with identical coordinates and different element connectivities may not be possible within the structure of many codes and likely requires introduction of an independent meshing algorithm in all codes.

Figure 5.1: Finite Element Mesh in the Vicinity of a Bond Zone
In order to represent the three dimensional bond-zone response within the limits of the two-dimensional model, the concept of a cover-concrete element is introduced into the model. The cover-concrete elements overlay the concrete and bond elements introduced in the previous model. The bond elements represent the response of the bond-zone concrete; this bond zone extends a limited distance around the reinforcing bar. The concrete elements in line with the bond elements represent a thickness of concrete equal to the thickness of the bond zone. The cover concrete elements represent the thickness of concrete outside the bond zone. If the radial forces associated with bond response are significant, the cover concrete could be expected to carry tension in the direction perpendicular to the axis of the reinforcing bar and develop splitting type cracks if this loading is severe. Figure 5.2 shows the finite element mesh in the vicinity of the reinforcing bar with cover concrete present. Because the cover concrete elements are not connected to the nodes that define the bond and reinforcing steel elements, deformation of the cover concrete elements is not compatible with deformation of these elements nor is it compatible with deformation of the concrete elements that are in-plane with the bond and reinforcing steel elements. This is shown in Figure 5.2

The simplest representation of concrete-steel bond is achieved by introducing a one-dimensional bond element. The one-dimensional bond element represents the bond stress versus slip response, including dependence on concrete and steel material state, but not the radial response mode. Given the limited data defining radial bond response, evaluation of such a model is appropriate in the current investigation. Figure 5.3 shows an idealization of the finite element mesh in the vicinity of such an element. It is relevant to note that introduction of the one-dimensional bond element requires introduction of only one additional, repeated node for each steel element node, while introduction of the two-dimensional bond element required two additional elements. Thus, the one-dimensional bond
element is somewhat more efficient than the two-dimensional element. Introduction of cover concrete is not appropriate in the case of a one-dimensional bond element.

5.2.2 Solution Algorithm for a System with Material Softening

The integration of strength-degrading material constitutive relationships into the global finite element model may result in a system that is defined by multiple solution paths. These paths characterize system response for various levels of damage localization. In general, solution of such systems requires modification of standard non-linear solution
algorithms to force system response onto the solution path that essentially represents the minimum total potential energy.

The occurrence and significance of multiple equilibrium states is explored most easily through evaluation of simple one-dimensional systems. Figure 5.4 shows two one-dimensional systems composed of strength-degrading elements in combination with elastic elements. Figure 5.5 shows the multiple solution paths that define the response of System 1 and System 2 in Figure 5.4 for the case of monotonically increasing displacement at Node A. For these systems, increasing displacement at Node A eventually results one or more of the elements beginning to lose strength and return a negative contribution to the global system tangent. This negative contribution to the global tangent results in a global tangent matrix that is not positive definite. Thus, at this point in the load history, the solution state is not unique and multiple solution paths exist. This is the bifurcation point.

The multiple solutions paths define various distributions of damage within the system. The Distributed Damage Path with maximum strength defines a system in which each of the strength-degrading elements exhibits the same local deformation and carries the same load. The Localized Damage Path defines system response for the case in which damage is localized in a single element. In this case, one element exhibits local element deformation in excess of that corresponding to the element strength while the remainder of the elastic and inelastic elements unload. The Localized Damage Path also may be envisioned as the response of the system if one element has an initial imperfection rendering it slightly weaker than the remainder of the elements. The Distributed Damage Paths that lie between that defining the maximum system strength and the Localized Damage Path represent system response for the case of damage distributed between two inelastic elements. Introduction of more elements into the system will result in the existence of more Distributed Damage Paths.
The Distributed and Localized Damage Paths do not necessarily follow the form suggested in Figure 5.5. A “snap-back” failure can occur if the elastic stiffness of the material is relatively soft. A “snap-back” failure defines a system in which initiation of damage results in reduced system elongation with reduced load (Figure 5.5b) rather than increased deformation. A “snap-back” response will occur if the increment of deformation associated with unloading of elastic and undamaged inelastic elements (this is a negative increment of deformation since it is in the direction opposite to that of the initial loading) is greater than the increment of deformation associated with elongation of the damaged inelastic element. Also, the existence of multiple solution paths is not limited to the case of systems with global strength degradation. Figure 5.5c shows the possible solution paths for System 2 in Figure 5.4. Here damage in the inelastic elements in parallel with the elastic element results in a system in which post-bifurcation points define an increase in system strength.

While numerical solution of the problem indicates that there are multiple solution paths, only one path represents the true response of the system that would be observed in the laboratory. This is the Localized Damage Path. The Localized Damage Path defines a system with minimum total potential energy. This system represents also the response of a system with a single imperfect element. The instability of the Distributed Damage Path and the experimentally observed response of brittle systems indicates that the Localized Damage Path defines the true system response and is the desired computed response of the finite element model.

Given the multiple solution paths that define response of these systems, it is of interest to consider the sequence of unconverged points that define the iterations between solution points. Typically, a full Newton-Raphson iteration scheme is the most efficient and robust algorithm for solution of non-linear boundary value problems with incrementally
advancing boundary conditions. Applying this solution method to the system load histories presented in Figure 5.5, may result in any combination of the solution histories shown in Figure 5.6. The computed solution path depends on the geometry of the problem, the material relationship and the load increment. Depending on the relationship of these parameters, the solution may converge to the correct state (Figure 5.6a), may converge to point on a distributed damage path (Figure 5.6b) or may oscillate indefinitely (Figure 5.6c). Further the system may not converge due to oscillation of the solution algorithm between multiple paths.

Solution of these systems depends on defining the characteristics of the bifurcation point and the possible solution paths. The behavior of these systems has been studied by a number of researchers. Crisfield [1982], De Borst [1987] and Crisfield and Wills [1988] provided exceptionally lucid and detailed discussions of the response of these systems. The bifurcation point is defined by a global system tangent with a zero eigenvalue. For most numerical applications, either the exact bifurcation point will not be discovered to be
Figure 5.5a: Multiple Solution Paths for System 1

Figure 5.5b: Multiple Solution Paths for System 1 Including a “Snap-Back” Response

Figure 5.5c: Multiple Solution Paths for System 2

Figure 5.5: Multiple Solution Paths for Systems Composed of Strength-Degrading Materials
Figure 5.6: Possible Iteration Paths for Systems with strength-degrading Elements
(Newton-Raphson Iteration Scheme)

Figure 5.6a: Convergence to Distributed Damage Path

Figure 5.6b: Convergence to Localized Damage Path

Figure 5.6c: No Converged Solution
a discrete point on the computed solution path or numerical error will prevent discovery of a zero eigenvalue. Points on the distributed damage paths are defined by a global tangent with one or more negative tangents. However, Crisfield does identify instances in which unloading and reloading of individual elements may produce solution points in the vicinity of the distributed damage path that have a positive-definite global tangent. Consideration of the simple systems shown in Figure 5.4 suggests that if all elements are exhibiting damage, then each element contribution to the diagonals of the global tangent will be negative with the result that the global tangent likely will have one or more negative eigenvalues. If damage is localized in a single element, the element adjacent to this element will make a positive contribution to the global tangent, typically out-weighing the negative contribution of the damaging element and resulting in a positive-definite global tangent. Thus, in practice, calculation of a negative eigenvalue is sufficient qualitatively to define a non-unique solution path. The existence of a negative eigenvalue in the global tangent implies the existence of a negative pivot in the \(LDL^T\) decomposition of the tangent, which is easily checked during the solution process. Crisfield and Wills proposes an additional stiffness parameter for characterizing system response; this is the ratio of the norm of the global displacement increment to the dot-product of the global displacement increment with the global load increment. Evaluation of this stiffness parameter in conjunction with the smallest eigenvalue of the global tangent defines loading on the localized or distributed damage paths, unloading, snap-back and snap-through response.

Given that the existence of non-positive definite global tangent is sufficient for the existence of non-unique solutions, improved system convergence may be realized through introduction of an algorithm to modify the global tangent to provide a positive-definite matrix. Considering the eigenvalue decomposition of the global tangent, for any interac-
tion in which the assembled global tangent has a negative eigenvalue, a rank-one update of the global tangent to remove the negative eigenvalue is a reasonable approach (Eq. 5-1):

\[
K^o = \sum_{i} \lambda_i v_i v_i^T 
\]

(5-1a)

\[
K^{updated} = K^o + \beta v_j v_j^T
\]

(5-1b)

where \( K^o \) is the original global tangent defined by eigenvalues \( \lambda \) and associated eigenvectors \( v \) for which \( \lambda_j \leq 0 \). From this it follows that \( \beta \) must be chosen such that \( \beta > \lambda \). This assumes a symmetric tangent, which will not be true for the case of the two-dimensional bond element; however, this approach can be extended for the unsymmetric case or an approximate, symmetric tangent can be used for the bond element. Given the orthogonality of the eigenvectors, it follows that the update of the tangent may be considered an update of the displacement increment. For a load increment \( \Delta R_k \) associated with a given time in the load history and a particular iteration in the Newton-Raphson solution scheme, \( k \), the original problem is to solve for the corresponding displacement increment, \( \Delta u^o_{k+1} \):

\[
K^o_k \Delta u^o_{k+1} = \Delta R_k
\]

(5-2)

If \( K^o_k \) is determined to have a negative eigenvalue, then the rank-one update of K defined in Eq. 5-1b is identical to modifying the displacement increment, \( \Delta u^o_{k+1} \), by a scalar multiple of the eigenvector associated with the negative eigenvalue:

\[
K^{update}_k (\Delta u^o_{k+1}) = \Delta R_k
\]

\[
(K^o_k + \beta v_j v_j^T) (\Delta u^o_{k+1}) = \Delta R_k
\]

\[
K^o_k (\Delta u^o_{k+1} + \frac{\beta + \lambda}{\lambda} v_j) = \Delta R_k
\]

(5-3)

\[
K^o_k (\Delta u^{updated}_{k+1}) = \Delta R_k
\]

Crisfield and Wills propose that the update to the displacement increment be defined as follows:
De Borst proposes that the update to the incremental displacement field be defined on the basis of two criteria: orthogonality of the update and the original displacement increment, and equality of the norm of the original and updated displacement increment. The result is as follows:

$$
\Delta u_{k+1}^{updated} = \Delta u_{k+1}^o \pm \left( \frac{\max(\Delta u_{k+1}^o)}{\max(v_j)} v_j \cdot 10^{-4} \right)
$$

(5-4)

In addition to modifying the global tangent, or the displacement field, to represent a system with a positive-definite tangent matrix, application of arc-length load control may improve convergence as well. A number of researchers have looked at application of arc-length control methods for systems characterized by strength-degrading materials. These include arc-length control on the basis of the global displacement field [Crisfield, 1983] as well as on the basis of local displacement and strain fields [de Borst, 1987; Chen and Schreyer, 1990; May and Duan, 1997]. In general arc-length control will be necessary to trace “snap-back” and “snap-through” response. However, in the absence of these response mechanisms, displacement control is sufficient. Further, it has been observed in the current investigation and in studies conducted by others [de Borst, 1987] that arc-length control may not improve convergence for systems with non-smooth response or highly localized damage.
The response mode identified in Figure 5.6c indicates an additional problem encountered in systems with strength-degrading materials. For a given load increment, it may not be possible to find a converged solution point. Evaluation of the iteration history shown in Figure 5.6c suggests that this situation is characterized by a sequence of iterations that show a repeating pattern of displacement, residual or energy increments. This situation has been encountered frequently by the author in relatively large systems and always in the presence of symmetric positive definite global tangent matrix. Evaluation of the unbalanced residual during the Newton-Raphson iteration sequence confirms oscillation between two distinct possible load paths that typically characterize two different localized failure modes. For the simple systems shown in Figure 5.4, this behavior might characterize oscillation between two computed responses, one in which damage is localized in Element 12 and one in which damage is localized in Element 13. In this situation, failure to converge is identified on the basis of a repeated sequence of unbalanced energy norms (dot product of the displacement increment and the unbalanced residual). Achievement of a converged solution state requires backing up to the previous converged solution and reattempting advancement of the solution using a load increment that represents either an increase or decrease from the previously used load increment.

Prior research and qualitative evaluation of the response of extended systems composed on strength-degrading elements supports the solution algorithm proposed for the current investigation. This algorithm is intended for use in analyzing relatively large systems (2500 dof) composed of strength-degrading materials that do not exhibit either snap-back or snap-through behavior. This characterizes the reinforced concrete beam-column connection sub-assemblies of interest to the current investigation. On the basis of the unbalanced energy norm and the pivots of the global tangent, the algorithm defines rules for modification of the load increment and modification of the global tangent. Modifica-
tion of the load increment follows a relatively simple formula that is an extension of that defined in FEAP [Taylor, 1998; Zienkiewicz, O.C. and R.L. Taylor, 1991 and 1987] in which the load increment is first increased to a maximum and then decreased to a minimum if the normalized difference between every second or every third unbalanced energy norm is sufficiently small during any one iteration sequence at a given load increment.

Starting from a converged solution point the algorithm is as follows:

**Algorithm 5.1:**

Apply displacement increment using time increment defined upon achieving previous converged solution

set flag that system is not oscillating (oscillation = false)

set number of iterations at which to begin checking that system is oscillating (n = N)

set flag to increase time increment if solution is oscillating (dir = +1)

**do while** no converged solution

begin Newton-Raphson iteration, store unbalanced energy norm at each iteration

\[ E_i = |\Delta u_i^T \Delta R_i| \]

\[ E_1, E_2, E_3, E_4, E_5, E_6 \text{ represent the last six unbalanced energy norms} \]

if solution has not converged after \( n \) iterations, check for oscillating solution on the basis of the last four and last six values of the energy norm:

\[ \Delta E_{21} = E_1 - E_3; \Delta E_{22} = E_2 - E_4 \]

\[ \Delta E_{31} = E_1 - E_4; \Delta E_{32} = E_2 - E_5; \Delta E_{33} = E_3 - E_6 \]

\[ \text{energy\_check}_1 = \frac{\Delta E_{21} \Delta E_{22}}{(E_1 - E_2 + E_3 - E_4)^2}; \]

\[ \text{energy\_check}_2 = \frac{\Delta E_{31} \Delta E_{32} + \Delta E_{32} \Delta E_{33} + \Delta E_{33} \Delta E_{31}}{(E_1 - E_2 + E_3 + E_4 - E_5 + E_6)^2}; \]

if the solution has been oscillating (oscillation = true) then

if solution is oscillating (energy\_check\_1 < 10^{-6} or energy\_check\_2 < 10^{-6}) then

set flag to backup to previous converged solution and resolve with new displacement increment defined by new time increment (backup = true)

end if

else if solution has not been oscillating then

if solution is oscillating (energy\_check\_1 < 10^{-4} or energy\_check\_2 < 10^{-4}) then

set flag that solution is oscillating (oscillating = true)

set flag to backup to previous converged solution and resolve with new displacement increment defined by new time increment (backup = true)

set number of iterations for checking that solution is oscillating (n = 6)
end if
end if

if system is oscillating (backup = true) or has not converge after N iterations then
define new time increment
if displacement increment is increasing (dir = +1)
dt = 1.1dt
if increment is too large (dt > dtmax) then
set time increment to be maximum allowed value (dt = dtmax)
set flag that time increment should decrease (dir = -1)
end if
else if displacement increment is decreasing (dir = -1)
dt = 0.5dt
if increment is too small (dt < dtmin) then
set time increment to minimum allowed value (dt = dtmin)
set flag for explicit solution algorithm (explicit = true)
end if
end if
backup to previous converged solution state and increase load on the basis of
new time increment
else (solution has converged)
check how many iterations were required to achieve converged solution
if the number of iterations is less than a defined minimum
increase dt (dt = 1.1dt)
end if
if the number of iterations is greater than a defined maximum then
decrease dt (dt = 0.5dt)
end if
end if
end do
if a converged solution cannot be achieved (explicit = true) then
go to an explicit solution scheme
dt = dtmin
end if

The last few lines of the algorithm are important to note. In the event that the above algorithm goes through the entire range of user defined load increments and still does not achieve a converged solution, implicit advancement of the incremental boundary value problem is abandoned, the time increment is set to the minimum defined by the user and a series of five to ten explicit advancements of the solution are made. Following this, the time increment is set to the maximum and the solution proceeds as before with implicit integration of the time dependent problem.
The author has found the above algorithm to be necessary for solution of most relatively large finite element problems that define the response of strength-degrading materials. This solution algorithm looks to find a converged solution state regardless of whether or not the state defines the minimum potential energy of the system. It has been observed both by the author and by others [de Borst, 1987] that the solution path will eventually converge to that corresponding to the minimum energy state even starting from points on non-localized damage paths. While the above algorithm has been shown to work for many systems, it may not be sufficient for all and further modification of the solution algorithm maybe necessary for systems in which non-unique solutions paths are encountered. For the current study a modification of the standard Newton-Raphson iteration scheme is implemented that follows the algorithm proposed by de Borst [1987]. In this algorithm, during any iteration, the pivots of the $\text{LDL}^T$ decomposition of the global tangent are monitored. In the event that a zero or negative pivot is encountered, the displacement increment is updated by a scalar multiple of the eigenvector associated with the negative eigenvalue that is closest to zero. A number of different criteria were considered in determining appropriate scale factors to define the contribution of the initial displacement increment and the eigenvector to the updated displacement increment. These criteria included those proposed by de Borst as well as 1) defining the updated displacement increment so that the rank-one contribution to the updated tangent matrix associated with the negative eigenvalue is slightly non-negative, and 2) defining the update to the original displacement increment to produce a positive-definite tangent and minimize the unbalance energy associated with the updated displacement increment. These last criteria proved to be most successful. In its most basic form, the modified solution algorithm consists of updating the global tangent or the displacement increment whenever the global tangent is determined to contain one or more negative eigenvalues:
Algorithm 5.2:

If during the LDL$^T$ decomposition of the global tangent $K$ a negative pivot is encountered do while no negative eigenvalues of $K$ are known
compute the 10 unknown eigenvalues of the matrix $K$ with the smallest absolute value
\[ \lambda = \text{absolute largest negative eigenvalue of } K, \; \mathbf{v} = \text{associated eigenvector} \]
end do
\[ \Delta u^{\text{updated}} = \alpha \Delta u^0 + \beta \mathbf{v} \] where $\alpha$ and $\beta$ are defined so that $\Delta u^{\text{update}}$ meets desired criteria (either normality of the displacement increment as proposed by de Borst or minimization of the energy norm as proposed by the author)
end if

However, application of Algorithm 5.2 in the solution of relatively complex systems results in an update of the tangent on the basis of the largest negative eigenvalue on iteration $n$, followed by a second update on the basis of the new largest negative eigenvalue on iteration $n+1$ and so on for a number iterations. Evaluation of the updates suggests that the eigenvalues associated with subsequent updates may or may not be related. Given the numerical expense of the eigenvalue decomposition as well as the need to force the system towards a single localized damage path, the final implementation of the algorithm includes an update of the displacement increment on the basis of the first negative eigenvalue encountered during a sequence of iterations at a given time increment. If this does not result in a converged solution, Algorithm 5.1 is employed.

The final solution algorithm proposed for this study includes application of both Algorithms 5.1 and 5.2 in the solution of systems that may have non-unique solution paths. This solution method was used successfully on a number of systems including one-dimensional problems such as those identified in Figure 5.4, uniform, two-dimensional, multi-element patches subjected to uniform shear stresses and non-uniform multi-element reinforced concrete panels subjected to shear loading. However, employment of this solution algorithm for more complex systems under general loading conditions met with substantially less success. It seems likely that for these systems, multiple solution paths are so closely aligned that it is impossible to force the solution onto a single path for conver-
gence. For some systems, application of Algorithm 5.1 allows for a converged solution to be achieved. However, for some systems, it appears that a plausible solution may not be achieved using either Algorithm.

5.3 Analysis of Plain Concrete Beams

Verification of the plain concrete element presented in Chapter 2 focussed on comparison of computed and observed behavior for relatively simple specimens with simple load histories in which the concrete stress fields are essentially uniform. Available experimental data also provide a means for evaluating the model’s capacity to represent more complex stress and damage fields. One standard experimental test method that produces moderately complex concrete material response is that used to determine concrete fracture energy. As discussed in Chapter 2 the standard test to determine concrete fracture energy requires loading a notched concrete beam in three-point bending to failure while controlling the applied load to produce a specific rate of crack width opening (Figure 5.7). Under these loading conditions, plain concrete exhibits progressive damage and accurate prediction of system response requires accurate representation of concrete response under uniaxial tension as well as appropriate representation of undamaged concrete response under two-dimensional loading.

![Figure 5.7: Plain Concrete Beam and Loading Conditions for Concrete Fracture Energy Test as Proposed by RILEM [985]](image)
To evaluate the concrete element under the loading conditions imposed by the fracture energy test, computed and observed response data are compared for tests conducted by several different researchers. For each experimental test, three finite element analyses are performed using progressively finer element meshes. Figure 5.7 shows dimensions suggested by the RILEM standard for testing plain concrete with a maximum aggregate diameter of 16 mm. Figure 5.8a defines dimensioning for the plain concrete specimens used in experimental investigations considered here. Figure 5.8b defines mesh parameters and boundary conditions for the finite element model used to represent these tests. The RILEM standard recommends controlled loading on the top surface of the specimen to produce a specific rate of crack width opening. Since such load control is not possible for typical finite element solution algorithms, loading in the finite element analysis is introduced by monotonically increasing vertical displacement (in the negative direction) at the point of load application. Table 5.1 lists specimen dimensions and relevant material properties cited by the investigators and used in the finite element analysis. Assumptions are made where necessary to define relevant data that are not provided by the investigators; parameters defining material response are computed using the relationships presented in
Chapter 2. Table 5.2 lists mesh parameters defining the coarse, fine and super-fine meshes used in analysis of each specimen.

<table>
<thead>
<tr>
<th>Investigation</th>
<th>x (mm)</th>
<th>y (mm)</th>
<th>z (mm)</th>
<th>h (mm)</th>
<th>t (mm)</th>
<th>Gf (N/m)</th>
<th>ft (MPa)</th>
<th>Ec (GPa)</th>
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<tbody>
<tr>
<td>Kozul and Darwin [1997]</td>
<td>152</td>
<td>100</td>
<td>100</td>
<td>25</td>
<td>10</td>
<td>183</td>
<td>4.2</td>
<td>23.8</td>
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<td>300</td>
<td>50</td>
<td>50</td>
<td>2</td>
<td>10</td>
<td>90</td>
<td>3.4</td>
<td>42.0</td>
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<tr>
<td>Malvar and Warren [1988]</td>
<td>394</td>
<td>101</td>
<td>5</td>
<td>50</td>
<td>10</td>
<td>76</td>
<td>3.1</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Note: Bold values are estimated by Lowes. Italic values are estimated by the investigator cited.

**Table 5.1: Parameters for Concrete Fracture-Energy Test Specimens**

<table>
<thead>
<tr>
<th>Investigator</th>
<th>number of elements</th>
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<td></td>
<td>a</td>
</tr>
<tr>
<td>Kozul and Darwin [1997]</td>
<td></td>
</tr>
<tr>
<td>coarse</td>
<td>3</td>
</tr>
<tr>
<td>fine</td>
<td>6</td>
</tr>
<tr>
<td>super-fine</td>
<td>12</td>
</tr>
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<td>Petersson [1980]</td>
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<tr>
<td>coarse</td>
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<tr>
<td>fine</td>
<td>5</td>
</tr>
<tr>
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</tr>
<tr>
<td>Malvar and Warren [1988]</td>
<td></td>
</tr>
<tr>
<td>coarse</td>
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<tr>
<td>fine</td>
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</tr>
<tr>
<td>super-fine</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 5.2: Finite Element Model Parameters**
Figures 5.9, 5.10 and 5.11 show the computed and experimentally observed load displacement history for a specimens tested by Malvar and Warren [1988], by Kozul and Darwin [1997] and by Petersson [1980]. Figures 5.9 and 5.11 show reasonably good correlation between computed and observed response histories. The model predicts a stronger and more ductile response in the vicinity of maximum stress than is observed in the real system. This is a function of the defined post-peak tensile stress-strain history for the concrete. Figure 5.10 shows a relatively poor correlation between computed and observed response. Here, this follows from the estimate of the concrete elastic modulus provided by Petersson. Modification of the elastic concrete models using the relationship proposed by ACI 318 provides a greatly improved prediction of the pre-failure response, but a less accurate prediction of strength and deformation in the vicinity of the maximum load. Figures 5.9 through 5.11 show increasing strength and deformation with increasing mesh refinement; though results for the fine and super-fine meshes are essentially the same for the Malvar test and the Kozul test. The increasing strength associated with increasing mesh refinement results from the reduction in concrete volume associated with individual quadrature points in the finer meshes. As individual element quadrature points accumulate damage and exhibit reduced tensile strength, the loss in total system strength is substantially larger for a coarse mesh than for a finer mesh. The fact that the fine and super-fine meshes provide essentially the same response, confirms that the solution is not exhibiting mesh-sensitivity but is converging towards an exact solution as the deformation field is more accurately represented by more elements.

5.3.1 Results of Analyses of Plain Concrete Beams

Evaluation of concrete fracture energy tests indicates that the proposed concrete material predicts behavior under Mode I fracture with an acceptable level of accuracy as well as representing well the plane stress elastic response.
Figure 5.9: Computed and Observed Response for Concrete Fracture Energy Test (Experimental Data from Malvar and Warren [1988])

Figure 5.10: Computed and Observed Response for Concrete Fracture Energy Test (Experimental Data from Petersson [1997])
5.4 Analysis of Idealized Reinforced Concrete Bond Zones

For the current investigation, the reinforced concrete bond zone provides a limited structural sub-system that includes all of the proposed material models. Thus, evaluation of the global finite element model appropriately begins with an evaluation of the model’s capacity to represent the response of an extended bond zone. This evaluation includes comparison of computed and observed response for the two types of bond zones found in reinforced concrete beam-column connections. Additionally, a parameter study is conducted to determine the effect of various model parameters on computed response. The results of this investigation identify model parameters that are appropriate for representation of bond-zone response in larger structural sub-assemblages.

Figure 5.11: Computed and Observed Response for Concrete Fracture Energy Test (Experimental Data from Kozul and Darwin[1997])
5.4.1 Definition of the Finite Element Models To Be Evaluated

5.4.1.1 Definition of the Prototype Specimens

Two prototype bond-zone sub-systems are identified to represent the two regions within a reinforced concrete bridge beam-column sub-assemblage in which bond between concrete and reinforcing steel has a significant effect on behavior of the system. These regions, identified in Figure 5.12, typically are designated flexural bond zones and anchorage bond zones. Figure 5.13 shows an idealization of the prototype finite element models used to predict the behavior of these regions. The flexural bond zone is idealized by the full-length finite element model shown in Figure 5.13a; however, to reduce computational effort the half-length finite element model is used in the analyses (Figure 5.13a). Material properties that complete definition of the finite element models are listed in Table 5.3. The flexural bond zone model is representative of the tension zone in a beam-column element.
subjected to at least moderate flexural loading. Under this loading, cracks develop at discrete intervals in the tension-zone concrete of the beam-column element. A segment of the flexural tension zone may be idealized as consisting of a reinforcing bar anchored within a plain or reinforced concrete prism and subjected to equal tensile loading on both exposed ends. Here the prototype flexural bond zone model consists of a 25 mm (1.0 in.) bar anchored in a plain concrete prism 500 mm long and 125 mm square (approximately 20 in. by 5 in. by 5 in.) and subjected to monotonically increasing elongation developed through tensile loading of each exposed end of the reinforcing bar. The dimensions of the concrete prism represent the flexural bond zone of typical building elements and the model is similar in scale to specimens used in experimental flexural bond-zone investigations.

The prototype model for the anchorage bond-zone structural sub-system shown in Figure 5.13b is representative of a beam or column longitude reinforcing bar anchored in a beam-column connection. Under moderate to severe earthquake loading, flexural demand on beam and column elements results in longitudinal reinforcement reaching the yield stress at the beam-connection or column-connection interface. Note that column reinforcement likely achieves significant post-yield strength under severe earthquake loading. Load on the reinforcement is transferred to the plain or reinforced concrete in the anchorage zone. Here the prototype anchorage bond-zone model consists of a 25 mm (1.0 in.) diameter reinforcing bar anchored in a plain concrete prism 500 mm long by 500 mm wide by 100 mm thick (approximately 20 in. by 20 in. by 4 in.) and subjected to monotonically increasing displacement developed through tensile loading of one end of the bar. For this specimen, the greater width of the anchorage bond zone represents the extended concrete region on either side of the anchored reinforcing bar, while the limited thickness represents the limited concrete cover of the anchorage zone. The scale of the proposed bond-zone model is representative of building system connections and of a similar scale to
concrete prism: 500 x 125 x 125 mm
reinforcing bar: 25 mm diameter
concrete prism: 500 x 125 x 125 mm
concrete element mesh is 10x20
Bond elements connect reinforcing steel to concrete prism

Figure 5.13a: Flexural Bond-Zone Model

concrete prism: 250 x 125 x 125 mm
concrete element mesh is 10x8

Figure 5.13b: Anchorage Bond-Zone Model

Figure 5.13: Finite Element Models of Flexural and Anchorage Bond Zones
bond-zone specimens tested in the laboratory. The bond-zone model represents approximately a half-scale bridge connection for which nominal reinforcing bar diameters may be as large as 57 mm (2.25 in for No. 18).

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete modulus and Poisson ratio</td>
<td>26 GPa</td>
<td>Elastic modulus</td>
<td>200 GPa</td>
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<tr>
<td>Tensile strength</td>
<td>3.0 MPa</td>
<td>Yield strength</td>
<td>470 MPa</td>
</tr>
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<td>Shear strength</td>
<td>12 MPa</td>
<td>Strain at which strain hardening initiates</td>
<td>2.2%</td>
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<tr>
<td>Compressive strength</td>
<td>30 MPa</td>
<td>Hardening modulus</td>
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<td></td>
<td></td>
<td>Tensile strength</td>
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</tr>
<tr>
<td></td>
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<td>Strain at which tensile strength is achieved</td>
<td>10%</td>
</tr>
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</table>

Table 5.3: Material Parameters for Bond-Zone Models

5.4.1.2 Parameters of Interest to the Investigation

Of interest to this study are both material model parameters and systems parameters that may determine bond response. These include the thickness of the concrete cover over the anchored reinforcing bar, angle between radial and tangential bond forces, concrete fracture energy, residual tensile and shear strength of cracked concrete and volume of reinforcement transverse to embedded reinforcing bar, as listed in Table 5.4. With the exception of the volume of transverse reinforcement, the effect of these parameters on computed response is evaluated with the goal of determining appropriate values for all subsequent analyses. It is assumed that the values of these parameter as determined from evaluation of the structural system for calibration of individual elements may not be appropriate when a composite model is used. A total of 17 models are analyzed for each of the bond-zone representations (flexural and anchorage). Each model represents a one- or two-parameter variation from the prototype model.
5.4.1.3 Evaluation of Bond-Zone Response

The response of a reinforced concrete bond zone is defined by a number of factors including the distribution and characteristics of concrete damage, the embedded reinforcing steel stress-strain distribution, the local bond-slip response, average bond strength and global and local load-displacement histories. These characteristics are considered in comparing the computed response for different representations of the bond zones and in comparing computed and observed response. For each finite element model considered in the parameter study, characteristics of response are defined by the following data: the concrete, steel and bond load along the length of the bar at intervals during loading; the concrete, steel and bar load versus time at intervals along the embedded length of the bar; the distribution of the concrete damage parameter in concrete elements adjacent to the bonded bar at intervals during loading; the stress-strain history in concrete elements adjacent to the bar and the local bond stress versus slip history at points along the length of the prism, and the load versus displacement history at both ends of the anchored reinforcing bar.

<table>
<thead>
<tr>
<th>parameter</th>
<th>values</th>
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<tbody>
<tr>
<td>thickness of cover concrete over bond element</td>
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<tr>
<td>bond model includes radial action</td>
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</tr>
<tr>
<td>angle of orientation of bond force resultant in degrees</td>
<td>0, 30, 45, 60</td>
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<td>concrete fracture energy</td>
<td>0.2 lb-in., 2.0 lb-in.</td>
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<td>residual concrete strength as a percentage of initial strength (tension, shear)</td>
<td>(0.1,0.1), (1,1), (0.001,0.001), (0.001,0.5), (0.5,0.5)</td>
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<tr>
<td>transverse steel ratio</td>
<td>0.0, 0.008 (flexural model), 0.04 (anchorage model)</td>
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<tr>
<td>bond elements</td>
<td>yes, no</td>
</tr>
</tbody>
</table>

Table 5.4: Parameters for Bond-Zone Model Investigation (Parameters for Prototype Specimen are Bold)
5.4.2 Behavior of the Prototype Flexural and Anchorage Bond-Zone Models and Comparison with Experimental Data

The prototype models are the most basic representation of the bridge connection critical bond zones that can be developed using the material models proposed in Chapters 2, 3 and 4. For these models, the concrete and bond material model parameters are calibrated on the basis of material testing of plain concrete and limited bond-zone specimens as presented in Chapters 2 and 4. Radial bond response, cover concrete and transverse reinforcement are excluded from the prototype model to maintain simplicity.

5.4.2.1 Computed and Observed Behavior of Flexural Bond-Zone Models

In comparing computed and observed behavior for flexural bond zone models the most appropriate data for consideration include the load-displacement history, the distribution of concrete cracking and average bond stress. Figure 5.14 shows the computed distribution of the primary concrete damage parameter at maximum loading. Data in Figure 5.14 show three zones of concentrated damage that are representative of discrete cracks along the span of the concrete prism. Figure 5.15 shows the average orientation of the fictitious crack surfaces within individual concrete elements. The distribution of the concrete damage shown in Figure 5.14 as well as the crack patterns shown in Figure 5.15 indicate that the discrete cracks in the concrete prism form approximately perpendicular to the axis of the reinforcing bar. Data in Figure 5.15 show that between discrete prism cracks concrete cracks in the local vicinity of the embedded bar on average form an angle of approximately 65 degrees with respect to the axis of the embedded bar. This indicates that concrete responds in shear and tension in this region.

The formation of three discrete cracks in the concrete prism is supported by the results shown in Figures 5.14 and 5.16 that show the distribution of steel strain along half of the model at various points during the load history. In Figure 5.16 the log of the steel
strain is plotted versus embedment depth to show the strain distribution throughout the entire load history, and individual strain distributions are identified in the legend on the basis of the maximum stress developed at the exposed end of the reinforcing bar. These data show also that the initial crack forms at mid-span of the full length model and that the second pair of cracks forms under increased loading. The data presented in Figures 5.14 and 5.16 show crack spacing of 110 mm and 140 mm (4.3 in. and 5.5 in.). These crack spacing data can be used to estimate average bond strength. Assuming a uniform bond stress distribution over an uncracked segment of a concrete prism and an average concrete tensile stress distribution that increases linearly from zero at the ends of the uncracked segment to a maximum at the mid-span, it follows that a crack forms at mid-span of the prism when the bond strength and segment length are such that the concrete tensile strength is achieved at mid-span. Here the computed crack spacing average indicates that maximum average bond strength is at least $0.78\sqrt{f_c}$ MPa with $f_c$ in MPa (9.4\sqrt{f_c}$ psi).

The global load displacement history for the flexural bond-zone model provides information about global system response. Figure 5.17 shows the relationship between applied tensile load and displacement of one end of the embedded reinforcing bar with respect to the center of the flexural bond-zone model. These data show two points of temporary loss of system strength that correspond to formation of discrete crack zones at load levels of approximately 66 kN (180 kips) and 148 kN (440 kips). Loss of load associated with crack initiation is a function of the interaction of the concrete, steel and bond elements. At a prescribed level of axial elongation, the local increase in steel strain, and thus steel stress, does not compensate for the reduction in concrete strength associated with concrete cracking. The cracked strength is determined also by the bond element since slip between concrete and steel provides a mechanism for localized increase in the steel strain, and stress, without an increase in the elongation of the adjacent concrete elements.
These data may be compared with the experimental data generated by Goto [1972]. The Goto study is discussed in detail in Chapter 4. In that study crack patterns in two reinforced concrete prisms were considered. In one specimen, a 19 mm (0.75 in.) diameter reinforcing bar was imbedded in 100 mm (3.9 in.) square prism. Observed crack spacing varied between 130 mm and 250 mm (5.1 in. and 9.8 in.); these spacings being determined in part by intentional pre-cracking of the prism surface concrete. With the minimum crack spacing just less than half of the maximum crack spacing, it is appropriate to define a maximum average bond strength for the specimen on the basis of an assumed uniform bond stress distribution and a maximum uncracked prism length equal to twice the minimum crack spacing. This corresponds to a maximum average bond strength of $0.67 \sqrt{f_{c}}$ MPa,
Figure 5.16: Strain Distribution for Embedded Reinforcing Bar (Note that bar stresses are listed in the order of increasing displacement at the point of loading)

Figure 5.17: Load Versus Displacement for Prototype Flexural Bond-Zone Model
with $f_c$ in MPa ($8.1 \sqrt[2]{f_c}$ psi, with $f_c$ in psi). For a second specimen consisting of a 32 mm diameter reinforcing bar embedded in a 120 mm (4.7 in.) square concrete prism, observed crack spacing varied between 140 mm and 240 mm (5.5 in. and 9.43 in.) indicating a maximum average bond strength of $0.53 \sqrt[2]{f_c}$ MPa, with $f_c$ in MPa ($6.4 \sqrt[2]{f_c}$ psi, with $f_c$ in psi).

Goto observes that cracks initiating at the concrete-steel interface and propagating towards the surface of the prism form an angle of approximately 60 degrees with respect to the axis of the reinforcing bar near the surface of the bar. Data presented by Goto suggest also that the localized concrete cracks that initiate at an angle of approximately 60 degrees at the concrete-steel interface become progressively steeper as they propagate towards the surface of the flexural zone.

Correlation between the flexural bond-zone behavior computed by the finite element model and that observed in laboratory specimens is good. The finite element model appears to represent well the fundamental characteristics of flexural bond zones in reinforced concrete elements. The model predicts crack spacing and average bond strength that is typical of that observed in the laboratory. Additionally, the model predicts well the orientation of localized concrete cracking at the bar-concrete interface that occurs between discrete cracks in the concrete prism. The model is less accurate in representing localized crack orientation along discrete crack surfaces. Computed localized crack orientation within the vicinity of discrete prism cracks is approximately perpendicular to the axis of the reinforcing bar along the entire crack plane. For the initial discrete crack that forms at mid-span, this is likely due to the boundary conditions of the half-length model. For subsequent discrete prism cracks, local crack surface orientation is somewhat less steep, but does not exactly represent the behavior observed by Goto. Finally, the model predicts temporary losses in systems strength, associated with initiation of significant discrete concrete cracks, that typically are not observed in the laboratory. Here, it is assumed that these
strength losses are not observed in the laboratory due to the stiffness of typical laboratory testing equipment and standard testing procedures\(^1\).

5.4.2.2 **Computed and Observed Behavior of Anchorage Bond-Zone Models**

In comparing computed and observed response for anchorage bond zone models, an appropriate data set includes the global load versus displacement history, distribution of the primary concrete damage parameter and the distribution of steel stress-strain response along the embedment length. The global load deformation response identifies critical points on the load path. The concrete damage and steel stress-strain response indicate the extent to which yielding of the reinforcing bar penetrates the anchorage zone as a result of bond deterioration in the vicinity of the applied loading.

There are only limited experimental data defining anchorage bond-zone response for idealized systems such as that represented by the finite element model in Figure 5.13b. Lab models tested by Viwathanatepa [1979a, 1979c] and Engström *et al.* [1988] are sufficiently similar to the finite element models in both configuration and loading, that data generated in these experimental investigations may be compared with that provided by the computer models. Engström *et al.* consider anchorage bond-zone models consisting of 16 mm (0.63 in.) diameter reinforcing bars anchored with 290 mm and 500 mm (11 in. and 20 in.) embedment lengths in concrete prisms of extended depth and width and with clear concrete cover of \(1d_b\), \(2d_b\) and \(~12d_b\). However, a system with a 500 mm (20 in.) anchorage length and clear concrete cover of \(2d_b\) is not evaluated. Thus, while the finite element model represents the loading conditions for the Engström specimens, the model does not represent exactly the geometry of any one specimen. With the exception of the extended

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1. Ideally the testing equipment should be included as part of the model; however, very few reports include sufficient information to adequately incorporate this part of the system into the finite element model. Similarly, the standard test procedure should be represented in the computer simulation; however, this may not be possible for systems tested under load control or using system feedback.
anchorage length, the finite element model represents well the geometry of the specimen with 290 mm (11 in.) anchorage length and $2d_b$ concrete clear cover, if it is assumed that bond response is somewhat symmetric with respect to the axis of the reinforcing bar and that only a portion of the extended depth of the concrete specimen contributes to bond strength. Figure 5.18 shows drawings of the finite element model and the model tested by Engstrom et al. with 290 mm (11 in.) anchorage length and $2d_b$ concrete clear cover. The shallow concrete cover for both the analytical and laboratory models emphasizes concrete damage in the bond zone. The reduced anchorage length for the laboratory model likely results in reduced anchorage strength once concrete damage progresses from the point of load application to the non-loaded end of the bar. However, this does not affect the distribution and characteristics of concrete damage at moderate load levels. Thus, computed concrete damage patterns may be compared with observed concrete damage for this Engström specimen. With the exception of the shallow concrete cover, the finite element model represents well the geometry of the laboratory specimen with 500 mm (20 in.) embedment length and $12d_b$ concrete cover. The increased concrete cover for this Engström specimen likely results in increased bond strength over that computed by the finite element model. Thus, the reinforcing bar stress-strain histories observed by Engström et al. for this specimen may be considered to bound those computed by the model.

The tests conducted by Viwathanatepa also provide data for comparison with computed response. This investigation is discussed in detail in Chapter 4 and in Section 5.5. Here it is sufficient to note that the Viwathanatepa specimens consist of a single reinforcing bar of 19 mm (0.75 in.) diameter anchored with 635 mm (24 in.) embedment length in a reinforced concrete prism with an extended reinforced concrete clear cover ($6d_b$). Loading of the Viwathanatepa model is such that at the loaded end of the reinforcing bar concrete carries tension in the direction perpendicular to the axis of the bar while at the non-
loaded end of the bar concrete carries compression. As discussed in Chapter 4, the concrete stress state results in reduced bond capacity at the loaded bar end and increased bond capacity at the non-loaded bar end. The slightly increased anchorage length as well as the increased, reinforced concrete cover of the Viwathanatepa model in comparison to the prototype finite element model would be expected to increase the bond strength of the lab-
oratory model in comparison to that computed by the computer model. However, the concrete stress state acts to reduce bond capacity in the vicinity of the load point. Due to these variations in system geometry and material state, the finite element model would be expected to predict only the trends in system behavior as observed by Viwathanatepa.

Figure 5.19 shows the computed distribution of concrete damage for the prototype anchorage bond-zone model. Figure 5.20 shows the orientation of fictitious crack planes developed in the bond-zone concrete elements. These data are smoothed over the mesh, and the length of individual lines indicates the extent of cracking in the vicinity of the node. These data indicate that concrete cracks form an angle of approximately 40 degrees with respect to the axis of the reinforcing bar and that angles become somewhat steeper as cracks propagate from the reinforcing bar to the supports. The orientation of the cracks at the bar-concrete interface indicates that bond force transfer results in shearing of the concrete. The crack pattern and damage parameter data indicate also that concrete damage is limited to approximately the initial $10d_b$ to $15d_b$ of the embedment length. The damage represented in these figures may be compared with the concrete crack patterns observed by Engström et al. as shown in Figure 5.21. Here cracks form at angles between 40 and 80 degrees with respect to the axis of the reinforcing bars with extended cracking near the loaded end of the bar (approximately the $10d_b$ closest to the point of load application) and limited cracking at the non-loaded end of the bar. The orientation and localization of cracking is represented well by the finite element model. However, the distribution of discrete cracking in the bond zone is not. Analyses of a more finely discretized mesh suggest that the highly distributed cracking computed by the finite element model is not a function of the relatively coarse mesh used in the prototype model. Instead, this appears to be a limitation of the global finite element model and interaction of the concrete, steel and bond material models.
Figure 5.19: Distribution of Concrete Damage in Finite Element Anchorage Bond-Zone Model with 500 mm (20 in.) Anchorage Length

Figure 5.20: Computed Orientation of Fictitious Crack Surfaces in Anchorage Bond Model (Note 500 mm Anchorage Length)
computed behavior of the anchorage bond-zone model is further defined by the distribution of steel stress, steel strain and bond strength along the length of the anchored reinforcing bar. Figures 5.22 and 5.23 show the computed distribution of average strain and stress in the steel reinforcing bar along the embedded length of the bar at various points in the load history. Figure 5.24 shows the associated local bond stress distribution. The legends in these figures indicate the maximum steel stress associated with an individual strain or stress distribution. Here it is important to note that prior to yield of the reinforcing bar the strain distribution is approximately linear with maximum bond stress depending on the applied loading. For the unyielded reinforcing bar, a maximum bond strength of 2 MPa or $0.4\sqrt{f_c}$ (290 psi or $4.4\sqrt{f_c}$ psi) is developed near the point of load application and a maximum bond strength of 8 MPa or $1.5\sqrt{f_c}$ (1160 psi or $17\sqrt{f_c}$ psi) is developed at a distance from the free surface. The yield strength of the reinforcing bar (470 MPa, 68 ksi) corresponds to an average bond strength of 5.9 MPa or $1.1\sqrt{f_c}$ MPa (850 psi or $13\sqrt{f_c}$ psi). Yielding of the reinforcing bar triggers rapid deterioration of bond strength along the...
yielded portion of the reinforcing bar. This is observed in both the bond stress distribution and in the strain and stress distributions (Figure 5.24 and Figures 5.22 and 5.23). Continued loading produces further yield penetration due to progressive loss of bond strength along the embedded length of the reinforcing bar. This is most obviously observed in the steel strain distribution shown in Figure 5.22. Data in this figure and Figure 5.24 indicate that yielding of the reinforcement and associated deterioration of bond strength penetrates to a depth of approximately 100 mm (4 in.). For the yielded reinforcing bar, a maximum local bond strength of 9 MPa or $1.6\sqrt{f_c}$ MPa (1350 psi or $20\sqrt{f_c}$ psi) is achieved at a distance from the applied load. At capacity (tensile strength of the reinforcing bar is defined to be 520 MPa), an average bond strength of 6.5 MPa or $1.2\sqrt{f_c}$ MPa (940 psi or $14.4\sqrt{f_c}$ psi).

The steel stress, strain and bond stress distribution may be compared with similar data provided by Engström et al. and Viwathanatepa. Figure 5.25 shows the steel stress

![Figure 5.22: Computed Distribution of Strain Along Anchored Reinforcing Bar](image)

The steel stress, strain and bond stress distribution may be compared with similar data provided by Engström et al. and Viwathanatepa. Figure 5.25 shows the steel stress
Figure 5.23: Computed Distribution of Stress Along Anchored Reinforcing Bar

Figure 5.24: Distribution of Average Bond Stress Along Anchored Reinforcing Bar
distribution observed by Engström et al. at a maximum load for a reinforcing bar with yield strength of 569 MPa (83 ksi) and tensile strength of 648 MPa (94 ksi) anchored with
a 500 mm (20 in.) embedment length and 12\(d_h\) clear concrete cover. The data shown in this figure indicate that the reinforcing bar yields to a depth of approximately 100 mm (4 in.). Figure 5.26 shows the steel strain distribution observed by Viwathanatepa for a No. 6 reinforcing bar (nominal diameter of 0.75 in (19mm)) with yield strength of 470 MPa (68 ksi) at a maximum bar stress of approximately 529 MPa (76 ksi). This stress level is comparable to the tensile strength of the reinforcement considered in the finite element analysis. Viwathanatepa notes that steel strain at the second measure point along the embedded length of the reinforcing bar is particularly large due to formation of a discrete crack in the concrete at this point. These data indicate yield penetration to a depth of approximately 125 mm (5 in.). The computed steel stress-strain history is approximately the same as that observed by Viwathanatepa and Engström et al. As previously suggested, variation of a number of critical system parameters between the laboratory and computer model would be expected to produce differences in the computed and observed response. However, the net effect of these parameters result in a computed yield penetration depth comparable to that observed in the laboratory.

### 5.4.3 The Effect of Cover Concrete Relative Thickness on Computed Response

As discussed in Section 5.2.1, a thickness of concrete extending over the reinforcing steel, bond elements and adjacent concrete elements is introduced into the global two-dimensional finite element model as a means of representing the volume of concrete that is outside the bond-zone in the true three dimensional structure. The thickness of this cover concrete defines the volume of the bond zone, is unique to each system and likely is determined on the basis of the geometry and reinforcement detailing of the system. Here the behavior of finite element models with variable concrete cover thickness is considered both to validate the proposed modeling technique as well as to determine the effect on
computed behavior of variation in cover concrete thickness. Results of this investigation support the definition of appropriate cover concrete thickness in subsequent analyses.

To promote activation of cover concrete in resisting radial bond forces, models defined by a bond angle of 45 degrees and cover concrete thicknesses of 0 mm, 5 mm, 25 mm and 50 mm (0 in., 0.2 in., 1.0 in. and 2.0 in.) are considered in addition to the prototype model that has no cover concrete. In real systems in regions of limited cover concrete thickness, development of radial forces in association with bond forces would be expected to produce splitting of cover concrete and subsequent deterioration of bond strength. For the proposed two-dimensional models, radial forces associated with bond response also would be expected to produce cracking of cover concrete; however, bond deterioration would only be expected if the cover concrete represented a substantial thickness of the total bond-zone concrete thickness. Radial cracking of cover concrete would be expected for the models with 5 mm (0.2 inch) and 25 mm (1.0 in.) cover concrete thickness and possibly for the models with 50 mm (2.0 in.) cover concrete thickness. Aside from this mode of response, for both flexural and anchorage bond zone models, concrete cover thickness would be expected to determine bond zone response only for systems in which reduced in-plane concrete thickness limits bond strength. Finally, the effect of cover concrete thickness might be expected to be more significant for anchorage rather than flexural bond zone systems. For anchorage bond zones, concrete in the vicinity of the anchored bar resists load primarily through shear action, and cover concrete is not activated in shear at the bar-concrete interface. However, for flexural bond zone, concrete the vicinity of discrete prisms cracks, responds primarily in axial tension and cover concrete is activated in axial tension.

Data generated as part of this study indicate that the proposed cover concrete model does not provide an accurate and robust representation of the observed three-dimensional
bond-zone response. Models with minimal concrete cover thicknesses of 5 mm (0.2 in.) are considered as a means of verifying activation of splitting-type bond response. For the flexural bond zone models, introduction of the thin concrete cover has no effect on observed behavior. A splitting type bond failure is not induced even under maximum loading. Instead cover concrete acts in unison with in-plane concrete and the dominant cracks form approximately perpendicular to the axis of the reinforcing bar. For the anchorage bond zone models, in-plane and cover concrete response is somewhat independent. Cover concrete does exhibit cracking that is more radial in orientation; however, here it appears that the orientation of cracking is determined by the orientation of cracking in the in-plane concrete elements rather than from the development of radial stresses associated with bond response. Essentially loading of the in-plane concrete produces shear action and subsequent damage of in-plane concrete. Reduced strength of the in-plane concrete causes load transfer to the cover concrete and subsequent damage to this concrete. For models with 25 mm (1.0 in.) and 50 mm (2.0 in.) cover concrete thicknesses, the behavior is essentially identical to that of the models with 5 mm (0.2 in.) cover. However for the models with thicker concrete cover, there is noticeable damage to in-plane concrete due to higher concrete stresses associated with bond force transfer. Also, for the models with higher concrete cover thickness some numerical instability is introduced into the system; this appears to result from oscillation between two solutions paths defined by accumulation of damage in either the in-plane or the cover concrete layer.

The results of this investigation indicate that in conjunction with the proposed bond model, introduction of the cover concrete does not provide an improved representation of the three dimensional bond zone over that provided by the in-plane two-dimensional model.
5.4.4 The Effect of the Ratio of Local Tangential Bond Stress to Radial Stress on Computed Response

Experimental investigation indicates that tangential bond stress is developed in combination with radial stress at the concrete-steel interface. This research suggests that the orientation of the bond-radial force resultant is a function of the orientation of concrete damage in the vicinity of the concrete-steel interface. Prior research indicates also that for typical bond-zone specimens the resultant force is oriented between 45 and 60 degrees with respect to the axis of the reinforcing bar (ratio of radial to bond stress of between 1.0 and 1.7). This angle is introduced as a parameter for investigation because the interaction of the proposed steel model, bond-zone model and concrete model with orthotropic damage necessarily defines an effective angle between the radial and tangential stresses developed at the perimeter of an extend bond zone. Consideration of the response of the composite system is necessary to determine the ratio of bond to radial force to be used in subsequent analyses and thereby complete calibration of the bond model. Four angles of orientation of the resultant bond element force are considered for each of the bond-zone models (Table 5.4). Additionally, for each angle of orientation and each bond-zone model the behavior of the system is evaluated for the case of a limited volume of transverse reinforcement introduced into the bond zone (Table 5.4).

For the case of the flexural bond-zone specimen, the bond model angle has relatively little effect on observed response. Increasing the ratio of radial to bond force increases slightly compression stress in the direction perpendicular to the reinforcing bar in the concrete element adjacent to the reinforcing bar. This retards initial damage to these elements, shifting the angle of orientation of fictitious crack surfaces developed in these concrete elements increases bond strength slightly. Additionally, for the systems with angles of 45 and 60 degrees, radial stresses promote shear action over axial action in the concrete ele-
ments adjacent to the reinforcing bar in the vicinity of discrete cracks. However these variations in response do not appear to be significant.

For the anchorage zone models, variation in the angle of the bond resultant force also is relatively insignificant. Here loading is such that the fictitious crack surfaces in the concrete elements adjacent to the anchored bar form at an angle with respect to the axis of the reinforcing bar. Thus, initial cracking is accompanied by shearing of the concrete elements adjacent to the reinforcing bar and temporary loss of specimen strength. Variation in bond model angle shifts slightly the angle of orientation of the cracks surfaces in adjacent concrete elements. As with the flexural bond model, an increased angle of inclination of the bond force resultant is accompanied by a retardation of initial damage.

For both the anchorage and the flexural bond-zone models, variation in bond force resultant has essentially no effect on maximum tensile stress developed in transverse reinforcement. Maximum tensile stress developed in transverse reinforcement for each of the flexural and anchorage bond zone models is listed in Table. On the basis of these data and evaluation of the concrete stress state, it appears that introduction of the radial stress in association with bond stress activates a radial mode of deformation in the bond elements. The radial stiffness of the bond element attracts the radial forces developed in association with bond, preventing substantial elongation of transverse reinforcement perpendicular to bonded reinforcing bar. This explains the relatively limited stresses that are developed in the transverse reinforcement in the flexural bond zone models; for these models, maximum tensile stress varies between 90 MPa and 12 MPa (13 ksi and 1.7 ksi). Maximum tensile stress is not a function of the bond model parameters but instead a function of proximity to severely damage concrete. For the anchorage bond-zone models maximum transverse reinforcement tensile stress is approximately 225 MPa (32.6 ksi) for all of the models with the exception of the model that does not represent radial bond response. For
this model, maximum tensile stress is 140 MPa (20.3 ksi). The higher stresses observed in the anchorage bond-zone models over the flexural bond zone model results from the dominant orientation of concrete damage in the models. For anchorage bond-zone models, concrete is activated primarily in shear; damage associated with shear response results in increased radial deformation and activation of transverse reinforcement. For flexural bond-zones, the dominant mode of concrete response is axial, and damage associated with axial tensile failure does not produce substantial radial deformation and thus does not activate transverse reinforcement. For the anchorage bond zone models, the higher steel stresses for the models that include radial bond response is attributed to activation of a radial response mode in the bond elements.

<table>
<thead>
<tr>
<th>anchorage (A) or flexural (F) bond model</th>
<th>thickness of concrete cover over bond element (mm)</th>
<th>bond model includes radial action</th>
<th>angle of orientation of bond force resultant (degrees)</th>
<th>concrete fracture energy (MPa-mm)</th>
<th>residual concrete strength as a percentage of initial strength (tension, shear)</th>
<th>transverse steel ratio</th>
<th>bond elements</th>
<th>maximum tensile stress in transverse reinforcement (Mpa)</th>
</tr>
</thead>
<tbody>
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<td>0 no</td>
<td>yes</td>
<td>0.2</td>
<td>(0.1,0.1)</td>
<td>0.04</td>
<td>yes</td>
<td>140</td>
<td>200</td>
</tr>
<tr>
<td>A</td>
<td>0 yes</td>
<td>yes</td>
<td>0.2</td>
<td>(0.1,0.1)</td>
<td>0.04</td>
<td>yes</td>
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<td>250</td>
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<tr>
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<td>0.04</td>
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</tr>
<tr>
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<td>0.2</td>
<td>(0.1,0.1)</td>
<td>0.008</td>
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<td>yes</td>
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<td>0.008</td>
<td>yes</td>
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<tr>
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<td>yes</td>
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<td>0.2</td>
<td>(0.1,0.1)</td>
<td>0.008</td>
<td>yes</td>
<td>20</td>
</tr>
</tbody>
</table>

Note: For transverse steel stress, single values imply that all reinforcement carries approximately the same stress; multiple values indicate wide variation in stress between individual bars.

Table 5.5: Tensile Stress Developed in Transverse Reinforcement as a Function of the Angle of Orientation of the Resultant Bond Force
The results of this study indicate that the introduction of radial bond stresses does not greatly enhance representation of bond-zone response. The radial bond model does not predict bond-zone damage and subsequent loss of bond strength associated with radial bond response in inadequately reinforced systems. The orientation of bond-zone damage typically associated with radial bond response arises naturally in association with orthotropic concrete damage and is affected only slightly by introduction of the radial mode of response within the bond element. Additionally, introduction of radial response establishes a dependence of the model on the bond-element radial stiffness and this stiffness cannot be calibrated on the basis of experimental data.

5.4.5 The Effect of Concrete Fracture Energy and Residual Strength on Computed Response

The proposed concrete constitutive model defines material behavior on the basis of concrete fracture energy and residual tensile and shear strengths. As discussed in Chapter 2, concrete fracture energy is the energy dissipated in creation of a unit area of fracture surface under Mode I cracking. Residual strength defines capacity for transfer of tensile and shear traction across open crack surfaces at extreme levels of deformation. Experimental data indicate that residual strength is essentially zero; concrete fracture energy is defined by experimental testing and typically ranges from 90 N/m to 230 N/m (0.5 lb/in. to 1.3 lb/in.). Comparison of observed and computed behavior for plain concrete systems indicates that determination of these model parameters on the basis of experimental data is appropriate for representation of plain concrete systems (e.g., notched plain concrete beams discussed in Section 5.3). However, in modeling the behavior of reinforced concrete systems, some modification of these factors may be desirable to more closely represent observed behavior of these systems or to facilitate numerical solution of the finite element problem. For example, a non-zero residual tensile strength and increased fracture
energy may be appropriate for representing tension stiffening for even moderately fine meshes for systems in which discrete cracks do not develop naturally. The low fracture energy of plain concrete is reflected in the highly brittle response of concrete specimens subjected to severe tensile and shear loading. For systems composed of plain concrete, reinforcing steel and bond-zone elements, this brittle response may introduce severe problems into standard solution algorithms. Increasing concrete fracture energy may not affect global response of reinforced concrete systems significantly but may enhance numerical solution of these problems.

Figures 5.27 and 5.28 show the load versus deformation history for the anchorage and flexural bond-zone models for the prototype specimen and for a system with ten times the concrete fracture energy of the prototype model. For the flexural bond-zone models, fracture energy acts to smooth the specimen response. As previously discussed, the temporary loss in strength exhibited by the prototype specimen is associated with shear action in the concrete elements adjacent to the reinforcing bar. For the models with high fracture energy, activation of the shear surface is apparently delayed. For the anchorage bond-zone model, concrete damage is dominated by shear action. Deterioration of concrete strength associated with shear transfer across open crack surfaces is much more rapid than that associated with tensile load transfer. Thus, until damage associated with shear response is initiated in the high fracture energy model, this model shows substantially more strength than the prototype specimen.

With the exception of the systems with significantly increased residual shear strength (5%), variation in residual strength has no effect on computed system response. Crack patterns, distribution and extent of damage and global load versus deformation response are similar for these systems. However, for the case of significantly increased residual shear
Figure 5.27: Maximum Bar Stress Versus Displacement Relationship, Flexural Bond-Zone Models with Different Levels of Concrete Fracture Energy

Figure 5.28: Maximum Bar Stress Versus Displacement Relationship, Anchorage Bond-Zone Models with Different Levels of Concrete Fracture Energy
strength, the anchorage bond-zone model shows significantly more strength, and a slightly smaller region of concrete damage than does the prototype specimen.

Investigation of concrete material model parameters indicates that significant variation in concrete fracture energy has a noticeable effect on computed behavior; however, concrete is sufficiently brittle that even increasing the fracture energy to ten times the appropriate value still defines a brittle material. Increase in fracture energy has the most significant effect on systems that exhibit concrete damage under shear action since shear action deteriorates strength more rapidly than does tensile action and this magnifies the variation in the model parameters. Relatively small variation in residual shear strength and relatively large variation in residual tensile strength has essentially no effect on computed response. Significantly increased residual shear strength does affect response, especially for systems in which shear action contributes significantly to accumulated concrete damage.

5.4.6 Effect of Bond Element on Observed Response

In evaluating the proposed bond model and the model’s capacity for representing bond zone response, it is appropriate to question the necessity of explicit representation of concrete-to-steel bond. For both the flexural bond zone and the anchorage bond zone the computed response is considered for finite element models in which concrete and reinforcing steel are connected directly, without introduction of the bond element. Figure 5.29 shows the discrete crack patterns for the flexural model without bond elements. This is comparable to the discrete crack pattern shown in Figure 5.19. Without introduction of the bond element, crack spacings are much smaller than those computed by the prototype finite element model (110 mm and 140 mm) and those observed in experimental investigation (minimum of 130 mm and 140 mm). The crack spacing for the no-bond-element model corresponds to an average bond strength of $1.2 \sqrt{f_c}$ MPa with $f_c$ in MPa ($15 \sqrt{f_c}$
psi). This is significantly higher than the average bond strength of $0.78 \sqrt[3]{f_c}$ MPa (9.4 $\sqrt[3]{f_c}$ psi) computed by the prototype model and the average bond strength of $0.67 \sqrt[3]{f_c}$ MPa (8.1 $\sqrt[3]{f_c}$ psi) and $0.53 \sqrt[3]{f_c}$ MPa with $f_c$ in MPa (6.4 $\sqrt[3]{f_c}$ psi) observed in laboratory models. Figure 5.30 shows the relationship between maximum bar stress and displacement of the loaded end of the bar. Here the computed response does not vary significantly with removal of the bond element from the model.

For the anchorage bond-zone model, variation in computed response between the prototype and the no-bond-element is evaluated on the basis of the concrete damage patterns, global load-displacement response and the strain distribution in the anchored bar. The effect of including the bond element is most obvious in the computed strain distribution. Figure 5.31 shows the anchored bar strain distribution at intervals during the load history. Data in this figure are similar to those presented in Figure 5.22. For the no-bond-element model, deterioration of bond strength near the point of load application is much less than that computed by the prototype model; this results in the anchored reinforcing bar yielding to only a limited depth. For the no-bond-element model, even at maximum loading, concrete damage near the point of load application is not the sufficient to deteriorate concrete stiffness to the point that a significant loss in bond strength, defined by load
transfer between the reinforcing steel and concrete, is observed. Figure 5.32 shows the
global load-displacement response for the prototype and the no-bond-element models in
the form of the relationship between maximum bar stress versus displacement of the
loaded end of the reinforcing bar. As was the case for the flexural bond zone model, the
global load displacement relationship does not show significant variation with introduc-
tion of the bond element. Figure 5.33 shows the distribution of concrete damage for the
model if bond response is not explicitly included in the model. The computed damage pat-
terns are very similar to those computed using the prototype finite element model (Figure
5.19). However, here a greater volume of concrete exhibits significant damage and dam-
age occurs at a greater depth into the concrete anchorage block. These characteristics of the
computed response follow from the fact that when the bond element is introduced into the
model, some displacement develops through deformation of the bond element, while
when the bond element is removed all displacement must develop through deformation of the concrete and steel elements.

5.4.7 Conclusions of Bond-Zone Study

The presented bond zone study provides an improved understanding of the issues associated with modeling anchorage and flexural bond zone regions. The study supports identification of characteristics of the proposed material models that are critical for accurate representation of bond zones. Additionally, data support modification of individual material models to enhance representation of reinforced concrete systems.

Results of the study indicate that the three-dimensional behavior of the bond zone cannot be represented using a two-dimensional finite element model with an acceptable level of accuracy and standardization. As discussed in Section 5.2.1, the two-dimensional bond element and accompanying layer of cover concrete are introduced to represent radial

![Figure 5.31: Computed Distribution of Strain Along Anchored Reinforcing Bar](image)
Figure 5.32: Bar Stress-Displacement Response for Models with and without Bond Elements, Anchorage Bond-Zone Model

Figure 5.33: Computed Concrete Damage Pattern for Anchorage Bond-Zone Model with No Bond Element to Connect Concrete and Reinforcing Steel
response observed in laboratory bond zone investigation. For systems with limited concrete cover and limited transverse reinforcement, this radial response is characterized by development of radial splitting cracks and deterioration of bond strength. For systems with adequate concrete cover and transverse reinforcement, radial response produces localized damage that activates passive confinement provided by transverse reinforcement. Also as discussed in Section 5.2.1, a stiff radial response for the bond element is assumed to ensure that concrete material response, rather than bond element response, determines the global response of the model in the direction perpendicular to a reinforcing bar. However, investigation of bond zone response using models with variable bond element angles, indicates that a substantial increase in the radial forces developed in associated with bond forces essentially has no effect on global model behavior. The most plausible explanation for this observed model response is that radial bond forces are equilibrated through radial action of the bond element. To address this undesirable mode of response in which radial bond forces load the bond element rather than surrounding concrete, the flexibility of the bond element in the radial direction could be increased greatly. This would ensure that radial bond forces be transferred to surrounding concrete. To ensure that this would not provide an inaccurate representation of the response of the system in the direction perpendicular to the reinforcing bar, increased bond element radial flexibility could be accompanied by increasing the relative thickness of the cover concrete in comparison to the in-plane concrete layer. However, reduction in the in-plane concrete layer thickness likely would be accompanied by reduced bond strength and increased damage to in-plane concrete. For any particular system, there is an appropriate distribution of in-plane and cover concrete thickness as well as an appropriate distribution of bond force transfer to cover and in-plane concrete. While this distribution may be represented by a two dimensional
idealization, only a full three-dimensional model provides a practical method for discovering the appropriate three-dimensional distribution of element stiffness.

While the proposed bond model does not represent well the three-dimensional radial bond zone response for general systems, the bond model in combination with the orthotropic concrete damage model does represent well a number of characteristics of this response. Variation in the resultant bond force angle has relatively little affect on the orientation of bond zone cracks in the vicinity of the anchored reinforcing bar, with the result that discrete flexural cracking occurs perpendicular to the bar axis rather than at 60 degrees as observed by Goto [1971]. However, between discrete crack zones or near free concrete surfaces, the transfer of axial bond forces produces shearing of the concrete in the vicinity of the reinforcing bar, and damage to this concrete is characterized by the development of local cracking that is oriented at approximately 45 degrees with respect to the reinforcing bar. As a result, increased deformation of these elements in the direction parallel to the axis of the reinforcing bar is accompanied by radial expansion. Thus, the concrete damage activates confinement of the bond zone. Finally, the bond-zone study data indicate that the global finite element model represents well the average response of the bond zone as characterized by average bond strength, concrete crack patterns and the embedded reinforcement strain history.

Thus, on the basis of the bond zone study, it appears that the most effective bond zone model for implementation in a two-dimensional finite element model likely is one in which the radial bond response is neglected. The prototype model used in the parameter study is used in subsequent analyses. Results of the bond zone study indicate that this model in conjunction with the proposed concrete model represent well observed bond zone response.
Questioning the necessity of an explicit bond element, naturally follows from the proposition that a one-dimensional bond model in conjunction with an orthotropic representation of the damaged concrete continuum adequately predict observed bond zone behavior. It has been suggested that explicit representation of the bond zone is of limited value if radial bond action is not included [Darwin, 1991]. Additionally, researchers have noted that the behavior of many reinforced concrete systems can be adequately computed without explicit representation of the bond zone. However, the bond-zone study indicates that explicit representation of the bond zone is necessary. Data provided by the study indicate that in the absence of bond elements, bond strength may be over-computed leading to computed concrete crack spacing that is significantly less than that observed in the laboratory. Additionally, data provided by the study indicate that the behavior of reinforced concrete systems that exhibit yielding of reinforcing steel many not be adequately computed unless explicit representation of observed deterioration of bond strength associated with yielding reinforcement is provided. From this it follows that the bond element is necessary to represent the characteristics of reinforced concrete system response that do not arise naturally from interaction of the proposed concrete and steel elements. Thus, it seems likely that the bond model is necessary to represent the observed increase in bond strength developed in concrete compression zones since load transfer between an undamaged concrete element and a steel element does not depend on the concrete stress state and load transfer between a damage concrete element and a steel element only depends of the concrete stress state if applied loading is such that concrete cracks are forced closed. These specific examples combined with numerous other likely examples provide adequate incentive for including one-dimensional bond zone representation in subsequent finite element models.
5.5 Analysis of Anchorage Bond Zones in Reinforced Concrete Building Beam-Column Connections

The anchorage investigation by Viwathanatepa [1979a, 1979c] provides data for evaluating fully the proposed bond model in combination with the concrete and steel material models. Figure 5.34 shows the Viwathanatepa test setup. The specimen and load configuration are representative of a building beam longitudinal reinforcing bar anchored in a beam-column connection. As in the actual system, load applied to the ends of the anchored reinforcing bar is accompanied by the development of a flexural compression zone near the compression end of the reinforcing bar and a flexural tension zone near the tension end of the reinforcing bar. Additionally, the anchorage length is sufficiently long for these models that the tensile and compressive strength of the reinforcing bar is developed at moderate to extreme slip levels. Thus, the concrete and steel material state varies substantially along the length of the anchored reinforcing bar. Finally, the Viwathanatepa investigation provides data characterizing anchorage bond-zone response for system subjected to reversed cyclic loading in the post-yield regime. Thus, comparison of observed and computed response for these models provides a means of evaluating the adequacy of the proposed bond model for predicting bond zone response under severe loading conditions similar to those that develop under earthquake loading of reinforced concrete bridges.

Figure 5.35 shows the finite element model used to predict the behavior of the Viwathanatepa models. The geometry of the Viwathanatepa model, as incorporated into the two-dimensional finite element model, is defined in Table 5.6. Concrete and steel material properties are listed in Table 5.7 with bold values identifying values approximated as part of the current study. In this finite element model, no radial bond response is considered and no cover concrete is included in the model. Loading is controlled by pre-
Figure 5.34: Loading of a Laboratory Model by Viwathanatepa [1979a, 1979c]

Figure 5.35: Finite Element Model of Loading of the Typical Laboratory Specimen Tested by Viwathanatepa [1979a, 1979c]
scribing the horizontal displacement history at one or both exposed ends of the bar. In the laboratory, for the case of push-pull loading, testing was controlled by prescribing a displacement history at the pull-end of the reinforcing bar and applying to the other exposed end of the bar a compressive load of equal magnitude to that developed in the pull-end actuator. Such load control is not possible for the finite element model; instead, for the case of push-pull loading, displacement histories are prescribed at both ends of the anchored reinforcing bar.

<table>
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<tr>
<th>design parameter</th>
<th>value</th>
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</tr>
<tr>
<td>height of block</td>
<td>48 in.</td>
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<tr>
<td>thickness of block</td>
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<tr>
<td>diameter of anchored reinforcing bar (nominal)</td>
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</tr>
<tr>
<td>diameter of longitudinal reinforcement (nominal)</td>
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<tr>
<td>diameter of transverse reinforcement (nominal)</td>
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<td>longitudinal steel ratio</td>
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<tr>
<td>transverse steel ratio (in-plane area ratio in the vicinity of the anchored bar)</td>
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Table 5.6: Design Parameters for Laboratory Models Tested by Viwathanatepa et al. [1979c]

In comparing the computed and observed response for the model defined in Figure 5.34, behavior appropriately is defined by both global and local response quantities. The most basic of these is the relationship between load applied to the exposed end of the reinforcing bar and slip of the reinforcement. In the laboratory Viwathanatepa defines slip as the difference in displacement of the exposed end of the reinforcing bar and the surface of the concrete anchorage block at a “small distance” from the reinforcing bar. Evaluation of the instrumentation set-up used by Viwathanatepa suggests that this distance is approximately 1 to 2 inches (25 to 50 mm). Measurement of the bar strain in the exposed portion
of the reinforcing bar provides a means for eliminating this deformation from the reported slip measurements. For the finite element model, slip of the reinforcing bar is defined by the displacement of the reinforcing bar at the surface of the anchorage block with respect to the concrete block at a distance of 1 in. from the bar. For both the laboratory and computer models, load is defined by the stress developed in the exposed ends of the reinforcing bar. The distribution of strain in the anchored reinforcing bar as a function of maximum applied load defines the response of the model. These data are generated by Viwathanatepa using strain gages attached to the anchored reinforcing bar at distributed locations.

<table>
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<tr>
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<th>value</th>
<th>steel material properties</th>
<th>value</th>
<th>transverse and longitudinal reinforcement</th>
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<td>elastic modulus</td>
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<tr>
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<td>strain at which strain hardening initiates</td>
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<td>1.79%</td>
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<tr>
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<td>1000 ksi</td>
<td>1000 ksi</td>
</tr>
<tr>
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<td>102 ksi</td>
</tr>
<tr>
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<td>strain at which tensile strength is achieved</td>
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<td>10%</td>
</tr>
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<td></td>
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<td>strain at which tensile fracture occurs</td>
<td>20%</td>
<td>20%</td>
</tr>
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</table>

Table 5.7: Material Properties for Laboratory Models Tested by Viwathanatepa et al. [1979c]
5.5.1 Comparison of Computed and Observed Response for Monotonic Loading in Tension

Comparison of computed and observed behavior for the specimen subjected to monotonic loading at one exposed end of the reinforcing bar provide a means of confirming the results of the previous anchorage zone study. Figures 5.36 and 5.37 show the observed bar stress versus slip relationships for an anchored bar subjected to displacement-controlled tensile loading at one exposed end of the bar. The applied load history is defined by monotonically increasing tensile loading with a single unload-reload cycle initiated at a relatively large slip level (Point A’ in the Figures). The data presented in Figures 5.36 and 5.37 define the capacity of the system (Points B and C). Additionally, assuming that the unload cycle initiating at Point C characterizes anchorage failure, the data define the loss in strength and resulting reduced bar elongation resulting from pull-out. The data presented in the Figures indicate significant bond strength at relatively large slip levels at both the loaded and unloaded ends of the bar. These data show also a brittle failure, with loss of bond strength accompanied by reduced slip. The capacity of the instrumentation and data acquisition system to accurately measure the brittle failure response shown in Figures 5.36 and 5.37 is unlikely, bringing into question the entirety of the response history beyond Points B and C. The behavior of the system under monotonic loading is defined also by the strain distribution along the anchored length of the reinforcing bar.

The response of a reinforcing bar subjected to monotonic loading is characterized also by the strain distribution along the length of the anchored bar. For the specimen under consideration, the strain distribution along the anchored bar at various stages of the load history are shown in Figures 5.38, 5.39 and 5.40. These data show progressive yield penetration associated with deterioration of bond strength under increased loading. Additionally, under increasing loading, the data show reduction in the slope of the strain
Figure 5.36: Bar Stress Versus Slip for Loaded End of Anchored Bar [Figure 4.9 from Viwathanatepa et al., 1979]

Figure 5.37: Bar Stress Versus Slip for Non-Loaded End of Anchored Bar [Figure 4.10 from Viwathanatepa et al., 1979]
distribution for the region of the distribution that falls between the approximately maximum strain and minimum strain. This reduced slope results in part from the reduced slope of the steel stress-strain relationship at increasing post-yield stress levels, but likely indicates reduced bond strength as well. Also, under increased loading, the data show the development of a region near the point of load application of approximately 4 inches (102 mm) in length along which the strain in the reinforcing bar is approximately constant. A constant strain region implies constant stress and thus zero bond strength. In addition to tensile yielding of the anchored bar and development of a flexural tension zone, this extreme deterioration in bond strength likely follows from the development of a pull-out cone focussed at the point of load application. Viwathanatepa notes that cone formation is evinced by surface crack patterns observed at a bar stress level equal to the yield strength of 68 ksi (470 MPa) and that the 4.5 in. (110 mm) radius cone is completely disconnected from the specimen to a depth of 3.0 in. (76 mm) at a bar stress of approximately 80 ksi (550 MPa). At bar stress levels approaching the tensile strength of the reinforcement, this region of approximately zero bond strength extends deeper into the specimen.

![Graph showing strain distribution along anchored reinforcing bar](image)

**Figure 5.38: Strain Distribution Along Anchored Reinforcing Bar, Load Corresponds to Yield Strength of the Bar [Figure 4.16c from Viwathanatepa et al., 1979c]**

The data presented in Figures 5.38, 5.39 and 5.40 show the development of lower strains and stresses in the strain gage that is closest to the point of load application than in
the adjacent gage that is further from the point of load application. If the first strain gage has an embedment length of zero, then the strain data from this gage would be expected to be the largest along the length of the reinforcing bar since the bar stress outside the
anchorage block is the entire applied load with no load distributed to the anchorage block. Viwathanatepa explains that the relative high strains at the point of the second gage result from the formation of a single crack in the vicinity of the second strain gage. Formation of a discrete crack at the location of a strain gage could be expected to result in high local stresses and strains since the reinforcing bar must carry all tensile load in the vicinity of the crack. However, the only way for the measured strain to be larger at the point of the second strain gage is if something restrains elongation of the bar outside the anchorage block resulting in a reduction in the measured strain or if the first gage has a small but finite embedment length such that the strains measured in the bar reflect some distribution of bar load to the surrounding concrete at the point of the first gage. Either explanation is equally plausible and might has a significant impact on the current investigation. Of interest to the current investigation is the depth of yield penetration and the fact that development of a pull-out cone can determine observed response.

The distribution of steel strain and stress along the embedded length of the reinforcing bar, the slip of both ends of the reinforcing bar and the concrete damage patterns may used as a basis for comparing the observed response with the response computed using the finite element model. The computed concrete damage patterns (Figure 5.41) indicate a triangular-shaped region, approximately 3 in. (80 mm) deep and 2 in. (50 mm) wide in the vicinity of the load point in which limited concrete cracking is initiated. Concrete damage is minor and does not appear to reduce concrete strength and stiffness substantially. While the computed damage pattern represents some of the damage observed in the laboratory (Figure 5.42), the finite element model does not show development of the splitting-type cracks observed along the anchorage length of the reinforcing bar nor does it represent completely the concrete cone pull-out. The limited concrete damage indicated by the computer model appears to follow from the two-dimensional representation of the three-
dimensional system. The computer model implies that the entirety of the concrete model thickness is activated in resisting bond response, whereas the formation of the pull-out cone in the three dimensional physical model indicates that only a portion of the thickness is activated.

Figures 5.43 and 5.44 show the computed and observed bar stress versus slip response for both the loaded and non-loaded ends of the reinforcing bar. In general, these data show a good correlation between computed and observed response. As previously discussed, deviation in the computed and observed failure path may be due to misrepresentation of the observed failure path rather than model inaccuracy. The computed failure path does deviate substantially from the observed response at slip levels in excess of 0.7 in. (17 mm) for the pull-end. For the computer model, bond failure is computed to initiate at a slip level of approximately 0.7 in. (18 mm) at the load-end and approximately 0.06 in.

Figure 5.41: Computed Concrete Crack Patterns for Anchorage Specimen Subjected to Monotonic Tension-Only Loading as Tested by Viwathanatepa et al. [1979c]
This is less slip at the loaded end of the bar than the 1.0 in. (25 mm) bond-failure slip observed in the laboratory and approximately the same slip at the no-load end of the bar as the 0.065 in. (1.7 mm) bond-failure slip observed in the laboratory. While the computed and observed failure slip at the no-load end of the bar are approximately the same, the data show that in general the computed slip at the no-load end of the reinforcing bar is greater than that observed in the laboratory. This is obvious
from consideration of the unload-reload cycle imposed in the computer simulation. The computer-model unload cycle occurs at a load-end slip level that is less than that used in the laboratory investigation; however, the data show that at the no-load end of the bar, this unload cycle occurs at slip levels in excess of that corresponding to the unload cycle in the laboratory. An explanation for the variation in the computed and observed response follows from consideration of the computed steel strain distribution as presented in Figure 5.45. These data show computed peak strain and corresponding depth of yield penetration that are approximately the same as that observed in the laboratory. However, the data presented in Figure 5.45 show also an approximately constant strain gradient along the yielded portion of the reinforcing bar. This distribution differs somewhat from the observed strain distributions (Figures 5.38, 5.39 and 5.40) that show a limited region near the point of load application for which the strain gradient is quite small. For a given level of bar stress, the difference in the post-yield strain distributions would be expected to result in substantially less slip at the loaded end of the bar for the computer model than for the laboratory model. However, for any particular level of slip, the variation in the strain distributions would be expected to produces greater slip at points along the embedded length of the bar for the computer versus laboratory model. Thus, the discrepancies between computed and observed response appear to follow from inadequate representation of the loss of bond strength in the vicinity of the applied load.

5.5.2 Comparison of Computed and Observed Response for Monotonic Simultaneous Loading in Tension and Compression

From the case of monotonic tensile loading, evaluation of the model continues to include comparison of computer and laboratory model behavior for the case of monotonic loading in both compression and tension. As previously discussed, in the laboratory, this load condition is established through displacement control at the tension-end of the bar
Figure 5.43: Observed and Computed Bar Stress versus Slip Response for the Loaded End of the Reinforcing Bar

Figure 5.44: Observed and Computed Bar Stress versus Slip Response for the No-Load End of the Reinforcing Bar
and load control to achieve the same bar stress at both ends of the bar. For the computer model, such “load-control” is not possible and loading is achieved through displacement control at both exposed ends of the bar. Several different relative displacement paths are used in the computer analysis. The computed and observed response histories for two of these displacement paths are shown in Figures 5.46 and 5.47. As would be expected, bar stress versus slip response is a function of the total load applied to the model. Model response observed in the laboratory is defined further by the strain distribution along the length of the anchored bar. Data in Figure 5.48 show the observed strain distribution for loading approximately equal to the maximum. These strain distributions may be compared with the computed strain distributions shown in Figures 5.49 and 5.50.

Figure 5.45: Strain Distribution Along Embedded Length of the Reinforcing Bar

The laboratory and computer generated data show the same general characteristics of model response. The limited slip levels developed at push-end of the bar in comparison with the pull-end of the bar indicate the greater bond strength at the push-end of the bar.
The greater bond capacity at the push-end of the bar is reflected also in the strain distribution data that show significantly less yield penetration at the push-end of the model. The increased bond strength at the push-end of the model is due to the enhanced bond conditions in the vicinity of push-end of the bar. For the computer model, the enhanced bond conditions at the push-end of the bar are defined by significant concrete compression perpendicular to the bar, the lack of open fictitious concrete crack surfaces perpendicular to the bar and compressive yielding of the bar.

Detailed comparison of computed and observed response is questionable given that the laboratory and computer models are subjected to different load histories; however, comparison of computed and observed strain distributions indicates two characteristics of response that are independent of the load history. As was the case for the monotonically applied pull-only loading, the observed strain distribution shows a localized region in the vicinity of the applied tension load that having substantial deterioration of bond strength. In Figure 5.48, the depth of this zone at maximum load is approximately the same as observed in the pull-only monotonic tests, 4.0 in. (100 mm). Also as observed for the monotonically applied tension-only loading, this phenomenon is not observed in the computer simulation. Consideration of the computed strain distributions indicates that the depth of strain penetration at the tension end of the model varies between 12.5 and 15 in. (320 and 380 mm) depending on the load path. This is somewhat larger than the 10 in. (250 mm) depth observed in the laboratory. It seems unlikely that a different load history might produce a strain distribution with less yield penetration than observed in the laboratory. Thus, it is reasonable to conclude that for this load case the computer over estimates the depth of yield penetration. Yield penetration on the push-end of the bar appears to be limited to the first interval of measurement for both the laboratory and computer models.
Figure 5.46: Observed and Computed Bar Stress versus Slip Response, Load Applied to Both Ends of the Bar - Simulated Displacement History One

Figure 5.47: Observed and Computed Bar Stress versus Slip Response, Load Applied to Both Ends of the Bar - Simulated Displacement History Two
Figure 5.48: Observed Strain Distribution Along the Embedded Length of the Anchored Bar for Maximum Load Applied at Both Ends of the Bar [Figure 4.19g from Viwathanatepa et al., 1979c]

Figure 5.49: Strain Distribution Along Embedded Length of the Reinforcing Bar, Load Applied at Both Ends of the Bar - Simulated Displacement History One
Comparison of the response of the laboratory and computer models indicates that the computer model predicts well the observed bond-zone behavior for this load case. The model predicts global response and strain distribution with acceptable accuracy. Discrepancy between computed and observed response appears to be limited to the finite element model predicting greater bond strength in the vicinity of tensile load application than is observed in the laboratory. This discrepancy likely is due to the two-dimensional versus three-dimensional representation of the system.

5.5.3 Comparison of Computed and Observed Response for Cyclic Loading

The experimental investigation by Viwathanatepa also provides data characterizing anchorage behavior under reversed cyclic loading. In the laboratory, the test consists of applying tension and compression loads of equal magnitude at the two exposed ends of the anchored reinforcing bar. The load history is defined by forcing one end of the anchored...
bar through a displacement path that consists of three full cycles each at an increasing amplitude. Figure 5.51 shows the observed bar stress versus slip history for the bar end that is subjected to tensile loading at the beginning of the test. These data are similar to those provided by other researches as discussed in Chapter 4. Defining characteristics of the response history include the form of the unload and reload paths, maximum bond strength, deterioration of bond strength under cyclic loading to a given maximum displacement and more rapid deterioration of bond strength at larger slip levels. In particular, these data show that peak bond strength under cyclic loading is approximately 80 percent of that observed under monotonic loading. These data may be compared with the computed bar stress versus displacement history (Figure 5.52). As was the case for the monotonic tests, the exact load history applied in lab cannot be reproduced in the computer model. Instead the data presented in Figure 5.52 define the computed response for a system subjected to the same reversed-cyclic displacement path at both exposed ends of the anchored bar. As shown in Figure 5.53, this simulated loading results typically in a slightly smaller tension load being applied than compression load. The computed response shows a peak bond strength under cyclic loading that is approximately equal to that developed under monotonic loading. For cycles to relatively large slip levels, the computed response history shows deterioration of bond strength that is approximately the same as that observed in the laboratory. The model predicts a less gradual re-load path than is observed in the laboratory.

5.5.4 Conclusions of Bond-Zone Study

Results of the anchorage bond-zone study show that the proposed finite element model represents well the observed behavior of reinforced concrete sub-systems. The proposed non-local representation of bond response as a function of the concrete and steel material state in the vicinity of the bar provides a robust method for predicting the bond
stress versus slip relationship in sub-systems subjected to general loading. In general, the model predicts the global load versus slip history and the local steel strain history for these systems. The analytical and experimental data do indicate that the finite element model
may not adequately represent the rapid deterioration of bond strength in a flexural tension zone in the vicinity of a free concrete surface. However, it is not obvious that this will be an issue for analysis of continuous systems. Investigation of the previously proposed pseudo-three-dimensional representation of the bond zone, with a two dimensional bond element that represents radial bond response and the introduction of cover concrete to represent the concrete volume outside the limited bond zone, suggests that such a model does not adequately represent the three dimensional bond response. Additionally the study suggests that the additional complexity of the pseudo-three-dimensional model does not warrant the limited improvement in the representation of bond response. Thus, the anchorage bond zone study indicates that the limited one-dimensional bond model is sufficient to represent bond response in a two-dimensional finite element model.

Figure 5.53: Computed Bar Stress at Both Exposed Ends of the Anchored Bar
5.6 Analysis of Reinforced Concrete Panels Subjected to Shear

Identification and evaluation of limited bond-zone sub-assemblies establishes the modeling technique and the model parameters that are appropriate for predicting the behavior of reinforced concrete systems subjected to tension-compression loading. The next phase of model verification and calibration focuses on predicting the response of reinforced concrete sub-systems subjected to shear type loading. In bridge sub-assemblies, accurate representation of shear response may be critical to prediction of beam-column connection response. In these systems, anchorage zones define the perimeter of the beam-column connection core. If the distribution of bond stress in these anchorage zones is approximately uniform, then the connection core is subjected to an approximately uniform shear loading (Figure 5.54a). If bond stress is significant only near the compression end of the bar, in the flexural compression zone, the connection core is subjected to shear in the form of diagonal compression loading (Figure 5.54b). Experimental investigation of building and bridge beam-column connections indicates that connection strength may be determined by the capacity of the connection core concrete to carry uniform shear and compression-strut shear. Thus, in evaluating the proposed finite element model, identification of appropriate model parameters for predicting the behavior of reinforced concrete continua to shear loading is necessary.

A number of researchers provide data characterizing the response of reinforced concrete panels subjected to shear-type loading. Here data provided by Vecchio and Nieto [1991] and by Stevens et al. [1991] are considered. These data define the response of relatively thin reinforced concrete panels subjected to uniform shear loading in the Shell Element Tester located at the University of Toronto.
Figure 5.54: Idealization of Uniform and Diagonal Compression-Strut Shear Loading of a Reinforced Concrete Beam-Column Connection

Figure 5.54a: Beam-Column Connection Core Subjected to Uniform Shear Loading for the Case of Uniform Bond Stress in Anchorage Zones

Figure 5.54b: Beam-Column Bridge Connection Core Subjected to Diagonal Compression Strut Shear Loading for the Case of Significant Bond Stress Only in Compression Zones

Figure 5.54: Idealization of Uniform and Diagonal Compression-Strut Shear Loading of a Reinforced Concrete Beam-Column Connection
5.6.1 Analysis of Panels Tested in Shear

Nieto [1989] presents the results of an experimental investigation of shear friction in reinforced concrete panels. The results of this study and a proposed constitutive model defining the behavior of reinforced concrete continua are presented by Vecchio and Nieto [1991]. The investigation considers the behavior of reinforced concrete panels subjected to uniform shear loading and subjected to uniform shear in combination with additional tension or compression loading in one direction. System response is defined by the average applied load and the shear deformation of middle of the panel. The data provided by the shear-only tests are appropriate for comparison with computed load versus deformation histories and provided a means of verifying the models capacity to represent the response of reinforced concrete sub-systems subjected to these types of load histories.

![Figure 5.55: Typical Reinforced Concrete Panel Tested by Vecchio and Nieto](image)

The focus of the Vecchio and Nieto investigation is the characterization of shear strength as determined by shear friction for reinforced concrete panels. Figure 5.55 shows
the geometry and reinforcing details for a typical panel tested by Vecchio and Nieto, and Figure 5.56 shows an idealization of the loads applied to a panel. Table 5.8 lists geometric and material parameters reported by the investigators and as estimated by the author for analysis (estimated values are bold). All but one of the panels are subjected to loading in axial tension to a maximum average tensile stress of 3.5 MPa to pre-crack the panel. The models are then unloaded and reloaded in shear combined with tension or compression in one direction. A group of forty actuators attached to steel friction keys embedded in a panel at five-points on each side of the panel are controlled to apply uniform shear and axial stress to the panel (Figure 5.56). Panel response is characterized on the basis of the average shear strain of the interior section of the panel as defined by the relative deformation of the instrumentation points shown in Figure 5.56.

Figure 5.56: Idealized Loading of Reinforced Concrete Panels Tested by Vecchio and Nieto
Behavior of the individual panels is determined by the tension/compression force applied in combination with shear (though all of the panels exhibit similar damage patterns), shear stress-strain histories and failure mechanisms. In particular, the response of the pre-cracked panel tested in pure shear is consistent with that exhibited by the remainder of the panels. For the panels that are pre-loaded in tension, a typical crack pattern developed as a result of the pre-load consists of four cracks perpendicular to the direction of the applied tensile loading and spanning the entire height of the panel; these cracks are located in the vicinity of the end of the short horizontal reinforcing bars (Figure 5.57). Pre-cracking appears to have relatively little effect on observed response under pure shear, as the cracks developed under initial tensile loading remain approximately at the unload-width under the second phase of loading (shear loading) and do not appear to determine the orientation of cracks that form under the shear loading. For the panels subjected to pure shear loading, Vecchio and Nieto note that in the outer strips of the panel, crack spacing is approximately 100 mm and data indicate that cracks are oriented at approximately 45 degrees. In the inner strips of the panel, Vecchio and Nieto note that crack spacing is

<table>
<thead>
<tr>
<th>concrete material properties</th>
<th>value</th>
<th>steel material properties</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>modulus</td>
<td>21 GPa</td>
<td>elastic modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.175</td>
<td>yield strength</td>
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</tr>
<tr>
<td>tensile strength</td>
<td>2.8 MPa</td>
<td>strain at which strain hardening initiates</td>
<td>2.2%</td>
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<tr>
<td>compressive strength</td>
<td>29.8 MPa</td>
<td>hardening modulus</td>
<td>6.9 GPa</td>
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<td>fracture energy</td>
<td>0.25 MPa-mm</td>
<td>tensile/compressive strength</td>
<td>520 MPa</td>
</tr>
<tr>
<td>residual tensile strength</td>
<td>0.1%</td>
<td>strain at which tensile strength is achieved</td>
<td>10%</td>
</tr>
<tr>
<td>residual shear strength</td>
<td>5%</td>
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Table 5.8: Material Parameters for Reinforced Concrete Panels Tested by Vecchio and Nieto
between 120 and 200 mm and cracks are oriented at 45 degrees. Figure 5.58 shows concrete crack patterns for the panel subjected to pure shear loading at a load level that is approximately equal to the panel strength. Figure 5.59 shows the average shear stress versus average shear strain observed for the panel subjected to pure shear loading following pre-cracking in tension and for the panel subjected to pure shear loading with no pre-cracking in tension. The investigators note that for the non-precracked panel, failure is due to pull-out of one of the shear keys and that a change in the detailing of the connection between the keys and the reinforcing steel mesh prevented shear key failure in subsequently tested panels.

**Figure 5.57: Typical Crack Pattern for Panel Subjected to Uniaxial Tensile Loading to 3.5 MPa**

Panel behavior is computed using the finite element model shown in Figure 5.60. The panel is represented as 900 by 900 by 70 mm (35.3 by 35.3 by 2.75 in.) and the concrete mesh is 18 by 18 elements. As a result, individual elements are 50 by 50 mm (2 by 2 in.) and in the most densely reinforced areas, individuals elements are bounded on both sides by reinforcing bars. All reinforcing steel elements are connected to the concrete elements via bond elements. Material parameters provided by Vecchio and Nieto and as estimated for the analysis are listed in Table 5.8
Evaluation of the Vecchio and Nieto panel response provides a means of assessing the proposed representation of shear transfer in concrete that has been damaged under tensile loading. The model proposed in Chapter 2 defines the capacity for shear transfer across an open fictitious crack on the basis of the following exponential function:

$$f = f_s - f_s(1 - \exp(-H\alpha))(1 - \beta_s)$$  \hspace{1cm} (5-6)
where \( f \) is the shear traction transferred across the open crack, \( f_s \) is the undamaged shear-friction strength defined on the basis of limited experimental data to be approximately \( 0.4f_c \), \( H \) is a damage parameter defined on the basis of the concrete fracture energy, \( \beta_s \) is the residual shear strength assumed to be approximately 0.05 and \( \alpha \) is the concrete damage parameter defining the extent of damage on a particular crack surface. In preliminary analysis of the panels it was observed that computed deterioration of panel strength and stiffness was significantly more rapid than was observed in the laboratory. Evaluation of the results of these preliminary investigations suggests that excessive loss of strength and stiffness may be attributed to inappropriate activation of the shear failure surface as defined by Equation (5-6). Over-activation of the shear traction surface may be addressed by introduction of a relatively high shear strength. If the shear strength is high, then the shear surface will not be activated initially when concrete damage is limited. Instead, damage will accumulated due to transfer of tensile tractions across the crack surface. Once significant damage has occurred (due to activation of the surface in tension), the capacity for shear transfer across the crack will be minimal, regardless of the original shear strength. Over-prediction of concrete deterioration due to activation of the shear surface could be addressed in the current model also by defining a relatively large residual shear strength. However, as observed in the bond zone study, increased residual shear strength may increase system strength considerably. Additionally, for several damaged concrete elements this provides substantially more capacity for transfer of shear traction across crack surfaces than is observed in the laboratory. Introduction of an independent concrete damage parameter to define concrete damage due to shear transfer across a fictitious crack surface likely is the most desirable approach to enhancing representation of system response; however, this severely complicates the proposed model, requires data defining concrete response under Mode II fracture and requires development of a relationship
between damage to the shear traction surface and the normal traction surface. Thus, for the current investigation increased capacity for shear transfer for undamaged concrete is taken as the most appropriate method for improving prediction of system response. Analyses are conducted using three different shear strengths for undamaged systems: $0.4f_c$, $f_c$ and $10f_c$.

Table 5.9 lists assumed material parameters for all of the finite element models included in the current study.

<table>
<thead>
<tr>
<th>finite element model identification</th>
<th>concrete shear strength</th>
<th>concrete fracture energy</th>
<th>loading frame</th>
<th>initial damage</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
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<td>VSN04</td>
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<td>0.25MPa-mm</td>
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</tr>
<tr>
<td>VSN10</td>
<td>$10f_c$</td>
<td>0.25MPa-mm</td>
<td>stiff</td>
<td>no</td>
</tr>
<tr>
<td>VSD1</td>
<td>$f_c$</td>
<td>0.25MPa-mm</td>
<td>stiff</td>
<td>yes</td>
</tr>
<tr>
<td>VFN1</td>
<td>$f_c$</td>
<td>0.25MPa-mm</td>
<td>flexible</td>
<td>no</td>
</tr>
</tbody>
</table>

**Table 5.9: Finite Element Model Parameters**

While laboratory loading of these panels consists of applying uniform tension followed by unloading and re-loading under uniform shear in combination with tension or compression, such loading is not possible in the computer simulation. For the computer model, initial pre-loading and the resulting concrete damage is introduced by pre-damaging concrete elements in the vicinity of the cracks observed in the laboratory specimen. Specifically, the two columns of concrete elements on the interior side of the short horizontal reinforcing bars are damaged initially (Figure 5.60). These elements are damaged minimally with the concrete damage parameter set to provide a concrete tensile stress that is approximately 99 percent of the uncracked strength. The goal of pre-cracking is simply to define the orientation of the fictitious concrete crack planes for the pre-cracked elements. It should be noted that if the concrete elements are initially cracked under tensile loading, the crack surfaces are established at 0 and 90 degree rotations from the horizon-
tal. As a result, another crack surface cannot develop at a 45 degree angle from the horizontal, as is observed in the laboratory. Table 5.9 identifies the finite element models for which the specified concrete element are initially damaged.

Load is applied through displacement control at two extreme nodes, stiff frame at the exterior of the panel provides distribution of load along panel edges

Load is applied through displacement control at two extreme points of a flexible frame connected to the panel. The flexible frame provides uniform distribution of the shear load to the edges of the panel

• The concrete mesh is 18 by 18 elements
• In the most heavily reinforced areas, reinforcement is 2 - 6 mm diameter bars spaced at 50 mm
• All reinforcing bars are connected to concrete elements via bond elements
• Grey concrete elements are damaged prior to application of shear load.

**Figure 5.60: Finite Element Models of Panels Tested by Vecchio and Nieto**

Laboratory loading of the panels in shear or shear combined with axial tension or compression is accomplished under load control. Vecchio and Nieto indicate that this loading is achieved through application of approximately uniform loads to the five shear keys embedded along each of the panel edges. Again, load control is not possible in the computer simulation nor is application of shear load in combination with a prescribed
level of tension or compression loading. In the computer simulation, only the case of pure shear load is considered, and this loading is applied under displacement control. Two different load frames are used to simulate the distribution of loading on the edges of the panel; these are shown in Figure 5.60. Load Frame A (Figure 5.60) consists of a relatively stiff frame composed of truss elements, connected to every node of the panel edge and subjected to point loading through displacement control at the two extreme nodes of the panel. The stiff frame provides for redistribution of the loads along the edges of the panel as a function of the panel damage. Given the stiffness of the exterior frame and the concrete damage pattern, significant redistribution of the load is observed and an absolutely uniform loading is not achieved. Load Frame B (Figure 5.60) consists of a relatively flexible frame composed of axial elements and connected to the panel at five points along each edge of the panel. Here the load frame is sufficiently flexible that relatively little redistribution of applied load occurs and an approximately uniform shear load is applied to each edge of the panel.

For the finite element models, average shear load and shear strain are defined differently for the two different load frames. For the stiff frame, shear stress is defined by the total load applied at the extreme nodes of the load frame and the shear surface of the panel. For the flexible frame, shear stress is defined on the basis of the load transferred from the flexible frame to the concrete panel at each node on the panel edge. For both frames, shear deformation is defined by the relative deformation of the model nodes at the measurement points used in the laboratory experiments.

The computed and observed behavior for the panels are compared on the basis of the concrete damage patterns as well as the global shear stress versus shear strain behavior. Table 5.9 lists the characteristics of the six finite element models used to compute the behavior of the panels tested by Vecchio and Nieto. Figures 5.61 through 5.63 show the
computed concrete damage patterns under moderate levels of shear loading for the three finite element models that have an initial shear friction strength equal to the concrete compressive strength. These data are intended to show the orientation of discrete cracking in the concrete panel prior to the development of highly localized failure mechanisms. Thus each figure shows a panel under a different level of shear load and at a different level of concrete damage. In general the data show the zones of discrete cracking to be oriented perpendicular to the direction of maximum principal tensile stress at an angle of approximately 45 degrees, as observed in the laboratory. The data presented in Figure 5.61 show an average crack spacing of approximately 200 mm; this compares well with the maximum crack spacing observed in the laboratory. Comparison of the data presented in Figures 5.61 and 5.62 shows the effect of initial cracking on the concrete damage state. For the system in which initial damage is imposed (Figure 5.62), the discrete damage zones show a change in orientation as the zone of initial damage is crossed. Additionally, within the most heavily damaged region of the panel, the concrete elements that are initially damaged accumulate less damage than do adjacent elements. In the laboratory, the initial concrete cracking does not appear to affect the orientation of concrete cracking developed under shear loading. The currently proposed concrete material model allows for development of two orthogonal crack surfaces, these panels represent a ‘worst-case’ scenario for computing response using this concrete model as the principal tensile stress in the post-crack state is oriented at exactly between the two fictitious crack surfaces at an angle of 45 degrees from the elastic stress state under which the fictitious crack surfaces are defined. Data in Figures 5.64, 5.65 and 5.66 show the orientation of the fictitious crack surfaces for finite element model VSN1 for which no initial damage is introduced, for model VSD1 in which the concrete is damaged initially to represent pre-loading in tension and for model VFN1 in which load is applied to the panel using a flexible frame and no initial concrete
damage is introduced. These data show the influence of initial cracking on the orientation of crack surfaces.

Figure 5.61: Discrete Concrete Cracking as Computed Using FE Model VSN1

The computed and observed panel behavior is compared also on the basis of the shear stress versus shear strain histories. As previously discussed, shear stress is assumed to be uniform for the panel and is defined on the basis of the total applied load and the total volume of the panel. The shear strain defines the engineering shear deformation of the interior zone of the panel that is reinforced only in the horizontal direction. Figure 5.67 shows the behavior observed in the laboratory as well as the computed response for each of the four finite element models in which load is applied to the panel through a stiff loading frame. The computed response is initially linear elastic. Once cracking of the concrete

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1. Note that the smoothed angle of orientation of the fictitious crack surface is presented with the result that for two adjacent element, one with fictitious crack surfaces oriented at 0 degrees and one with crack surfaces oriented at 45 degrees, the smoothed angle of orientation calculated at the node is 23 degrees
Figure 5.62: Discrete Concrete Cracking as Computed Using FE Model VSD1

Figure 5.63: Discrete Concrete Cracking as Computed Using FE Model VFN1
initiates, shear strength initially decreases with increasing shear strain as damage distributes throughout the panel. Eventually, one or more discrete crack zones dominate the response and shear strength begins to increase with increasing shear strain. Failure of the system occurs when all of the reinforcing steel crossing the center of the panel yields and the model becomes unstable. Since the laboratory testing is conducted under load control, it is not possible to observe a region of response in which shear strength deteriorates. Further, even if the panels were tested in the laboratory under displacement control, it is likely that this region of the response curve might not be observed due to insufficient stiffness of the testing frame or due to limitations in the test procedure. That the computed response recovers strength to match the observed response suggests that the momentary strength loss is necessary for the model to represent the post-cracking response of the panel and that it may represent the behavior of physical models under ideal loading conditions. For

Figure 5.64: Orientation of Fictitious Crack Surfaces as Computed Using Finite Element Model VSN1
Figure 5.65: Orientation of Fictitious Crack Surfaces as Computed Using Finite Element Model VSD1

Figure 5.66: Orientation of Fictitious Crack Surfaces as Computed Using Finite Element Model VFN1 (Flexible Exterior Frame is Shown)
finite element model VSN04, the computed response does not progress to the point of strength recover. Instead failure of the system occurs at a strength below that of the cracking strength. Here it is assumed that deterioration of concrete strength and stiffness due to activation of the shear surface produces the premature failure of the system. Finite element model VSD1 indicates a failure strength that is 25 percent less than that observed in the laboratory and a failure shear strain that is 23 percent of that observed in the laboratory. The gross inaccuracy of this model is attributed to the introduction of the fictitious crack surface that is not aligned with the principal stresses that determine system strength. Initially, this misalignment acts to stiffen the system, with the result that initial post-cracking strength exceeds that observed in the laboratory. However, as a result of this misalignment, shearing of the pre-damaged elements must initially activate both normal traction damage surfaces and eventually all three damage surfaces (two surfaces defining transfer of tensile stress and one defining transfer of shear stress) in order to effect reduced shear strength and increased shear deformation. in the concrete all three failure surfaces As a result, deterioration of the element stiffness and strength is relatively rapid and a failure state is reached at a shear strain that is significantly less than that observed in the laboratory.

Finite element models VSD1 and VSD10 indicate the observed response with acceptable accuracy. Both models indicate the observed failure strain and model VSD1 predicts a panel shear strength that is within 10 percent of the observed strength. Using the finite element model for which the initial shear friction strength is defined to be 10 times the concrete compressive strength, the computed shear strength of the panel exceeds that observed in the laboratory by almost 25 percent. Once a concrete element accumulates significant damage, the element becomes so soft that it is possible for the element to carry significant load in shear if the shear-traction damage surface is not activated. Thus, these
data confirm that it is necessary for the shear-traction surface to be activated once significant damage has occurred. The reasonably accurate prediction of panel strength provided by the finite element model for which initial shear friction strength is defined to be equal to the concrete compression strength (VSN1) suggests that this is a reasonable estimate for subsequent analyses.

Figure 5.67: Computed and Observed Shear Stress Versus Shear Strain History for Panels - Finite Element Models include Stiff Loading Frame

The data presented in Figure 5.68 suggest the effect of the loading frame on the computed and observed panel response. In this Figure, the shear stress-strain response is computed using a finite element model in which no initial concrete damage is introduced into the model and for which the initial shear-friction strength is defined to be equal to the compressive strength. The response computed on the basis of the flexible loading frame model shows a strength that is significantly less than that observed in the laboratory in the post-crack regime. Here the loss of strength that is observed in all of the models immediately following crack initiation is accompanied by significant shear strain and is not
accompanied by subsequent strength recovery. Observation of stress and damage distribution within the model during this phase of loading indicates that this behavior is a result due to the limited redistribution of the panel edge loads. Eventually, the entire panel is sufficiently soft in comparison with the loading frame that some redistribution of the load does occur and a substantial increase in shear strength is computed. Final panel shear strength and strain are similar to that computed assuming a stiff load frame.

Figure 5.68: Computed and Observed Shear Stress Versus Shear Strain History for Panels - Finite Element Models include Both Stiff and Flexible Load Frames

5.6.1.1 Conclusions of the Shear Panel Study

Data collected from the shear panel study provide insight into a number of aspects of the proposed model. Probably the most critical observation to be drawn from this study pertains to the proposed representation of shear transfer across open fictitious crack surfaces. Results of the current study indicate that the originally proposed friction shear strength of $0.4f_c$ is inadequate for systems that are controlled by shear response and introduction of this model results in a system that exhibits far less strength and deformation
than is observed in the laboratory. On the basis of the data provided by Vecchio and Nieto, definition of an initial shear-friction strength equal to the concrete compressive strength appears to produce computed response that is acceptably accurate. This definition of initial shear friction strength is proposed for use in subsequent analyses. As discussed in Chapter 2, there are very few data defining concrete shear-friction strength and the energy dissipated under Mode II fracture. Given the effect of the shear damage surface on computed response of the current system, additional experimental and analytical research focussed on improved representation of shear-friction response is appropriate. The data provided by the current shear panel study identify also the limitations of the proposed two-crack model. While the current study considers a somewhat contrived system in which the orientation of the principal stresses at failure is rotated 45 degrees from that at initial cracking, comparison of the computed response for the undamaged and damaged systems indicates that introduction of two orthogonal cracks is not sufficient to represent behavior defined by damage on a single crack surface lying direction between the two cracks. While such variation in loading is unlikely, result of the current study do support introduction of a third and final crack surface oriented at 45 degrees with respect to the initial surface to improve accuracy in predicting such response. Finally, data from this study highlight the importance of accurate representation of system boundary conditions. While the investigators suggest that the applied loading of the panel is uniform, evaluation of reinforcement and shear key detailing in the panels as well as the computed response indicates that either is not the case or that uniform loading from the actuators is distributed along the panel edge by reinforcing steel, shear keys and shear key anchorage dowels.

5.7 Analysis of Reinforced Concrete Flexural Elements

Evaluation of reinforced concrete bridge components requires predicting the behavior of beam and column elements subjected to flexural loading. Additionally, accurate pre-
diction of element flexural response is particularly critical for prediction of beam-column connection response, as the loads applied to the connection are defined by the flexural response of the beams and columns that frame into the connection. The proposed finite element model is used to predict the behavior of a series of reinforced concrete beams subjected to monotonic and cyclic flexural loading. The computed response is compared with behavior observed in the laboratory on the basis of the global load-displacement response, concrete damage patterns and the distribution of strain in longitudinal reinforcement.

5.7.1 Computed and Observed Behavior of Lightly Reinforced Beams

The lightly reinforced concrete beam elements tested by Burns and Seiss [1962] represent one of the simplest flexural systems. As such, these elements have been used by a number of researchers to verify analytical models [Kwak and Filippou, 1999; Govindjee and Hall, 1998]. These elements are very lightly reinforced with a longitudinal reinforcement ratio of 0.007 and no transverse reinforcement. The ratio of the flexural length to the depth of the longitudinal reinforcement is approximately 4. Thus behavior of the element is controlled entirely by flexure and the lack of transverse reinforcement does not adversely affect behavior. Figure 5.69 shows the geometry of the specimen and the idealized load pattern. Table 5.9 lists material properties for the concrete and reinforcing steel, material parameters estimated as part of the current study are bold.

![Figure 5.69: Reinforced Concrete Beam Tested by Burns and Seiss [1962]](image-url)
Eight finite element models are used to predict the response of these elements. These models represent two levels of mesh discretization (a coarse mesh in which the largest concrete elements are 4 in. by 4 in. in plan view and a fine mesh in which the concrete elements are 2 in. by 2 in.), two representations of bond response (a model with perfect bond between the concrete and the reinforcing steel in which no bond elements are included in the mesh and concrete and steel elements are connected directly, and a model with imperfect bond in which the previously presented one-dimensional bond element is included), and two different initial strengths for shear transfer across an open fictitious crack surface. The models include the previously discussed concrete, steel and bond models. As previously suggested, where included the bond model does not include radial bond response and no cover concrete is included in the model. Table 5.11 defines characteristics of the individual finite element models.

The computed response is defined by the local concrete crack patterns, the concrete damage patterns, the steel strain distribution and the global load-displacement history. Data from model bs_fine are presented as representative of the four finite element models used to predict behavior. Figure 5.70 shows the orientation of the fictitious crack surfaces.

<table>
<thead>
<tr>
<th>concrete material properties</th>
<th>value</th>
<th>steel material properties</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>modulus</td>
<td>3.5E+06 psi</td>
<td>elastic modulus</td>
<td>29.5E+06 psi</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.175</td>
<td>yield strength</td>
<td>44.9 ksi</td>
</tr>
<tr>
<td>tensile strength</td>
<td>350 psi</td>
<td>strain at which strain hardening initiates</td>
<td>1%</td>
</tr>
<tr>
<td>compressive strength</td>
<td>4820 psi</td>
<td>hardening modulus</td>
<td>0.1E+06 psi</td>
</tr>
<tr>
<td>fracture energy</td>
<td>1.5 lb/in.</td>
<td>tensile strength</td>
<td>47.0 ksi</td>
</tr>
<tr>
<td>residual tensile and shear strength</td>
<td>0.001, 0.05</td>
<td>strain at which tensile strength is achieved</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 5.10: Material Parameters for RC Beam Tested by Burns and Seiss

Eight finite element models are used to predict the response of these elements. These models represent two levels of mesh discretization (a coarse mesh in which the largest concrete elements are 4 in. by 4 in. in plan view and a fine mesh in which the concrete elements are 2 in. by 2 in.), two representations of bond response (a model with perfect bond between the concrete and the reinforcing steel in which no bond elements are included in the mesh and concrete and steel elements are connected directly, and a model with imperfect bond in which the previously presented one-dimensional bond element is included), and two different initial strengths for shear transfer across an open fictitious crack surface.

The models include the previously discussed concrete, steel and bond models. As previously suggested, where included the bond model does not include radial bond response and no cover concrete is included in the model. Table 5.11 defines characteristics of the individual finite element models.

The computed response is defined by the local concrete crack patterns, the concrete damage patterns, the steel strain distribution and the global load-displacement history. Data from model bs_fine are presented as representative of the four finite element models used to predict behavior. Figure 5.70 shows the orientation of the fictitious crack surfaces.
that develop in the concrete elements. Along the bottom edge of the beam at mid-span, these cracks are vertical reflecting the fact that tension developed under flexure dominates the response of the beam in this region. Moving towards the supports and towards mid-depth of the beam, the orientation of the cracks become more shallow reflecting the more significant contribution of shear to the beam load. These crack patterns are typical of those observed in the laboratory.

<table>
<thead>
<tr>
<th>finite element model identification</th>
<th>number of concrete elements</th>
<th>dimension of largest and smallest concrete element</th>
<th>representation of bond</th>
<th>initial shear strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>bs_coarse</td>
<td>180</td>
<td>4 by 4.5 in.; 4 by 2.0 in.</td>
<td>imperfect</td>
<td>$f_c$</td>
</tr>
<tr>
<td>bs_coarse_nb</td>
<td>180</td>
<td>4 by 4.5 in.; 4 by 2.0 in.</td>
<td>perfect</td>
<td>$f_c$</td>
</tr>
<tr>
<td>bs_fine</td>
<td>720</td>
<td>2 by 2 in.</td>
<td>imperfect</td>
<td>$f_c$</td>
</tr>
<tr>
<td>bs_fine_nb</td>
<td>720</td>
<td>2 by 2 in.</td>
<td>perfect</td>
<td>$f_c$</td>
</tr>
<tr>
<td>bs_coarse_shear</td>
<td>180</td>
<td>4 by 4.5 in.; 4 by 2.0 in.</td>
<td>imperfect</td>
<td>$0.4f_c$</td>
</tr>
<tr>
<td>bs_coarse_nb_shear</td>
<td>180</td>
<td>4 by 4.5 in.; 4 by 2.0 in.</td>
<td>perfect</td>
<td>$0.4f_c$</td>
</tr>
<tr>
<td>bs_fine_shear</td>
<td>720</td>
<td>2 by 2 in.</td>
<td>imperfect</td>
<td>$0.4f_c$</td>
</tr>
<tr>
<td>bs_fine_nb_shear</td>
<td>720</td>
<td>2 by 2 in.</td>
<td>perfect</td>
<td>$0.4f_c$</td>
</tr>
</tbody>
</table>

Table 5.11: Finite Element Model Parameters - Burns and Seiss Beam

that develop in the concrete elements. Along the bottom edge of the beam at mid-span, these cracks are vertical reflecting the fact that tension developed under flexure dominates the response of the beam in this region. Moving towards the supports and towards mid-depth of the beam, the orientation of the cracks become more shallow reflecting the more significant contribution of shear to the beam load. These crack patterns are typical of those observed in the laboratory.

Figure 5.70: Orientation of Fictitious Crack Surfaces in Beam Tested by Burns and Seiss Computed Using Finite Element Model bs_fine

Evaluation of the computed distribution of discrete concrete damage zones provides a means of assessing the computed stress distribution and failure mechanisms for the system. Figure 5.71 shows the computed distribution of concrete damage in the beam. Here discrete cracks in the concrete form at a spacing that varies between 11 in. and 4.5 in. with an average value of approximately 7 in. (180 mm). Assuming a bond zone with width
equal to that of the beam and a minimum depth equal to twice the cover over the reinforcing steel, on the basis of the average minimum crack spacing, the distribution of discrete cracks corresponds to a minimum average bond strength of \(5.5 \sqrt{f_c}\) psi (\(0.4 \sqrt{f_c}\) MPa). This minimum average bond strength is less than the average bond strengths of \(8.1 \sqrt{f_c}\) psi and \(6.4 \sqrt{f_c}\) psi (\(0.67 \sqrt{f_c}\) MPa and \(0.53 \sqrt{f_c}\) MPa) observed by Goto [1972] for flexural-type bond zone models. However, the bond-zone area is assumed to be a minimum value and as a result the bond average bond stress is minimum. Assuming that the bond zone has an effective depth equal to half the depth of the beam, approximately the depth to which substantial concrete damage penetrates the beam, the computed average bond strength is calculated to be \(14 \sqrt{f_c}\), larger than that observed by Goto. Figure 5.72 shows the stress distribution for the longitudinal reinforcing steel at specific points in the load history. These data show the formation of discrete crack zones along the length of the beam as well as a zone of approximately 8 inches in length, in the vicinity of the applied load, along which the longitudinal reinforcing steel yields.

![Figure 5.71: Distribution of Concrete Damage in Beam Tested by Burns and Seiss as Computed Using Finite Element Model bs_fine](image)

Finally, the global load-displacement history as computed using the eight finite element meshes with imperfect and perfect bond are compared with the response as observed in the laboratory. These data are presented in Figures 5.73, 5.74 and 5.75. The data in Figure 5.73 show the computed and observed load versus displacement history for the finite
element models with a coarse mesh and variable representation of both the bond zone and the concrete shear strength. These data show that the finite element models predict a slightly stiffer and slightly stronger response than is observed in the laboratory. Prior to yielding of the reinforcing steel, the data show that representation of imperfect bond between concrete and reinforcing steel results in a slightly softer/weaker system. However, following yield of the reinforcing steel, representation of perfect bond between concrete and steel produces a slightly weaker system. Likely this is a result of perfect bond forcing equal strain in the concrete and steel. If concrete and steel strains are equal, the concrete exhibits higher tensile strain and lower tensile stress and the strength of the system is reduced. If bond is imperfect, concrete strain is slightly less than steel strain, this increases concrete stress in comparison to the case of perfect bond and results in increased system strength. Additionally, the data in Figure 5.73 show that defining the initial con-
crete shear transfer strength to be $f_c$ rather than $0.4f_c$ results in a slightly higher computed post-yield strength.

Figure 5.74 shows the observed load-displacement history for the beam and the response computed using the more finely discretized meshes. These data show the same trends as is observed for the coarse meshes. There is some difference in strength between the models for loads corresponding to initial cracking of the concrete. However, for these systems as the load increases the difference in strength between the various models becomes negligible. There is essentially no difference between the individual models for loads in excess of approximately two-thirds of the yield strength.

Figure 5.75 shows the observed behavior and the computed response using the two most appropriate finite element models, the coarse and fine mesh with explicit, imperfect representation of bond between concrete and steel and an initial shear friction strength equal to the concrete compressive strength. These data show that the finite element models predict a yield displacement that is 10 percent less than that observed in the laboratory and a yield strength that is approximately equal to that observed in the laboratory and a post-yield strength that is at most 6 percent greater than the observed strength. These data indicate also that the coarsely discretized finite element mesh predicts a slightly weaker and softer system than the more finely discretized mesh. Differences in computed response resulting from variation in mesh refinement likely follow from the fact that peak strength corresponds to both tensile and compressive failure of the concrete. If the mesh is coarse, softening of a single element due to severe tensile or compressive strain demands produces a greater reduction in system strength and stiffness than does softening of a single element in a more finely discretized mesh. These data show computed system strengths and stiffnesses that are greater than those observed in the laboratory. This discrepancy likely results from several factors. Here the estimated concrete fracture energy is 1.5 psi-
in. (260 MPa-mm); this is slightly higher energy than the typically observed values of 0.5 psi-in to 1.3 psi-in. (90 MPa-mm to 230 MPa-mm) [Monteiro et al., 1993; Kozul and Darwin, 1997]. An increase in concrete fracture energy would be expected to result in increased stiffness and strength of individual concrete element, and thus the system, for a given strain distribution. Inaccuracy of the computed response also could result from the use of the basic element formulation to represent concrete behavior. This formulation is known to introduce ‘parasitic’ shear when used to model beams subjected to flexural loading and thus increase system strength and stiffness. The increased strength of the fine mesh as compared with the coarse mesh likely follows from the fact that for the coarse mesh, damage to a single node in a concrete element produces a much larger drop in system strength than for the fine mesh and the fact that the fine mesh has the capacity to represent a much smoother damage distribution than can be achieved with the coarse mesh. It is likely that for a given level of beam deformation, the coarse mesh predicts a greater number of more severely damage nodes than does the fine mesh, with the result that the coarse mesh predicts a global response that is softer and weaker than the fine mesh.

Evaluation of the observed and computed response for a lightly reinforced concrete beam subjected to flexural loading indicates that the proposed finite element model represents well the behavior of these systems. The model predicts local concrete cracking that reflects the distribution of stress within the flexural member as is oriented as observed in the laboratory. Additionally, the model predicts distributed discrete concrete crack patterns (Figure 5.70) that are both typical of crack patterns observed in the laboratory as well as consistent with typically observed average flexural bond strength. Finally, the model predicts a global load displacement response that is within 10 percent of the observed response. Evaluation of model response indicates that explicit representation of concrete to steel bond improves prediction of system response in the pre-yield regime but does not
Additionally, evaluation of the data indicate that an estimate of an initial concrete shear friction strength that is equal to the compressive strength produces accurate results for flexural systems in which concrete shear strength does not determine response.

5.7.2 Computed and Observed Behavior of Reinforced Beams Subjected To Monotonic and Cyclic Loading

Ma, Bertero and Popov [1976] provide data that are appropriate for evaluating the computed behavior for flexural elements subjected to simulated earthquake loading. The prototype specimen is a cantilevered beam and attached column segment (anchorage block) subjected to reversed cyclic flexural loading developed through application of a point shear load at the cantilever tip. The investigation considers behavior of a number of beams. However, of interest to the current investigation are data for two beams with the
same design, one of which is subjected to essentially monotonic loading and one of which is subjected to reversed-cyclic loading with increasing amplitude. Data provided by the investigators that are appropriate for comparison with computed response include information about the concrete damage patterns and observed crack spacing in the beam, the global load-displacement history and slip of the reinforcement out of the anchorage block.

The beam tested by Ma et al. is representative of a beam in a typical reinforced concrete building frame designed in accordance with modern standard practice. Figure 5.76 shows the beam of interest to the current study as well as an idealization of the test frame. The beams are reinforced with longitudinal steel ratios of 1.4% (top of the beam) and 0.74% (bottom of the beam) and a volumetric transverse steel ratio of 1.0%. The specimens are intended to be representative of building beam sections with longitudinal reinforcement anchored in a building beam-column connection. However, the longitudinal

Figure 5.74: Computed and Observed Load-Displacement History for Burns and Seiss Beam, Finite Element Model has Fine Concrete Mesh
reinforcement is anchored with a minimum hooked embedment length of $38d_b$, in a reinforced concrete block that does not exhibit the same load distribution as is developed in a beam-column connection. Thus, much better anchorage conditions are provided in the laboratory than would be observed in a the beam-column connection of a building or bridge frame. A point shear load is applied at the tip of the cantilever under displacement control. The ratio of the shear span to beam depth is 4.5 and the shear load at yield corresponds to a gross section average nominal shear stress of approximately $3.2\sqrt{f_c}$ psi, with $f_c$ in psi ($3.2\sqrt{f_c}$ kPa with $f_c$ in kPa). Thus, beam response is controlled by flexure and probably shear deformation is minimal. One specimen, identified by the investigators as R4, is subjected to reversed cyclic loading with the amplitude of the first cycle defined to be approximately 10 times the yield displacement; data from the first half of the first cycle define the monotonic response of the model. The second specimen, identified as R3, is subjected

Figure 5.75: Computed and Observed Load-Displacement History for Burns and Seiss Beam, Finite Element Model has Imperfect Concrete-Steel Bond and Initial Shear Friction Strength Equal to the Compressive Strength
to a reversed-cyclic load history consisting of three cycles each to increasing maximum displacement levels.

Figure 5.76: Reinforced Concrete Beam Tested by Ma et al. [1976]

The finite element model developed to compute the response of the laboratory specimens is shown in Figure 5.77. Tables 5.12 and 5.13 list the material data defining the model. The finite element model does not represent exactly the geometry of the cantilever beam and anchorage zone in the laboratory model; however, the differences between the computer and laboratory model are not expected to affect appreciably the computed response. The discretization of the mesh is such that individual concrete elements in the critical region range in size from 1 in. by 1 in. (25 mm by 25 mm) in the beam flexural tension/compression zone to 2 in. by 4 in. (51 mm by 102 mm) in the column segment (anchorage block) adjacent to the anchorage zone. The finite element model is loaded by forcing the tip of the cantilever beam through a prescribed displacement path that matches that used in the laboratory. To improve correlation between laboratory and computer
model displacement at the point of initial cracking, the flexibility of the post-tensioned anchorage is represented explicitly in the finite element model.

![Finite Element Mesh for RC Beams Tested by Ma et al. [1979]](image)

**Figure 5.77: Finite Element Mesh for RC Beams Tested by Ma et al. [1979]**

<table>
<thead>
<tr>
<th>concrete material properties</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic modulus</td>
<td>3.3E+06 psi</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.175</td>
</tr>
<tr>
<td>tensile strength</td>
<td>460 psi</td>
</tr>
<tr>
<td>compressive strength</td>
<td>4380 psi</td>
</tr>
<tr>
<td>initial shear strength</td>
<td>4380 psi</td>
</tr>
<tr>
<td>fracture energy</td>
<td>1.5 lbf-in.</td>
</tr>
<tr>
<td>residual strength (tension and shear)</td>
<td>0.1%, 5%</td>
</tr>
</tbody>
</table>

**Table 5.12: Concrete Material Properties for RC Beam Tested by Ma et al. [1976]**

The finite element model is used to predict the response of the beam subjected to monotonic loading (comparable to laboratory specimen R4) and reversed cyclic loading
Figures 5.78 and 5.79 show sketches of the observed crack patterns for the two specimens following severe loading. For specimen R4, this global concrete cracking is approximately perpendicular to the axis of the beam in the vicinity of the extreme concrete fibers, where the orientation of the elastic tensile principal stress would be expected to be approximately parallel to the axis of the beam. As cracking propagates towards the center of the beam, the orientation of the cracks follows the orientation of the principal stress and cracks form at an angle of approximately 45 degrees with respect to the beam axis at beam mid-height. The stress distribution results also in the development of cracks parallel to the axis of the beam in the concrete compression zone. For specimen R3, reversed cyclic loading results in the development of the crack pattern observed in specimen R4 for both directions of loading. The observed crack patterns show that at a particular location, the concrete cracks developed under positive loading are approximately perpendicular to those developed under negative loading. Ma et al. do not specifically identify the spacing of individual cracks; however, evaluation of the presented sketches and photographs sug-

<table>
<thead>
<tr>
<th>steel material properties</th>
<th>#2 Reinforcing Bar</th>
<th>#5 Reinforcing Bar</th>
<th>#6 Reinforcing Bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic modulus (psi)</td>
<td>28.17E+03</td>
<td>28.30E+06</td>
<td>29.11E+06</td>
</tr>
<tr>
<td>yield strength (psi)</td>
<td>60.0E+03</td>
<td>66.5E+03</td>
<td>65.5E+03</td>
</tr>
<tr>
<td>strain for strain hardening</td>
<td>0.023</td>
<td>0.0116</td>
<td>0.0134</td>
</tr>
<tr>
<td>hardening modulus</td>
<td>0.75E+06 psi</td>
<td>1.05E+06</td>
<td>1.05E+06</td>
</tr>
<tr>
<td>tensile strength (psi)</td>
<td>83.0E+03</td>
<td>95.5E+03</td>
<td>94.2E+03</td>
</tr>
<tr>
<td>strain for tensile strength</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>fracture strain</td>
<td>0.17</td>
<td>0.18</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 5.13: Steel Material Properties for RC Beam Tested by Ma et al. [1976]
gests a flexural cracking spacing of approximately 3.5 in. (90 mm) in the vicinity of the maximum flexural loading. The observed crack patterns may be compared with the computed global damage patterns in Figures 5.80 and 5.81. The computed damage patterns show localized zones of severe damage that are comparable to discrete crack zones. The orientation of these patterns follows that observed in the laboratory with cracking near the top (or bottom) face of the beam developing approximately perpendicular to the axis of the beam and becoming more inclined as the ‘crack’ propagates towards mid-height of the beam. Crack patterns developed under reversed-cyclic loading are similar to those observed in the laboratory with similar crack patterns forming at the top and bottom of the beam and with individual cracks intersecting orthogonally. Also, computed crack spacing in the vicinity of the maximum flexural loading is approximately 4 in (100 mm). Figures 5.82 and 5.83 show the orientation of local fictitious crack planes; these are similar in orientation to the global damage zones, but show greater inclination near mid-height of the beam.

![Observed Concrete Crack Pattern for Beam R4 Tested by Ma et al. [1979]](image)

**Figure 5.78: Observed Concrete Crack Pattern for Beam R4 Tested by Ma et al. [1979] (Observed Damage Following Monotonically Increasing Loading)**

At a more global level, computed and observed behavior may be compared on the basis of the load-displacement history. Figures 5.84 and 5.85 show the observed displace-
ment history for specimens R3 and R4. Figures 5.86 and 5.87 show the computed displacement histories for the models and the observed monotonic displacement history for specimen R4. For specimen R4, the computer model is subjected only to monotonically
Figure 5.81: Computed Concrete Damage Pattern for Beam R3 Tested by Ma et al. [1979] (Computed Damage Following Loading to a Maximum Absolute Displacement Equal to the Nominal Yield Displacement)

Figure 5.82: Computed Orientation of Concrete Element Fictitious Crack Surfaces for Beam R4 Tested by Ma et al. [1979] (Computed Damage Following Loading to a Maximum Displacement Equal to the Nominal Yield Displacement)
increasing column displacement. The computed pre-yield response is approximately the same as observed in the laboratory. This response is determined in part by the flexibility of the anchorage that is introduced into the model. However, because the observed response is significantly more flexible that is computed assuming a ‘cracked-section’ modulus for the length of the cantilevered beam, introduction of anchorage flexibility is appropriate. The computer model predicts a yield strength that is approximately equal to the observed strength and a post-yield strength that is greater than that observed in the laboratory. For the case of reversed cyclic loading, the computed response is similar to the observed behavior, but the model does not provide results at larger displacement levels.

The analytical results presented in Figure 5.86 show that while pre-yield response is adequately represented, the computed post-yield response is stronger and less ductile than observed in the laboratory. The greater strength of the computer model may be attributed
Figure 5.84: Observed Load-Displacement History for Beam Specimen R-4 (Figure 4.9(d) from Ma et al. [1976])

Figure 5.85: Observed Load-Displacement History for Beam Specimen R-3 (Figure 4.9(c) from Ma et al. [1976])
Figure 5.86: Computed and Observed Load-Outplacement History for Beam R4 as Tested by Ma et al. [1979]

Figure 5.87: Computed and Observed Load-Displacement History for Beam R3 as Tested by Ma et al. [1979]
to the computed concrete behavior. The Druker-Prager concrete compressive yield surface with a pressure coefficient of 0.2 (Section 2.3.2.1) predicts a concrete compressive strength under confinement in one direction that may be as much as 25 percent greater than the uniaxial compressive strength. For the concrete elements directly next to the beam-block interface, loading is such that the elements develop some additional compressive strength. Figure 5.88 shows the computed stress-strain response ($\sigma_{xx}$ versus $\varepsilon_{xx}$, $\sigma_{yy}$ versus $\varepsilon_{yy}$, and $\tau_{xy}$ versus $\gamma_{xy}$) for the extreme element in the concrete compression zone at the beam-block interface. Here the maximum compressive stress is approximately 13 percent greater than the defined compression strength due to the compression stress that develops in the perpendicular direction. The computed over-strength may be attributed also to the representation of bond response. If computed bond strength is too great, then the computed length of yielding bar is reduced and the post-yield rotation of the beam at the anchorage block is greatly reduced. While bond response and the steel strain distribution in the laboratory models are not known, the steel stress distribution if Figure 5.89 indicates that there is substantial bond stress developed within the anchorage block and some limited bond stress developed within the beam section. Thus, it is reasonable to conclude that the slip of the reinforcement out of the anchorage block, and thus the rotation at the beam-block interface, is under computed. The computed premature failure of specimen R4 under monotonic loading results for several reasons including the inability of the finite element model to transfer substantial shear across the beam-block interface once substantial tensile and compressive damage has occurred and the numerical instability introduced when all of concrete elements at the-beam-block interface begin to soften.

Figure 5.87 shows the computed response for the reinforced concrete cantilever beam subjected to reversed cyclic loading (Specimen R3). These results may be compared with the observed response presented in Figure 5.85. The computed response under
reversed-cyclic loading follows an envelope that is approximately the same as that computed under monotonic loading and is slightly stiffer and stronger than the observed response in the post-yield range. The computed and observed response histories are most similar for the range of loading corresponding to unloading from the positive direction and reloading in the negative direction. Here the final computed cycle indicates the slight drop in stiffness under increasing negative load that can be attributed to a delay in crack closure. The model does not predict the more substantial representation of the response mode that is observed in the opposite load direction (Figure 5.85), but this is because the model

1. Since specimen R3 has substantially more reinforcing steel on the top of the beam than on the bottom, crack opening under negative load (when the bottom reinforcing steel is in tension) is substantially greater than under positive loading. Thus, more displacement in the positive direction is required to close cracks that open under negative loading. In fact for many cycles, it appears that cracks may not close with the result that most of the compression load is transferred through the reinforcing steel.
fails prior to substantial yielding of the bottom reinforcing steel. For the case of reversed cyclic loading, the model fails upon unloading from the post-yield range. Here, rather than predicting a substantial loss in strength upon unloading, the computed response is essentially hyperelastic following the loading curve. This behavior results from numerical instability within the model. Rather than triggering unloading of the reinforcing steel elements, deformation associated with unloading is accommodated by the interaction of severely damaged concrete elements with bond elements that have relatively little stiffness. Figure 5.90 shows an exaggerated view of the mesh deformation at the point of model failure. In addition to the instability under load reversal from the post-yield range. Stability of the system is lost also in the vicinity of zero-displacement. Here, for a single iteration within a Newton solution algorithm, steel elements and bond elements rattle between the load and unload state while concrete elements rattle between an open and closed crack state. The

Figure 5.89: Computed Top Steel Stress Distribution Near the Beam-Block Interface for Specimen R4
states between which the model rattles have substantially different stiffnesses and strengths and the Newton iteration scheme does not converge. As discussed in Section 5.2.2, the global solution is advanced temporarily by switching to an explicit solution algorithm. The instability of the global model at the zero-displacement state is reflected also in the individual elements. Figure 5.91 shows the stress versus strain response ($\sigma_{xx}$ versus $\varepsilon_{xx}$) for an extreme-fiber concrete element at the beam-block interface. The response indicates an instance in which the element exhibits crack-closure, represented by increasing compressive stress, at a positive strain levels. This is due to the fact that the concrete element defines the crack closure on the basis of the stress state assuming that the crack is closed.

![Computed Deformation of Reinforced Concrete Beam Following Reversed-Cyclic Loading](image)

**Figure 5.90: Computed Deformation of Reinforced Concrete Beam Following Reversed-Cyclic Loading**

### 5.7.3 Results of Analysis of Flexural Elements

The analyses of flexural elements subjected to monotonically increasing a reversed cyclic loading indicate that the model represents well the response of systems subjected to
monotonically increasing load to moderate levels of deformation demand. The model ade-
quately predicts flexural strength and yield deformation; though strength and stiffness is always over-estimated. The model represents well the distribution of global and local con-
crete damage. The model does not provide a robust solution for the case of reversed cyclic loading. While all of the elements contribute to difficulties with the global solution, the greatest contribution appears to be the proposed concrete constitutive model for the case of crack opening-closing and for the case of severe deformation. During the course of this project, attempts were made to address this issue by limiting the level of damage accumu-
lated by individual elements, by introducing some residual elastic response and by intro-
ducing element viscosity. However, none of these approaches produced entirely satisfactory results. Instead the problem appears to require some smoothing of response between the tensile and compressive regimes. One approach appears to be the introduction

Figure 5.91: Typical Computed Stress-Strain History for a Concrete Element in Specimen R3 at the Beam-Block Interface
of isotropic damage into the proposed concrete constitutive model. This would allow for improved modeling of concrete compressive response as well as smooth the compression-tension transition. However, at this stage such modification of the model is inappropriate given the limited experimental data available for model calibration and development. Further modification of the model is left for future projects.

5.8 Analysis of Reinforced Concrete Beam-Column Connections

The goal of the current investigation is development of a model to predict the response of reinforced concrete beam-column connections subjected to simulated earthquake loading. Experimental investigation of these connections suggests that behavior may be determined by a number of mechanisms including failure of flexural and anchorage bond zones, development of a shear panel mechanism and flexural response. Evaluation of computed and observed response for these various sub-systems indicates that the proposed model provides relatively accurate prediction of observed system behavior. Here computed and observed response are compared for a series of three beam-column connections that define the range of typically observed connection response to simulated earthquake loading. Results of the this comparison complete identification of the proposed models capabilities and limitations.

5.8.1 Experimental Investigation of Reinforced Bridge Beam-Column Connection Response

The results of an experimental investigation include a proposed method for evaluation and retrofit of older, reinforced concrete bridge beam-column connections [Lowes and Moehle, 1999]. Results of the investigation include also data defining the response of a beam-column connection, typical of those constructed in California in the 1950’s and 1960’s, as well as two retrofit connections to simulated earthquake loading. Figure 5.92
shows the three sub-assemblages tested in the laboratory. Figure 5.93 shows the as-built connection in the test frame that is representative of an interior column, the interior beam-column connection and approximately half the length of the two adjacent beams in from a multi-column bridge frame. Gravity loading of the connection is simulated through application of a constant compressive force at the column base that is reacted by two-point loads on the beam span located to provide an appropriate shear and flexural loading at the beam-connection interface. Earthquake loading is simulated by forcing the base of the cantilever through a prescribed reversed-cyclic displacement path perpendicular to the axis of the column. The shear load applied at the column base is reacted by shear loading at the ends of the beam segments and an axial compression load in one of the two beam segments. Figure 5.94 shows the observed column shear load versus column tip displacement history. Under simulated earthquake loading, the as-built connection exhibits a rapid loss of strength at a maximum load that corresponds to approximately of the nominal flexural strength of the system. Failure is attributed to a splitting-type bond failure for column reinforcement anchored in the beam-column connection. Figure 5.95 shows damage in the vicinity of the connection during testing. A second as-built connection is retrofit by adding reinforced concrete bolsters on either side of the existing beam segments and the connection. The bolsters are designed to provide beam over-strength to prevent flexural yielding of the beam, to increase the volume of the beam-column connection and thereby reduce connection shear loading and to provide additional cover concrete and reinforcing steel to prevent development of a splitting-type bond failure. Figure 5.96 shows the observed load-displacement history for the reinforced concrete retrofit connection. Here the nominal flexural strength of the column is developed; however, some damage is observed in the beam-column connection. Upon further loading, damage accumulates in the connection and eventually an anchorage-type bond failure develops at a relatively limited displace-
ment demand. Figure 5.97 shows damage in the vicinity of the connection near the end of the test. The final connection is retrofit with prestressed concrete bolsters cast on either side of the existing beam segments and connection. The bolster dimensions and pre-stress loading are defined to limit the average principal tensile stress in the connection to $3.5 \sqrt{f_c}$ psi with $f_c$ in psi ($9.2 \sqrt{f_c}$ kPa with $f_c$ in kPa). Figure 5.98 shows the observed load-displacement history for this connection. Response of the pre-stressed concrete retrofit connection is controlled by flexural yielding of the column; no external cracking of the connection concrete, limited inelastic shear response of the connection concrete and minimal slip of column reinforcement anchored in the connection is observed. Figure 5.99 shows damage in the vicinity of the connection at the end of testing.

Behavior of the reinforced concrete bridge sub-assemblies is computed using the finite element model shown in Figure 5.100. The model discretization is finest in the regions that determine response, for which concrete element dimensions are approximately 1 in. by 1 in. (25 mm by 25 mm). Connectivity between concrete and reinforcing steel elements is established via bond elements within the zones labeled bond-zones and directly throughout the remainder of the model. Connectivity between concrete and transverse steel elements is established directly throughout the model. For the computer model, the simulated gravity load is introduced as constant point loads applied to the base of the column and on the beam surface. Simulated earthquake loading is applied by forcing the base of the column through a prescribed displacement path perpendicular to the column axis in the undeformed configuration. Simulated earthquake load is reacted by shear loading at the ends of the beam segments and through axial tension and compression in the beam segments. Beam axial load is reacted through idealized springs that are flexible in tension and stiff in compression with the result that essentially all of the earthquake load is reacted through compression in one of the beam segments. The ratio of tension to com-
pression load is a function of the spring stiffnesses as well as the subassembly deformation and thus is not constant throughout the test. The finite element model is defined completely by the material and material model parameters listed in Tables 5.14 and 5.15. In the laboratory, the beam and beam-column connection of the retrofit subassemblies are constructed from two batches of concrete. Mix One (Table 5.14) defines the material properties for the column section in each of the retrofit models. The weaker of Mixes One and Two

Figure 5.92: Reinforced Concrete Bridge Frame Sub-Assemblages Tested by Lowes and Moehle [1999]
Table 5.14 is assumed to determine the response of the beam sections and beam-column connections. Evaluation of the computed response for the reinforced concrete cantilever beams tested indicates that the proposed finite element model does not provide consistently accurate results for the case of reversed cyclic loading. Here the response of the connections to simulated earthquake loading is evaluated by subjecting the sub-assemblages to monotonically increasing lateral loading applied through displacement control at the base of the column. The case of the as-built connection is considered first. Figures 5.101 and 5.102 show the computed global concrete damage patterns and the orientation of local concrete fictitious crack surfaces. These distributions indicate discrete cracking associated with flexural loading of the column and the beam segments. The interaction of gravity and earthquake loading results in different flexural loads in the two beam segments at the

Figure 5.93: Idealized Loading of the Bridge Frame Sub-Assemblage [Lowes and Moehle, 1999]

(Table 5.14) is assumed to determine the response of the beam sections and beam-column connections.

Evaluation of the computed response for the reinforced concrete cantilever beams tested indicates that the proposed finite element model does not provide consistently accurate results for the case of reversed cyclic loading. Here the response of the connections to simulated earthquake loading is evaluated by subjecting the sub-assemblages to monotonically increasing lateral loading applied through displacement control at the base of the column. The case of the as-built connection is considered first. Figures 5.101 and 5.102 show the computed global concrete damage patterns and the orientation of local concrete fictitious crack surfaces. These distributions indicate discrete cracking associated with flexural loading of the column and the beam segments. The interaction of gravity and earthquake loading results in different flexural loads in the two beam segments at the
Figure 5.94: Observed Load-Displacement History for As-Built Bridge Sub-Assembly
[Experimental Data from Lowes and Moehle, 1999]

Figure 5.95: Laboratory Model of As-Built Connection During Testing
Figure 5.96: Observed Load-Displacement History for RC Retrofit Bridge Sub-Appliance, [Experimental Data from Lowes and Moehle, 1999]

Figure 5.97: Laboratory Model of RC Retrofit Connection During Testing
Figure 5.98: Load-Displacement History for Pre-Stressed Retrofit Bridge Sub-Assembly [Experimental Data from Lowes and Moehle, 1999]

Figure 5.99: Laboratory Model of Pre-Stressed Retrofit Connection Approaching the End of Testing
Figure 5.100: Finite Element Model of As-Built Bridge Sub-Assembly

Table 5.14: Concrete Material Parameters for Bridge Beam-Column Subassemblies (Values Estimated by Lowes are Bold)

<table>
<thead>
<tr>
<th>Concrete Material Properties</th>
<th>As-Built Model</th>
<th>Reinforced Concrete Retrofit</th>
<th>Prestressed Concrete Retrofit</th>
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<tbody>
<tr>
<td></td>
<td>Mix One</td>
<td>Mix Two</td>
<td>Mix One</td>
</tr>
<tr>
<td>compressive strength (psi)</td>
<td>5100</td>
<td>5190</td>
<td>5060</td>
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<tr>
<td>tensile strength (psi)</td>
<td>552</td>
<td>463</td>
<td>431</td>
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<td>elastic modulus (psi)</td>
<td>3.28E+06</td>
<td>3.06E+06</td>
<td>2.98E+06</td>
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<tr>
<td>fracture energy (lbf-in.)</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
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<tr>
<td>Poisson’s ratio</td>
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<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td>Residual strength (tension and shear)</td>
<td>0.1%, 5.0%</td>
<td>0.1%, 5.0%</td>
<td>0.1%, 5.0%</td>
</tr>
</tbody>
</table>

Notes:
1. Reinforcement is placed as indicated in specimen layout.
2. Longitudinal reinforcement is connected to concrete elements via bond elements within ‘Bonded- Zones’.
3. Transverse reinforcement and longitudinal reinforcement outside Bonded-Zones is connected directly to concrete elements.
beam-connection interface, only the more heavily loaded beam segment exhibits flexural damage at this load level. Crack patterns in the column segment show the interaction of flexure and shear with crack orientation perpendicular to the column axis at the edges of the column and shifting to wars a more parallel orientation as cracks propagate towards mid-depth of the column. The distribution of concrete damage and cracking indicates also accumulation of damage in the connection core. Cracking in the connection indicates the localized demand associated with anchorage of column reinforcement (left side of the connection) as well as damage associated with joint-shear demand towards the interior of the connection. Cracking indicates also damage associated with bond demand for anchored top beam reinforcement. These damage patterns are consistent with those observed in the laboratory (Figure 5.102) as well as the force transfer mechanism identified from laboratory testing [Lowes and Moehle, 1999]. Figure 5.103 shows the computed load-displacement history under monotonic loading as well as the observed cyclic response history. The computed response is stronger and stiffer than the observed response; this is consistent with the results of the previous analyses of flexural systems. For this particular case, some discrepancy between the computed and observed response follows from the differences in the load histories. While the computed load-displacement

<table>
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<tr>
<th>Steel Material Properties</th>
<th>GR40-#3</th>
<th>GR40-#5</th>
<th>GR40-#6</th>
<th>GR60</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic modulus (ksi)</td>
<td>30.0E+06</td>
<td>31.0E+06</td>
<td>30.0E+06</td>
<td>30.0E+06</td>
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<tr>
<td>yield strength (ksi)</td>
<td>59.0</td>
<td>51.0</td>
<td>47.0</td>
<td>60.0</td>
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<tr>
<td>strain for strain hardening</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>hardening modulus</td>
<td>1.0E+06</td>
<td>1.0E+06</td>
<td>1.0E+06</td>
<td>1.0E+06</td>
</tr>
<tr>
<td>tensile strength (ksi)</td>
<td>82.0</td>
<td>74.0</td>
<td>68.0</td>
<td>80.0</td>
</tr>
<tr>
<td>strain for tensile strength</td>
<td>0.10</td>
<td>0.10</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>fracture strain</td>
<td>0.27</td>
<td>0.27</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 5.15: Steel Material Parameters for Bridge Beam-Column Sub-Assemblages
response of the system does not follow the envelope of the observed response, the model does predict system strength with reasonable accuracy. Here computed strength is 28 kips compared with an observed strength of 25 kips. Additionally, the computed failure mode for the system is failure of the beam-column connection. The failure is triggered both by damage in the anchorage zone of the column reinforcement as well as by failure of connection core concrete in shear. Given the limitations of the two-dimensional finite element model to predict splitting-type bond failure, the computed failure is the best possible representation of the observed splitting bond failure followed by global connection failure.

Figure 5.103 shows the computed shear stress versus shear strain response for the interior of the connection, shear stress is computed be representing the flexural load of the framing elements as a compression-tension couple with load located at the centroid of the extreme reinforcing bars [Lowes and Moehle, 1999] while shear strain is computed from the nodal displacement mesh nodes on the interior of the connection. This response indicates the non-linear shear response of the connection resulting from accumulation of damage within the connection core concrete.

Figure 5.101: Computed Concrete Damage for As-Built Connection
Figure 5.102: Orientation of Fictitious Crack Surfaces for As-Built Connection

Figure 5.103: Load-Displacement Histories for As-Built Connection
Figures 5.105 and 5.106 show the computed damage patterns for the reinforced concrete retrofit connection. These damage patterns are similar to those observed for the as-built connection. However, here there is substantially more damage associated with flexural response of the column. Figure 5.107 shows the computed load-displacement history for the this connection as well as the observed response (Note that the sign of loads and displacements for the observed response is switched to facilitate comparison). Here the addition of reinforced concrete to the beam elements increases local connection strength required to anchor column longitudinal reinforcement and global connection shear strength. Thus, the flexural strength of the connection is developed and column reinforcing steel yields. However, as suggested by the concrete damage patterns, failure results from failure of the connection in shear rather than from flexural failure. This is supported by the results presented in Figure 5.108 that show the non-linear connection response in shear. Again this is a good representation of the failure mode observed in the laboratory.

Figure 5.104: Computed Connection Shear Stress Versus Shear Strain

Figures 5.105 and 5.106 show the computed damage patterns for the reinforced concrete retrofit connection. These damage patterns are similar to those observed for the as-built connection. However, here there is substantially more damage associated with flexural response of the column. Figure 5.107 shows the computed load-displacement history for the this connection as well as the observed response (Note that the sign of loads and displacements for the observed response is switched to facilitate comparison). Here the addition of reinforced concrete to the beam elements increases local connection strength required to anchor column longitudinal reinforcement and global connection shear strength. Thus, the flexural strength of the connection is developed and column reinforcing steel yields. However, as suggested by the concrete damage patterns, failure results from failure of the connection in shear rather than from flexural failure. This is supported by the results presented in Figure 5.108 that show the non-linear connection response in shear. Again this is a good representation of the failure mode observed in the laboratory.
Figure 5.105: Computed Concrete Damage for the Reinforced Concrete Retrofit Connection

Figure 5.106: Computed Orientation of Fictitious Crack Surfaces for Reinforced Concrete Retrofit Connection
Figure 5.107: Load-Displacement History for the Reinforced Concrete Retrofit Connection

Figure 5.108: Computed Connection Shear Stress-Strain Response for Reinforced Concrete Retrofit Connection
The final connection considered as part of this investigation in the prestressed retrofit connection. Figures 5.109 and 5.110 show the computed damage patterns for the prestressed retrofit connection. Here damage is isolated essentially that associated with flexural yielding of the column. Figure shows the development of some cracking in connection within the vicinity of the column reinforcement anchorage zone. Figure 5.111 shows the computed load-displacement history. The computed strength of the system is 46.5 kips compared with an observed maximum strength of 42 kips. The computed deformation is significantly less than that observed in the laboratory. Figure 5.112 shows the computed and observed connection shear-stress response. In the laboratory the connection displays essentially no inelastic shear deformation until the final displacement cycles. The computed response also indicates essentially no inelastic shear deformation within the connection.

Figure 5.109: Computed Concrete Damage for Pre-Stressed Retrofit Connection
Figure 5.110: Computed Orientation of Concrete Cracking for Pre-Stressed Concrete Connection

Figure 5.111: Load-Displacement History for Pre-Stressed Retrofit Connection
The proposed model accurately predicts the observed response mechanisms of reinforced concrete bridge beam-column connections. The model predicts the observed brittle connection failure for a system with limited reinforcement and high shear stress. For the laboratory model with moderate transverse reinforcement in the connection and reduced connection shear demand, the computer model predicts the observed development of a flexural mechanism followed by shear failure of the connection. Finally for the prestressed retrofit connection in which principal tensile stresses in the connection are relatively low, the model predicts the observed flexural mechanism. While the model predicts the observed failure mechanism and predicts observed strengths within 15 percent, the model predicts a much stiffer system than is observed in the laboratory and does not predict response for the case of significant post-yield deformation. These results support the
results of the previous investigation of the flexural response of reinforced concrete cantilever beams and indicate that additional research is required to improve the robustness of the model for the case of significant concrete damage and large localized strains.

### 5.9 Conclusions

The results of the analyses presented here indicate that the proposed model with represents well the response of general reinforced concrete structural components. Analyses of general reinforced concrete systems indicate that non-standard solution techniques are required to solve systems that represent material softening. The results of analyses of increasingly complex reinforced concrete systems and comparison of computed and observed response indicate that some modification of the model parameters is required to represent the response of general reinforced concrete systems. The most significant of these includes use a one-dimensional bond element, introduction of a relatively high initial shear strength for cracked concrete surfaces and moderate residual concrete shear and tensile strengths. Comparison of computed and observed response indicates that the proposed model represents well localized response phenomena such as the orientation of concrete cracking within flexural and anchorage bond zones. The model represents well the behavior of relatively systems such as plain concrete beams, bond zone models and shear panels. Additionally, the model predicts with acceptable accuracy the strength of more ductility systems such as reinforced concrete flexural elements. However, result of these analyses indicate that future research should focus on improved representation of the post-yield deformation of ductile systems. Further, the results of this investigation indicate that the robustness of the model for predicting the response of reinforced concrete systems subjected to reversed cyclic loading is also an appropriate topic for future research.