CHAPTER 3:
REINFORCING STEEL MATERIAL MODEL

3.1 Introduction

The behavior of reinforcing steel may control the response of reinforced concrete structural elements subjected to earthquake loading. Thus, it is necessary to develop an analytical model that predicts the fundamental characteristics of steel within a range of loading that is appropriate for these structural systems. Typical load histories for reinforcing steel follow from consideration of the observed response of reinforced concrete structural elements. Characteristics of steel response are established through laboratory testing of steel coupons. Previous research provides theories and techniques for development of an analytical model that predicts steel response. Experimental data provide information for final calibration and refinement of the proposed model.

The following sections present the analytical model used in this investigation for finite element analysis of reinforced concrete beam-column connections. The characteristics of reinforcing steel are defined by the experimental data presented in Section 3.2. Analytical models proposed for use in previous investigations are presented in Section 3.3. Section 3.4 provides a discussion of the model developed for use in this investigation. Comparison of the predicted and observed response of reinforcing steel is presented in Section 3.5.

3.2 Steel Material Properties Established Through Experimental Testing

In predicting the response of reinforcing steel for analysis of reinforced concrete structures subjected to earthquake loading, it is necessary that an analytical model be developed on the basis of a representative material data set. An appropriate data set encompasses the range of force and deformation demands as well as the range of load rates that the material may experience in an actual structure subjected to earthquake loading.

3.2.1 Criteria for Experimental Investigation of Reinforcing Steel

Reinforcing steel in reinforced concrete structures carries load primarily along the axis of the bar. Reinforcement is relatively strong and stiff when loaded along the bar axis, and reinforced concrete members are designed to exploit this. Typically, loading of a volume of reinforced concrete results in cracking of the concrete and transfer of the tensile load to the reinforcement along the bar axis, perpendicular to the plane of the crack. In regions of variably oriented loading or relatively high shear loading, or both, cracked reinforced concrete may be loaded perpendicular to the axis of a reinforcing bar and parallel to an established crack surface. This loading of the cracked reinforced concrete volume is referred to as shear friction, and activation of reinforcement perpendicular to the bar axis at a crack surface is referred to as *dowel action*. Most research into the shear-friction response of reinforced concrete elements indicates that even for this type of loading axial rather than dowel action dominates the response of the reinforcement perpendicular to the crack surface [Laible *et al.*, 1977; Paulay and Loeber, 1977]. Under shear friction loading, a concrete crack must open substantially for sliding to occur. However, crack opening produces tensile stress in the reinforcing steel crossing the crack. These forces are equilibrated by a clamping force in the concrete that pushes the crack closed. Increased clamping force results in increased sliding frictional resistance along the crack surface. Thus, axial stress in the steel rather determines response. While model represents the response observed by most researchers, results of one experimental investigation suggest that the dowel action of reinforcing steel may not be negligible and my contribute between twenty-five and thirty-five percent of the slip resistance at a crack surface [Hofbeck *et al.*, 1969].

For the current investigation, the dowel action of reinforcing steel is neglected and reinforcing steel is modeled as an uniaxial element. As discussed above, the results of experimental investigation indicate that this is an appropriate model. Additionally, as the response of reinforced concrete bridges typically is not controlled by shear-friction resistance, the contribution of steel dowel action has a limited effect on system response. Further, experimental and analytical data are not sufficient for development and calibration of this behavior. If reinforcing steel is modeled as an axial element, then experimental data defining the response of reinforcing steel to general, uniaxial loading are required for model development, calibration and verification.

In members that primarily carry flexure, uniaxial strain demands under earthquake excitation may be significant. Typically, reinforcing steel is subjected to significant tensile strain demands; while the compressive strength and stiffness provided by surrounding concrete limits compressive strain demands (Figure 3.1). Thus, an appropriate experimental strain history for use in development of a material model for longitudinal reinforcement includes significant tensile strain demands accompanied by moderate compressive stress demands as well as significant tensile strain demands accompanied by moderate compressive strain demands (and significant compressive stress demand). It is important to note that the asymmetry of the load history is more extreme for reinforcement designed to provide confinement or to carry shear load. This reinforcement typically experiences minimal compressive stress demand. While a strain history with severe tensile stress demand and limited compressive stress demand may be representative of the strain history for reinforcement in an actual structure, examination of steel response to more symmetric strain histories may provide additional information that is not obviously observed in the more representative strain histories.

Often, failure of ductile reinforced concrete flexural members results from buckling of longitudinal reinforcement. Buckling typically occurs following unloading from a point of severe tensile strain demand. However, the load and deformation at which buckling occurs depends on a number of system parameters including the strain history of the reinforcing steel bar, the volume and distribution of transverse reinforcement and concrete cover. This mode of response cannot be investigated through experimental testing of independent reinforcing steel bar.

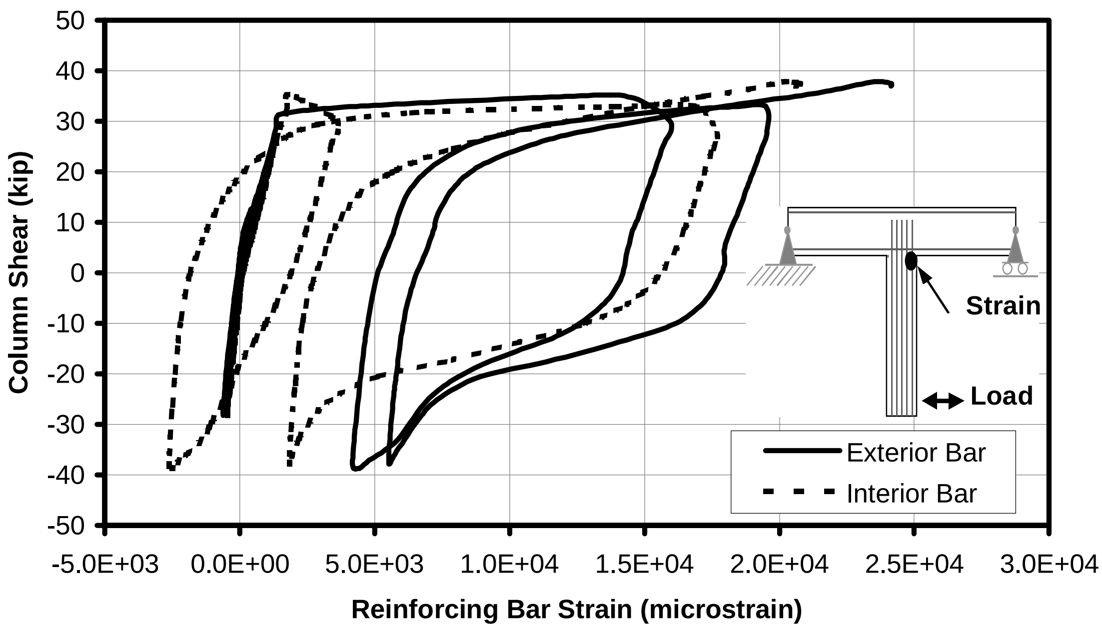

Figure 3.1: Typical Load Versus Strain History for Reinforced Concrete Column Subjected to Simulated Earthquake Loading (Data from Lowes and Moehle, 1995)

The engineering stress-strain history for reinforcing steel loaded in tension exhibits an elastic-plastic response with moderate strain hardening to a relatively high strain demand. However, severe strain demands ultimately result in necking of the reinforcement and reduced engineering (Cauchy) stress capacity. Characterization of steel response at large strains requires an experimental test procedure that includes a closed-loop system in which loading can be achieved at a fixed strain rate. Additionally, because necking is a localized phenomenon, characterization of this mechanism of response requires consideration of gage length.

Typically, experimental testing of reinforcing steel is completed at strain rates that are pseudo-static. However, some research has focussed on investigation of steel response at dynamic strain rates. Results of dynamic testing of reinforcing steel may be used to adjust the results of pseudo-static tests to account for dynamic load effects.

Evaluation of the effect of dynamic loading on steel response requires estimated maximum strain rate demands developed in structures subjected to earthquake loading. As previously discussed (Section 2.2.8) appropriate maximum strain rates for reinforced concrete bridges subjected to earthquake loading may be estimated on the basis of the dynamic properties of the bridge and the concrete and steel material parameters. For reinforcing steel, maximum strain rate may be estimate for both the elastic load range and the inelastic load range. For the case of unyielded reinforcing steel, maximum strain rate may be estimated assuming that maximum loading is applied during a time interval equal to $0.25T_{bridge}$. Here T_{bridge} defines the period of the bridge assuming some cracking of concrete under service level and environmental loading and typically ranges between 0.25 sec. and 1.0 sec. Assuming that maximum load corresponds to the yield strength of the reinforcing bar, this implies a maximum strain rate of 0.03 per second. For the case of yielded reinforcing steel, it is reasonable to assume that the maximum load corresponds to the steel developing ultimate strength at a strain demand of approximately 0.15. Further, it may be assumed that this load is applied during an interval equal to $0.25T_{eff}$. Here T_{eff} defines the effective period of the bridge with inelastic deformation and ranges between 0.5 sec. and 2.0 sec., assuming an average ductility demand of 4. On the basis of these assumptions it follows that the maximum strain rate for reinforcing steel is 1.2 per second.

In order to improved the accuracy of experimental results, laboratory testing of reinforcing steel typically is carried out using coupons that have been machined to a uniform diameter over a given range and then notched to localize the failure mechanism. This facilitates measurement of steel area. Ma *et al.* [1976] tested both unmachined and machined bars and noted that machined specimens had slightly higher yield strength and slightly higher initial hardening modulus. However, strength variations between the machined and unmachined bars were less than 5 percent. Thus, experimental testing of either unmachined or machined reinforcing bars is appropriate; however, accurate measurement of initial steel area is necessary.

3.2.2 Behavior of Reinforcing Steel Subject to Axial Loading

Experimental data define the response of reinforcing steel subjected to uniaxial, pseudo-static monotonic tensile loading, cyclic tensile loading and reversed cyclic loading This information is extended by monotonic and cyclic testing of reinforcement at variable load rates.

Figure 3.2 shows a typical engineering stress-strain history for reinforcing steel subjected to monotonically increasing strain demand. Important characteristics of this response include the following:

1. Initial response is linear-elastic for stress demand less than the initial yield strength.
2. For strain demand exceeding that corresponding to the initial yield strength, there is a slight drop in strength below the initial yield strength. Strength is maintained at this lower yield strength for moderate increase in strain demand. This range of response is referred to as the yield plateau or the Lüders plateau, and the material yield strength typically is defined to be the average strength for loading within this strain range.
3. Increasing strain demand results in increased strength. This strain-hardening regime is maintained to a peak strength that typically exceeds the yield strength by

thirty to sixty percent (Figure 3.3). The ratio of peak strength to nominal strength is a function of the steel specification, grade and batch composition.

4. At severe tensile strain demand, reinforcement begins to neck and strength is reduced.
5. At a maximum strain demand, the steel reinforcement fractures and load capacity is lost.

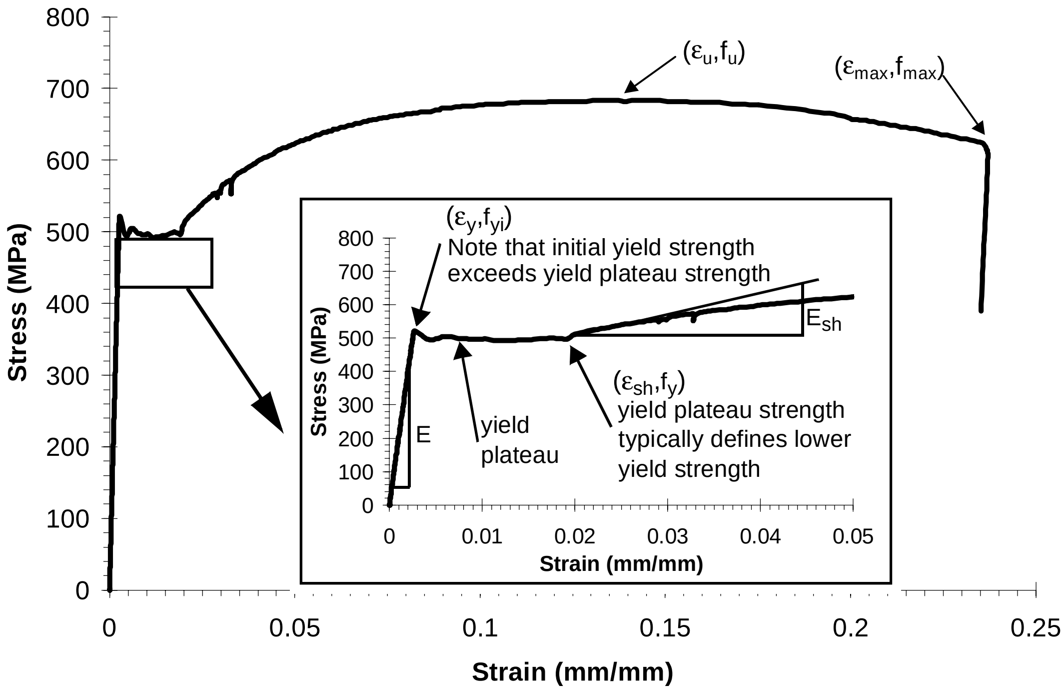

Figure 3.2: Tensile Monotonic Stress-Strain History for Typical Reinforcing Steel Bar (Data for A706 Grade 60 Reinforcement [Naito, 1999])

This monotonic steel response may be defined by a few material parameters as identified in Figure 3.2. These include the elastic modulus, E; the lower yield strength, f_y; the strain at which strain hardening initiates, ε_{sh}; the strain at which peak strength is achieved, ε_u; the peak strength, f_u; the strain at which fracture occurs, ε_{max}, and the capacity prior to bar fracture, f_{max}.

Figures 3.3 presents the behavior of samples of reinforcing steel subjected to monotonically increasing tensile strain demand. Some variation in response results from variability in steel specification and steel grade. However, these data show that two steel batches with the same specification and grade may have significantly different post-yield behavior (response of #4 and #6 A706 reinforcement as shown in Figure 3.3).

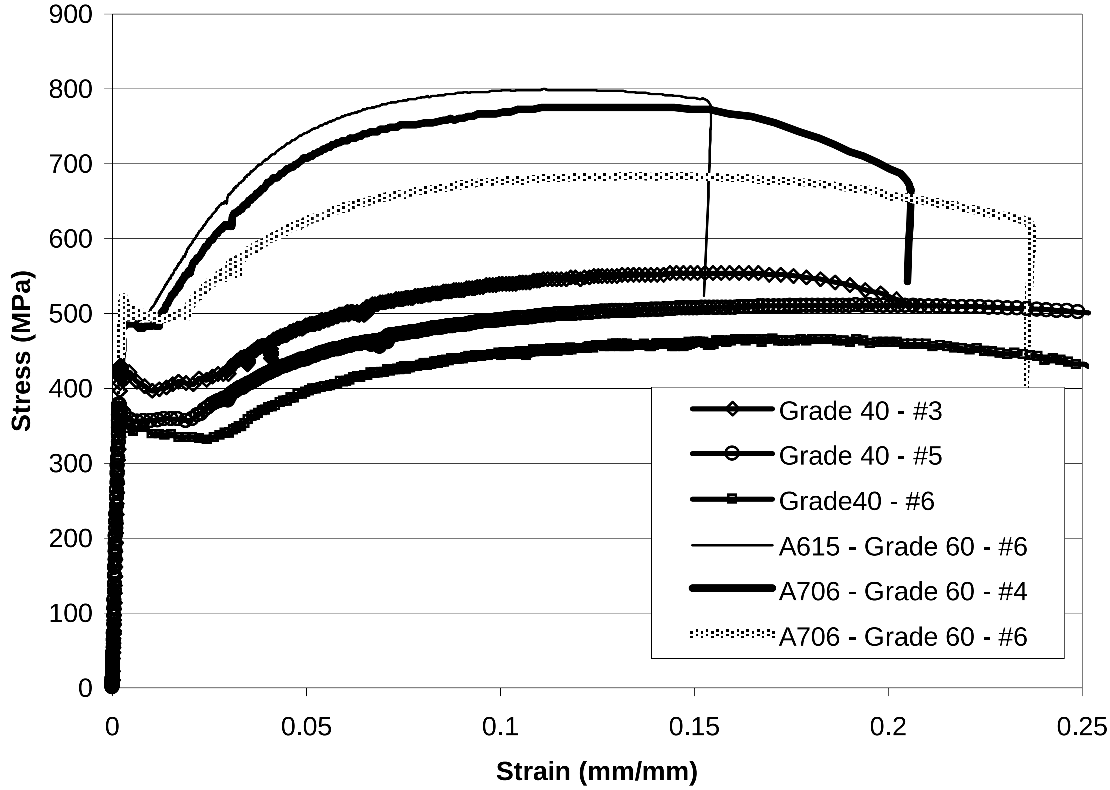

Figure 3.3: Stress-Strain Histories for Specifications and Grades of Reinforcing Steel Typically Used in the United States (Data from Lowes and Moehle, 1995; Mazzoni,

Figures 3.4, 3.5 and 3.6 present data for steel reinforcement subjected to reversed cyclic loading. The strain histories used in these material tests are representative of the strain history observed in actual structures. The extent of compressive stress/strain demand is a function of the load history. Figure 3.4 provides data for a system with limited compressive stress demand while Figure 3.5 provides data for steel subjected to moderate compressive strain demand. Figure 3.6 shows data for reinforcing steel subjected to a reversed cyclic strain demand with symmetric inelastic strain increments. While this is not representative of strain histories in actual systems, these data exhibit particular characteristics of response that are not obviously revealed by the previous data sets. Important characteristics of response shown in Figures 3.4, 3.5 and 3.6 include the following.

1. Upon unloading, steel exhibits a loss of linearity prior to achieving the yield strength in the opposite direction. This loss of linearity is referred to as the Bauschinger effect [Bauschinger, 1887]. Researchers have noted that this effect becomes more pronounced with increased strain demand [*e.g.*, Ma, 1976].
2. The initial tangent to the unloading stress-strain response is slightly less than the initial elastic stiffness. This characteristic has been observed by a number of researchers [*e.g.*, Bauschinger, 1887; Panthaki, 1991]
3. Reinforcing steel exhibits isotropic strain hardening, characterized by increasing strength under increasing inelastic strain demand. This is observed under cyclic as well as monotonic loading (Figures 3.3 through 3.6).
4. Reinforcing steel exhibits cyclic strain softening, defined by Ma *et al.* [1976] as reduced tangential stiffness under multiple cycles to particular strain limits (Figures 3.4, through 3.6). This loss of strength under cyclic loading to a prescribed strain limit is particularly evident in Figure 3.6.

These characteristics will be considered in development of a material model for this investigation.

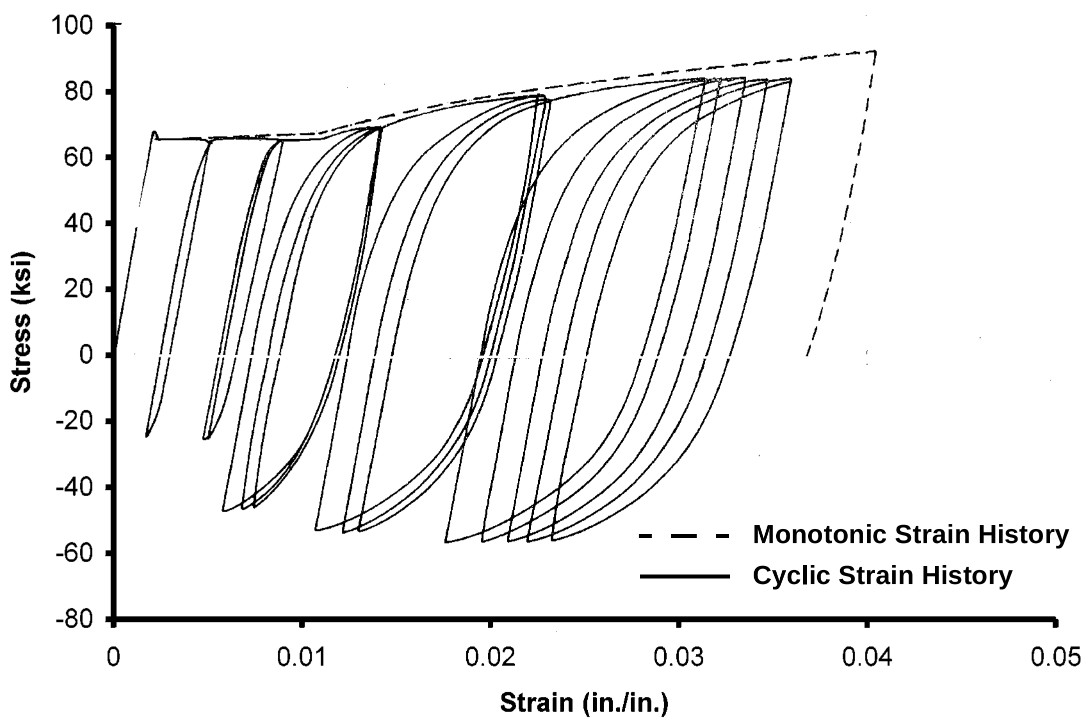

Figure 3.4: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Moderate Compressive Stress Demand (Experimental Data from Ma *et al.*, 1976)

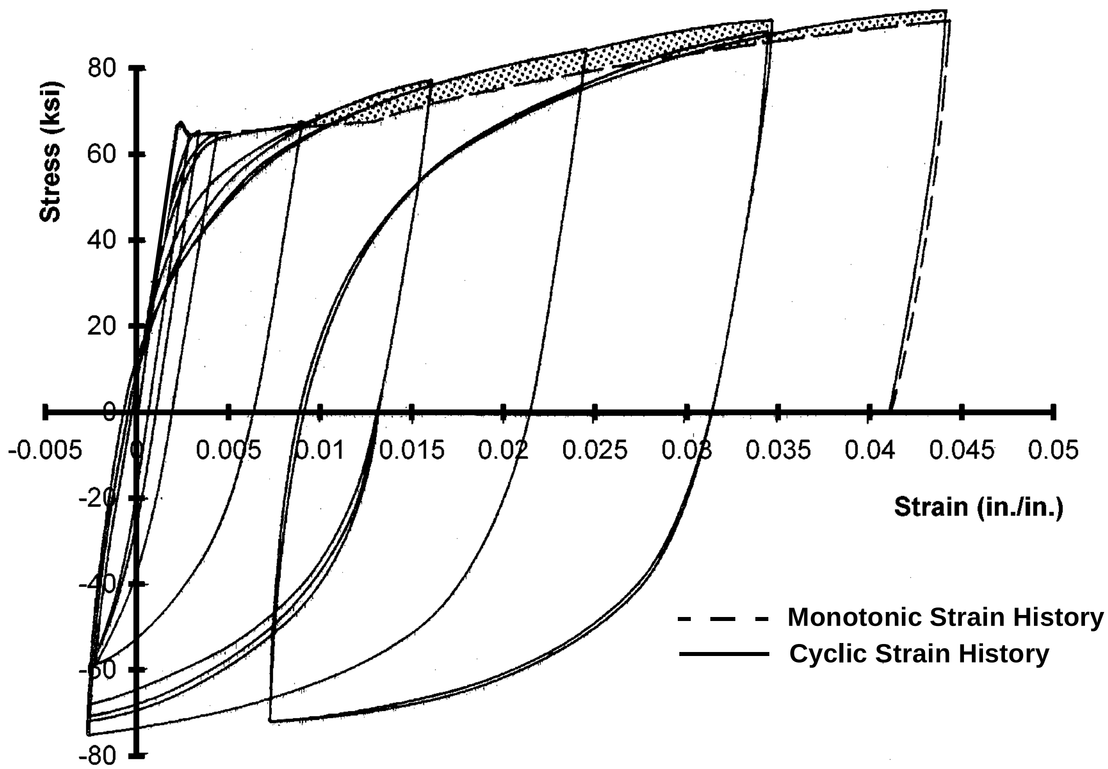

Figure 3.5: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Moderate Compressive Strain Demand (Experimental Data from Ma *et al.*, 1976*)*

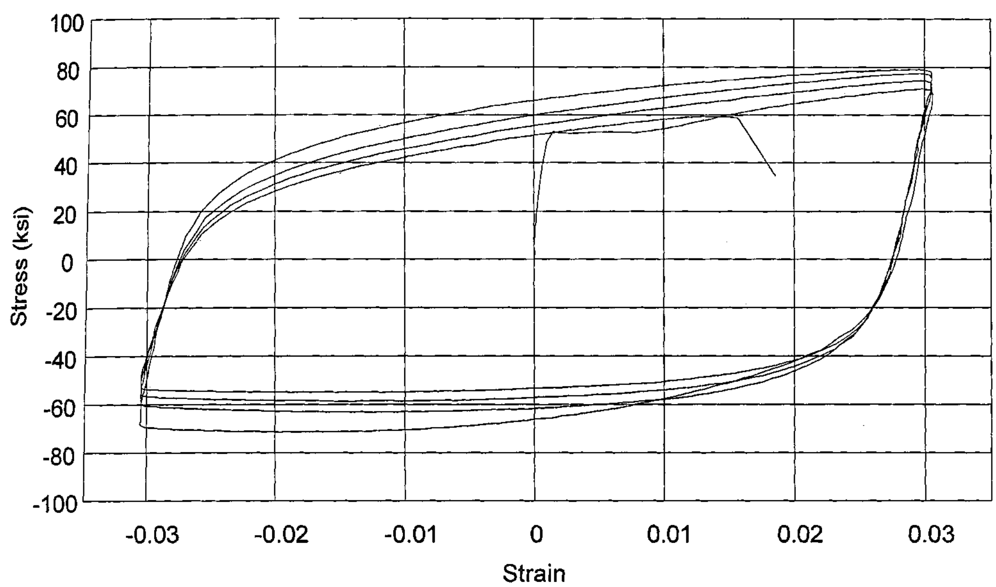

Figure 3.6: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Compressive and Tensile Strain Demands (Experimental Data from Panthaki [1991] as presented by Chang and Mander [1994])

3.2.3 Effect of Load Rate

Experimental data collected during previous investigations define the effect of increased strain rate on steel material response. These data may be used to adjust steel strength for loading in the range of strain rates developed in structures subjected to earthquake loading. For reinforcing steel, maximum strain rates developed under earthquake loading are approximately 0.3 per second for elastic steel approaching yield strength and 1.2 per second for yielding steel developing ultimate strength. Figure 3.7 shows the ratio of dynamic to static tensile yield strength as a function of strain rate. These data are for "mild structural steel" [Manjoine, 1944] and A36 steel [Chang and Lee, 1987] as well as steel grades typically used for reinforcing steel [Cowell, 1965] and A615 steel [Mahin *et al.*, 1972]. These data show that for typical reinforcing steel, dynamic yield strength is approximately 10 percent larger than the static yield strength within the range of load rate that is appropriate for structures subjected to earthquake loading. Additionally, these data show that the increase in yield strength observed under dynamic loading is less significant for higher grade steels. Investigations conducted by Mahin *et al.* [1972] show that increased strain rate has a negligible effect for steel loaded beyond the initiation of strain hardening and for steel subjected to reversed cyclic loading. Similarly, data collected by Manjoine [1944] show a maximum increase in ultimate strength of mild structural steel of approximately 4 percent within the range of strain rates appropriate for this investigation. Results of several investigations indicate that increased strain rate has relatively little effect on elastic modulus [ACI Committee 439, 1969; Mahin *et al.*, 1972; CEB, 1988].

The experimental data show that dynamic loading has a relatively limited effect on the response of reinforcing steel. Increased strain rate will result in increased yield strength; however, the strength increase beyond that observed under pseudo-static load

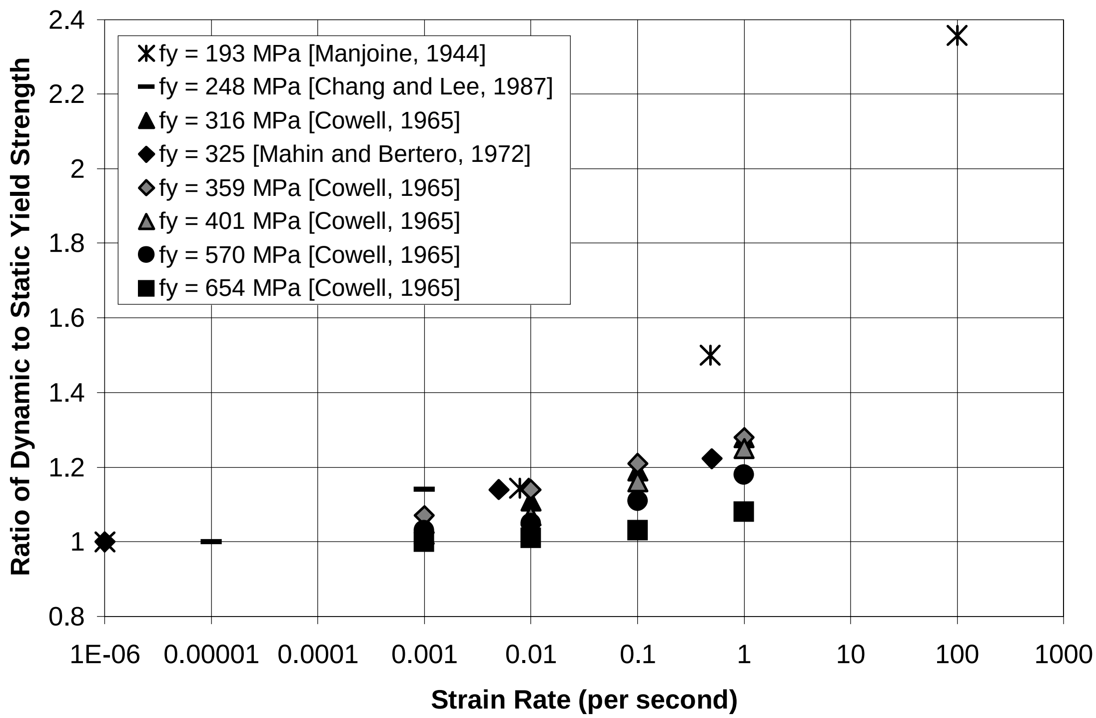

Figure 3.7: Effect of Strain Rate on Steel Yield Strength (Data from sources cited in the Figure)

conditions is at most ten percent within the range of steel grades and strain rates of interest to this investigation.

3.3 Steel Constitutive Models

An number of researchers have proposed models to characterize the response of reinforcing steel for use in analysis of reinforced concrete structures. Some of these models are developed on the basis of material constitutive theories; however, the majority of these are phenomenological models that characterize the macroscopic response on the basis of experimental data. The fundamental characteristics of the one-dimensional steel response are relatively simple. Thus, the appropriate material model not only predicts this response with a reasonable level of accuracy but is also calibrated to fit experimental data with relative ease and is computationally efficient.

3.3.1 Material Constitutive Theory Applied to Modeling Steel Behavior

Various theories have been proposed to characterize the response of reinforcing steel subjected to reversed cyclic loading on the basis of microscopic material response. A number of these models are identified and discussed by Cofie [1983]. The simplest and most computationally efficient model for predicting steel behavior is that developed on the basis of modern plasticity theory. The one dimensional behavior of reinforcing steel is representative of an *elastic-plastic* material. In particular, results of experimental testing show the accumulation of unrecoverable, plastic deformation and an unloading stiffness that is approximately equal to the initial elastic material stiffness. Additionally, steel exhibits isotropic strain hardening, characterized by increased strength under increased inelastic strain demand. Further, the premature yielding associated with the Bauschinger effect may be characterized by a plasticity model that incorporates kinematic strain hardening.

A one-dimensional constitutive relationship developed on the basis of plasticity theory and incorporating linear isotropic and kinematic strain hardening is defined by the following set of equations:

$$f(\sigma, \alpha_p) = (|\sigma - \beta| - q(\alpha_p)) \tag{3-1a}$$

$$\dot{\varepsilon}_p = \gamma \operatorname{sgn}(\sigma - \beta) \tag{3-1b}$$

$$\dot{\alpha}_p = \gamma \tag{3-1c}$$

$$\dot{\beta} = \gamma H \operatorname{sgn}(\sigma - \beta) \tag{3-1d}$$

$$q(\alpha_p) = \sigma_Y + K\alpha_p \tag{3-1e}$$

$$\gamma \ge 0 \qquad \dot{f} \le 0 \qquad \gamma\dot{f} = 0 \tag{3-1f}$$

where all plasticity variables follow the previous definitions (Section 2.3.2), σ_Y is the yield strength, K and H, respectively, are the isotropic and kinematic hardening parameters and the function *sgn(.)* is defined to be 1.0 for a positive operand and -1.0 for a negative operand.

Integration of these equations and implementation in an incremental solution algorithm results in an explicit solution algorithm. Additionally, explicit equations govern the determination of the material parameters from experimental data. Figure 3.8 shows a typical stress-strain history as predicted by Equations (3-1a) through (3-1f). While this model can be calibrated to predict steel strength in the vicinity of cyclic peak strain demands, the model does not represent well the observed steel response along the path between points of peak strain demand (Figure 3.4). However, for some applications, the inaccuracy of this model is acceptable given the simplicity of the formulation and the ease with which it may be calibrated to best fit observed response.

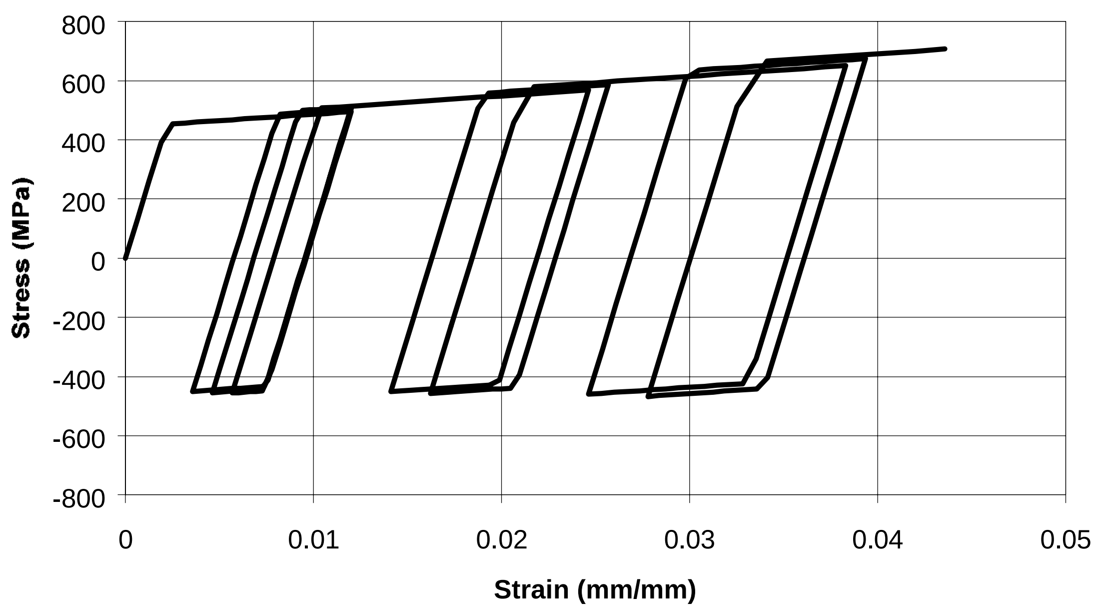

Figure 3.8: Steel Stress-Strain History as Predicted on the Basis of Plasticity Theory

The accuracy with which the plasticity model predicts the observed response may be improved through use of a more sophisticated hardening rule and additional hardening variables. However, this may not be the most efficient model for characterizing steel response.

3.3.2 Phenomenological Model

A more representative model for the response of reinforcing steel subjected to reversed cyclic loading can be achieved through the use of phenomenological models in which non-linear equations are calibrated on the basis of experimental data. One of the first models of this type is that proposed by Ramberg and Osgood [1943]. Various other models have followed. Recently, a number of models have been developed on the basis of work done by Menegotto and Pinto [1973]. Also recently, several models have been developed that characterize behavior of the basis of the natural steel stress and strain rather than the engineering stresses and strains.

3.3.2.1 Characterization of Steel Response Using the Ramberg-Osgood Equation.

The model proposed by Ramberg and Osgood [1943] uses a single non-linear equation to characterize the observed curvilinear response of reinforcing steel subjected to monotonic loading. This model defines the normalized strain to be a function of the normalized stress (stress and strain increments are normalized with respect to twice the yield value):

$$\varepsilon_{norm} = \beta\sigma_{norm}(1 + \alpha|\sigma_{norm}|^{n-1}) \tag{3-2}$$

The model may be extended for the case of reversed-cyclic loading by introduction into Equation (3-2) of the stress and strain at which load reversal occurs. While this model has been shown to predict the one-dimensional steel response with acceptable accuracy, the explicit dependence on the stress reduces the ease with which this model is implemented in a typical strain-driven finite element analysis program. Additionally, using the model to represent monotonic response does not provide for description of the yield plateau, a characteristic of response that may control system behavior. These issues are addressed by more recent models in which stress is defined to be an explicit function of the strain and the monotonic response is characterized more accurately.

3.3.2.2 Characterization of Steel Response Using the Menegotto-Pinto Equation

Menegotto and Pinto [1973] propose a model for characterizing reinforcing steel in which the response is defined by the following non-linear equation:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{\left(1+{\varepsilon^*}^{R}\right)^{1/R}} \tag{3-3}$$

where the effective strain and stress (ε^*, σ^*) are a function of the unload/reload interval, b is the ratio of the initial to final tangent stiffnesses and R is a parameter that defines the shape of the unloading curve. In this implementation, it is assumed that the reference curves (stress-strain curves that bound the cyclic response) as well as unloading and reloading response may be characterized by Equation (3.3). This implementation also neglects characterization of the yield plateau. In recent years, a number of researchers have proposed material models that use the Menegotto-Pinto equation to characterize the unloading-reloading response of reinforcing steel. One such model is that proposed by Stanton and McNiven [1979]. This model uses an approximate version of the Menegotto-Pinto equation (Equation 3-3) to improve computational efficiency and assumes that the reference curves for steel subjected to cyclic loading follows the monotonic envelope. A second model is that proposed by Filippou *et al.* [1983]. This model incorporates Equation (3-3) exactly to describe unloading response. The model follows from the assumption that the reference curve defining the cyclic stress-strain response is tri-linear. Isotropic, cyclic strain hardening is incorporated through shifting of the reference curve as a function of the plastic strain increment. Recently, Equation (3-3) has been incorporated into a sophisticated model proposed by Chang and Mander [1994]. This model assumes that the shape of the reference curve is defined by the monotonic stress-strain response. The model accounts for cyclic strain hardening through shifting of the reference curve as a function of strain history. Additionally, the model incorporates variability in initial unloading stiffness, cyclic strain softening and memory of multiple load-unload cycles. Each of these four models predicts with acceptable accuracy the observed cyclic response of reinforcing steel subjected to strain histories typical of those observed in reinforced concrete structures subjected to simulated earthquake loading. Of these models, that proposed by Chang and Mander [1994] provides quite accurate prediction of steel response while that proposed by Filippou *et al.* [1983] provides both reasonably accurate prediction of response and relatively simple implementation and calibration.

All of the previously discussed models assume a symmetric response for loading in compression and tension. However, data suggest that this may be not be an appropriate assumption (Figure 3.9). Data also show that the monotonic response in compression and tension are essentially the same if the response is characterized by the natural strains and stresses ($\bar{\varepsilon}$, $\bar{\sigma}$), defined as follows:

$$\bar{\varepsilon} = \ln(1 + \varepsilon) \tag{3-4a}$$

$$\bar{\sigma} = \ln(1 + \sigma) \tag{3-4b}$$

Recently two models have been proposed that define material response on the basis of the natural stress-strain history [Dodd and Restrepo-Posada, 1995; Balan *et al.*, 1998]. These models assume a shape for the cyclic reference curve as defined by the monotonic natural stress-strain history. These models predict various aspects of cyclic response including the Bauschinger effect, reduced elastic modulus, isotropic strain hardening, and cyclic strain softening. The models differ in the non-linear equations used to model individual characteristics of material response.

The symmetry of compression and tension response as characterized in the natural stress-strain system is conceptually pleasing; however, it is not obvious that this is necessary for modeling the response of reinforcing steel in reinforced concrete structures subjected to earthquake loading. For this steel, the load history typically is not symmetric

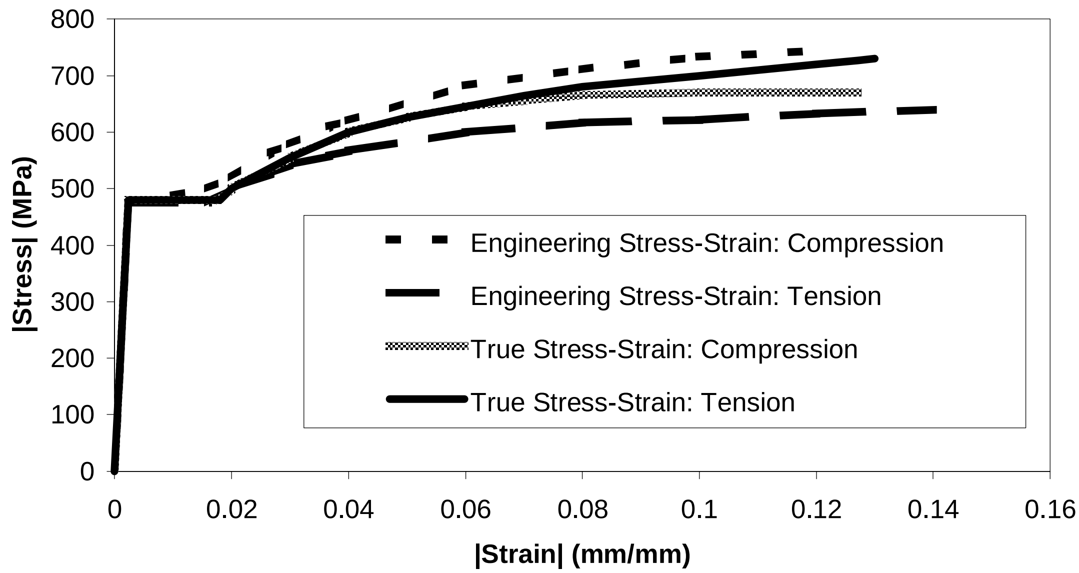

Figure 3.9: Engineering Versus Natural Stress-Strain History for Reinforcing Steel Subjected to Monotonic Compression and Tension Loading (Data from Dodd and Restrepo-Posada [1995])

with load histories showing significant tensile strain demand and limited compressive strain demand. For these cases, characterization of the model on the basis of the tensile monotonic response is perhaps appropriate. Additionally, it is not clear that the enhanced accuracy of these model justifies the additional complexity and computational effort.

3.4 Characterization of the Response of Reinforcing Steel

For this investigation, a material model is developed that defines those fundamental characteristics of steel behavior that control the response of reinforced concrete structures subjected to earthquake loading. This model follows from several previously proposed models and predicts the uniaxial steel material response as defined by the presented experimental data. Consideration of past research indicates that a macroscopic material model is most appropriate for prediction of steel response. Further, the results of past research show that steel behavior may be characterized with acceptable accuracy on the basis of engineering strains and stresses. Here, it is assumed that the observed moderate increase in steel yield strength as a function of increased strain rate may be neglected as this increased strength will only be observed during short periods of rapid loading, will not control development of a specific failure mechanism in the beam-column joint system and will facilitate correlation with observed laboratory tests in which loading is pseudo-static. Strain hardening may determine system strength and necessarily is incorporated into the model. Here strain hardening is characterized through the assumption that plastic deformation in both tension and compression results in nonlinear hardening as defined by the experimentally observed monotonic stress-strain response. Finally, experimental data show that the effect of cyclic strain softening is limited, and this characteristic of response is not incorporated into the material model.

The proposed model defines the response of reinforcing steel subjected to reversed cyclic loading on the basis of three characteristic stress-strain response curves: a reference curve, an unloading curve and a reloading curve. For this investigation, the monotonic stress-strain histories define the model reference curves. Experimental data show that the monotonic tensile stress-strain history bounds the tensile response of reinforcing steel subjected to reversed cyclic strain histories with severe tensile strain demands and moderate compressive demand. For reinforcing steel subjected to severe compressive and tensile strain demands, the monotonic stress-strain histories, shifted to account for accumulated plastic deformation provide a reasonably accurate bound for the observed response. Also for this investigation, unloading and reloading curves are defined using the Menegotto-Pinto equation and calibration parameters provided by previous researchers. Previous research shows that the Menegotto-Pinto equation represents well the unloading and reloading response of reinforcing steel subjected to cyclic loading. Additionally, previous research provides calibration parameters for this equation that are appropriate for modeling grades of reinforcing steel typically used in construction of reinforced concrete structures.

The reference curve is defined on the basis of the monotonic stress-strain history as determined from experimental testing. Here the equation proposed by Chang and Mander [1994] is used to describe the monotonic response in compression or tension:

$$\sigma = \frac{E(\varepsilon - \varepsilon_{shift})}{\left(1 + \left(\frac{E(\varepsilon - \varepsilon_{shift})}{\sigma_y}\right)^{10}\right)^{0.1}} + (\sigma_u - \sigma_y)\left(1 - \left|\frac{\varepsilon_u - (\varepsilon - \varepsilon_{shift})}{\varepsilon_u - \varepsilon_{sh}}\right|^{p}\right) \tag{3-5a}$$

where

$$p = E_{sh}\frac{\varepsilon_u - \varepsilon_{sh}}{\sigma_u - \sigma_y} \tag{3-5b}$$

and σ is the engineering stress, ε is the engineering strain, ε_{shift} is a function of the plastic deformation in either compression or tension and steel material parameters are as previously defined. The model variable, ε_{shift}, is defined, as proposed by Chang and Mander, to be a function of the strain at which strain hardening occurs in compression (or tension) and of the extreme strain experienced in compression (or tension). For the tensile reference curve, this parameter is defined as follows (here material parameters characterize the monotonic compressive response):

$$\varepsilon_{shift}^{tension} = k\left(\varepsilon_{sh} - \frac{f_y}{E}\right) + (1-k)\left(\varepsilon_{min} - \frac{f_{min}}{E}\right) + \varepsilon_{shift}^{compression} \tag{3-6}$$

where k is a weighting parameter defined as follows:

$$k = \exp\left(\frac{\varepsilon_{min}}{5000(\varepsilon_y)^2}\right) \tag{3-7}$$

It is important to note that the model variable ε_{shift} provides a measure of plastic deformation; this variable is analogous, but different in definition, to the plastic deformation defined in classical plasticity theory.

Unloading and reloading curves are defined using the Menegotto-Pinto equation. A curve is defined completely by the strain-stress point at which there is load reversal, $(\varepsilon_o, \sigma_o)$; the target strain-stress point on a reference curve, $(\varepsilon_t, \sigma_t)$, and the tangent to the curve at either end point. The target strain-stress point is taken equal to the extreme strain in the target direction, shifted to account for plastic deformation, and the stress is defined by the reference curve. Here it is assumed that the initial tangent to the curve upon a load reversal is equal to the initial elastic modulus. The model relationship proposed by Chang and Mander is used to predict the tangent to the stress-strain history as the material approaches the reference curve, E_t:

$$E_t = \frac{1}{\frac{1}{E} + \left(\frac{\varepsilon_{ex} - \varepsilon_y}{\varepsilon_{sh} - \varepsilon_y}\right)\left(\frac{1}{E_{sh}} - \frac{1}{E}\right)} \tag{3-8}$$

where ε_{ex} is the extreme strain previously achieved in tension or compression. Thus, for unloading or reloading, stress as a function of strain is defined as follows:

$$\sigma = \sigma_o + E(\varepsilon - \varepsilon_o)\left(b + \frac{(1-b)}{\left(1 + \left|A\frac{\varepsilon - \varepsilon_o}{\varepsilon_t - \varepsilon_o}\right|^R\right)^{1/R}}\right) \tag{3-9}$$

where A and b are model parameters that constitute a modification to the original Menegotto-Pinto equation. Chang and Mander derive the following relationship between the parameters:

$$A = \left(\left(\frac{E(1-b)}{E_{sc} - bE}\right)^R - 1\right)^{1/R} \tag{3-10}$$

where E_{sc} is the secant modulus to the unload-reload curve. The modification of these model parameters from the initial implementation by Menegotto and Pinto ensures the curve passes through both the initial and final stress-strain points and achieves the target slope at the final point. Chang and Mander propose a solution in which these models parameters are derived at each load reversal. However, here the requirement that the final slope be the target slope is relaxed allowing for explicit definition of Equation (3-9) and Equation (3-10). Thus, in this model b is defined as follows:

$$b = \frac{E_t}{E} \tag{3-11}$$

From Equations (3-9), (3-10) and (3-11), it follows that the steel stress and the algorithmic tangent are defined as follows:

$$\sigma = f_o + E(\varepsilon - \varepsilon_o)\left(b + \frac{1-b}{\left(1 + \left|\varepsilon - \varepsilon_o \frac{E - E_t}{E_{sc} - E_t}\right|^R\right)^{1/R}}\right) \tag{3-12a}$$

$$\frac{d\sigma}{d\varepsilon} = Eb + \frac{E(1-b)}{\left(1 + \left|\varepsilon - \varepsilon_o \frac{E - E_t}{E_{sc} - E_t}\right|^R\right)^{(1 + 1/R)}} \tag{3-12b}$$

3.5 Comparison of Material Model with Experimental Data

The proposed steel material model is implemented in the finite element program FEAP [Taylor, 1998; Zienkiewicz and Taylor, 1987 and 1991]. This implementation is used to analyze the response of a several-element mesh of reinforcing steel subjected to various load histories. The behavior of steel as predicted by the material model is compared with the experimentally observed response for a variety of load histories including monotonic tension, cyclic tensile loading, reversed cyclic loading with moderate compressive stress demand and reversed cyclic loading with severe strain demand in both tension and compression.

Figure 3.10 shows the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile strain demand and moderate compressive stress demand. Differences between the two histories primarily result from differences in the prescribed strain histories. The proposed model represents well the observed response. Figure 3.11 shows the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile and moderate compressive strain demand. The proposed material model represents well the fundamental characteristics of the observed response. However, for this strain history, in which cyclic strain increments are relatively large in both tension and compression, the model under represents the observed strain hardening and does not represent the reload curves accurately. Given that this inaccuracy is relatively small and that this strain history is extreme for an actual system, the errors in model are acceptable. Figure 3.12 presents the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile and compressive strain demands. Here, the proposed model represents well the fundamental characteristics of the response. However, the model does not represent the loss in strength that is observed following cycling to a fixed strain. Since this loss of strength is minimal and most significant for the case of multiple cycles to a fixed strain in which there is no accumulation of additional plastic strain and thus no additional material strain hardening, this inaccuracy is not significant.

3.6 Conclusions

The material model proposed for use in this investigation represents with acceptable accuracy the behavior of reinforcing steel within the range of loading that is appropriate for reinforced concrete bridges subjected to earthquake loading. The model employes a macromodel framework to describe the engineering stress-strain history of reinforcement subjected to reversed cyclic loading. The model uses nonlinear equations and calibration factors established by others and is readily calibrated to represent the response of typical reinforcing steel on the basis of parameters established through monotonic stress-strain histories. This model is appropriate for predicting the response of typical reinforcing steels subjected to variable reversed cyclic loading including load histories with severe tensile strain demand and moderate to extreme compressive stress and strain demands.

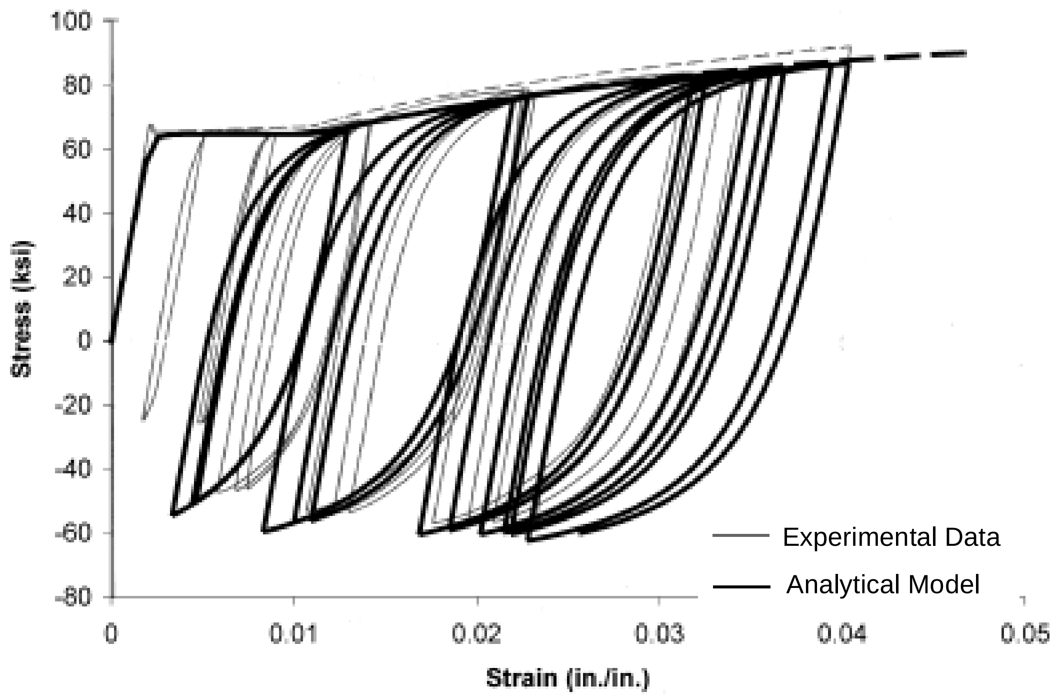

Figure 3.10: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile Strain Demands and Moderate Compressive Stress Demands as Predicted and as Observed (Data as Presented in Figure 3.4)

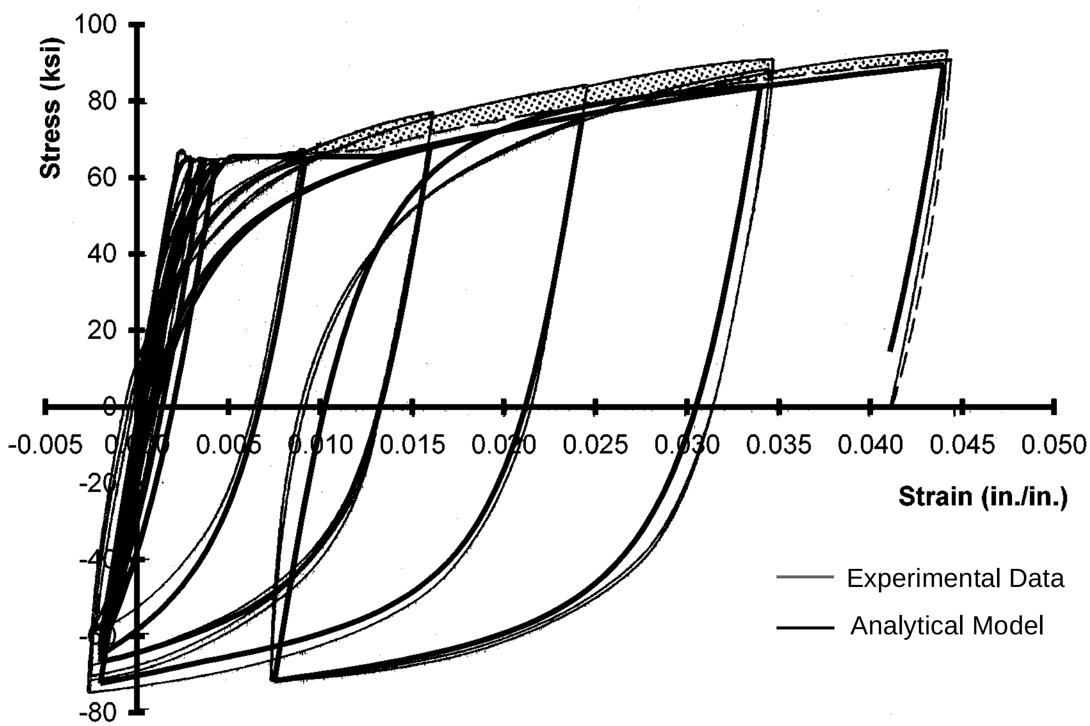

Figure 3.11: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile Strain Demands and Moderate Compressive Strain Demands as Predicted and as Observed (Data as Presented in Figure 3.5)

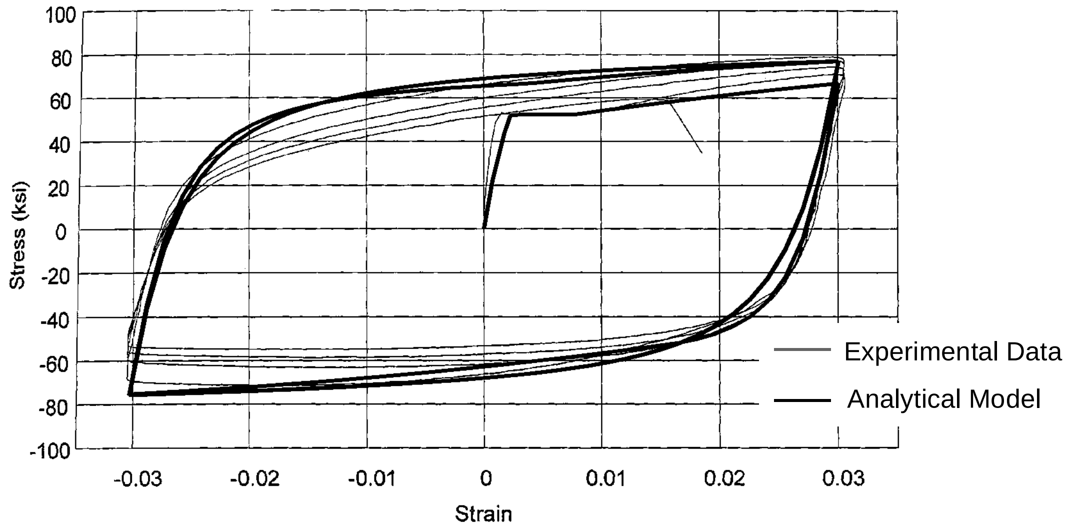

Figure 3.12: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile and Compressive Strain Demands as Predicted and as Observed (Data as Presented in Figure 3.6)

CHAPTER 3:
REINFORCING STEEL MATERIAL MODEL

3.1 Introduction

The behavior of reinforcing steel may control the response of reinforced concrete structural elements subjected to earthquake loading. Thus, it is necessary to develop an analytical model that predicts the fundamental characteristics of steel within a range of loading that is appropriate for these structural systems. Typical load histories for reinforcing steel follow from consideration of the observed response of reinforced concrete structural elements. Characteristics of steel response are established through laboratory testing of steel coupons. Previous research provides theories and techniques for development of an analytical model that predicts steel response. Experimental data provide information for final calibration and refinement of the proposed model.

The following sections present the analytical model used in this investigation for finite element analysis of reinforced concrete beam-column connections. The characteristics of reinforcing steel are defined by the experimental data presented in Section 3.2. Analytical models proposed for use in previous investigations are presented in Section 3.3. Section 3.4 provides a discussion of the model developed for use in this investigation. Comparison of the predicted and observed response of reinforcing steel is presented in Section 3.5.

3.2 Steel Material Properties Established Through Experimental Testing

In predicting the response of reinforcing steel for analysis of reinforced concrete structures subjected to earthquake loading, it is necessary that an analytical model be developed on the basis of a representative material data set. An appropriate data set encompasses the range of force and deformation demands as well as the range of load rates that the material may experience in an actual structure subjected to earthquake loading.

3.2.1 Criteria for Experimental Investigation of Reinforcing Steel

Reinforcing steel in reinforced concrete structures carries load primarily along the axis of the bar. Reinforcement is relatively strong and stiff when loaded along the bar axis, and reinforced concrete members are designed to exploit this. Typically, loading of a volume of reinforced concrete results in cracking of the concrete and transfer of the tensile load to the reinforcement along the bar axis, perpendicular to the plane of the crack. In regions of variably oriented loading or relatively high shear loading, or both, cracked reinforced concrete may be loaded perpendicular to the axis of a reinforcing bar and parallel to an established crack surface. This loading of the cracked reinforced concrete volume is referred to as shear friction, and activation of reinforcement perpendicular to the bar axis at a crack surface is referred to as *dowel action*. Most research into the shear-friction response of reinforced concrete elements indicates that even for this type of loading axial rather than dowel action dominates the response of the reinforcement perpendicular to the crack surface [Laible *et al.*, 1977; Paulay and Loeber, 1977]. Under shear friction loading, a concrete crack must open substantially for sliding to occur. However, crack opening produces tensile stress in the reinforcing steel crossing the crack. These forces are equilibrated by a clamping force in the concrete that pushes the crack closed. Increased clamping force results in increased sliding frictional resistance along the crack surface. Thus, axial stress in the steel rather determines response. While model represents the response observed by most researchers, results of one experimental investigation suggest that the dowel action of reinforcing steel may not be negligible and my contribute between twenty-five and thirty-five percent of the slip resistance at a crack surface [Hofbeck *et al.*, 1969].

For the current investigation, the dowel action of reinforcing steel is neglected and reinforcing steel is modeled as an uniaxial element. As discussed above, the results of experimental investigation indicate that this is an appropriate model. Additionally, as the response of reinforced concrete bridges typically is not controlled by shear-friction resistance, the contribution of steel dowel action has a limited effect on system response. Further, experimental and analytical data are not sufficient for development and calibration of this behavior. If reinforcing steel is modeled as an axial element, then experimental data defining the response of reinforcing steel to general, uniaxial loading are required for model development, calibration and verification.

In members that primarily carry flexure, uniaxial strain demands under earthquake excitation may be significant. Typically, reinforcing steel is subjected to significant tensile strain demands; while the compressive strength and stiffness provided by surrounding concrete limits compressive strain demands (Figure 3.1). Thus, an appropriate experimental strain history for use in development of a material model for longitudinal reinforcement includes significant tensile strain demands accompanied by moderate compressive stress demands as well as significant tensile strain demands accompanied by moderate compressive strain demands (and significant compressive stress demand). It is important to note that the asymmetry of the load history is more extreme for reinforcement designed to provide confinement or to carry shear load. This reinforcement typically experiences minimal compressive stress demand. While a strain history with severe tensile stress demand and limited compressive stress demand may be representative of the strain history for reinforcement in an actual structure, examination of steel response to more symmetric strain histories may provide additional information that is not obviously observed in the more representative strain histories.

Often, failure of ductile reinforced concrete flexural members results from buckling of longitudinal reinforcement. Buckling typically occurs following unloading from a point of severe tensile strain demand. However, the load and deformation at which buckling occurs depends on a number of system parameters including the strain history of the reinforcing steel bar, the volume and distribution of transverse reinforcement and concrete cover. This mode of response cannot be investigated through experimental testing of independent reinforcing steel bar.

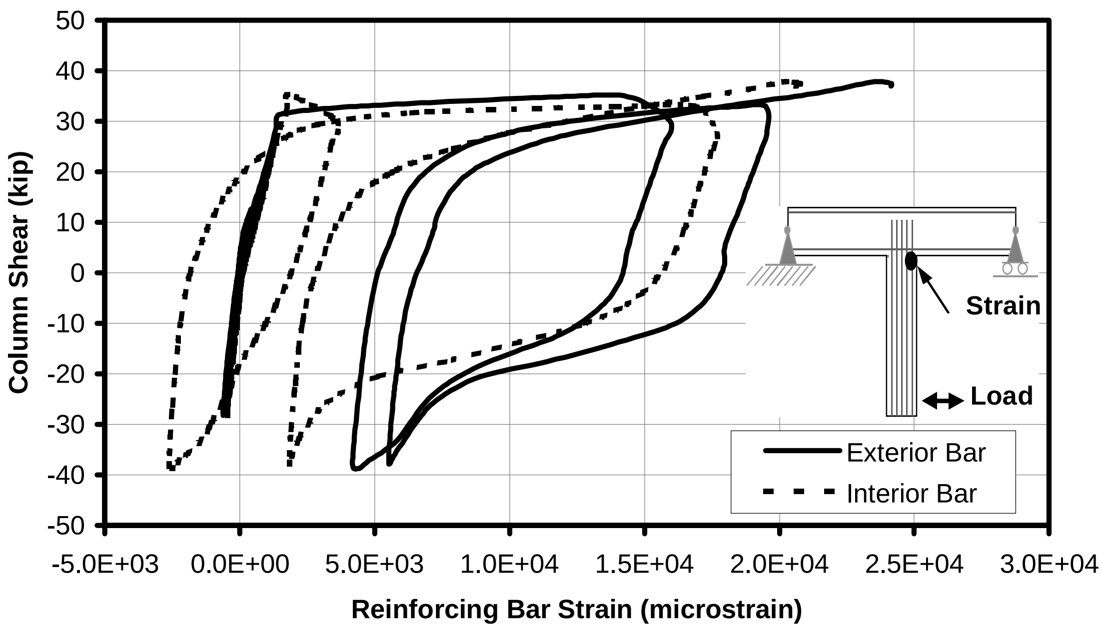

Figure 3.1: Typical Load Versus Strain History for Reinforced Concrete Column Subjected to Simulated Earthquake Loading (Data from Lowes and Moehle, 1995)

The engineering stress-strain history for reinforcing steel loaded in tension exhibits an elastic-plastic response with moderate strain hardening to a relatively high strain demand. However, severe strain demands ultimately result in necking of the reinforcement and reduced engineering (Cauchy) stress capacity. Characterization of steel response at large strains requires an experimental test procedure that includes a closed-loop system in which loading can be achieved at a fixed strain rate. Additionally, because necking is a localized phenomenon, characterization of this mechanism of response requires consideration of gage length.

Typically, experimental testing of reinforcing steel is completed at strain rates that are pseudo-static. However, some research has focussed on investigation of steel response at dynamic strain rates. Results of dynamic testing of reinforcing steel may be used to adjust the results of pseudo-static tests to account for dynamic load effects.

Evaluation of the effect of dynamic loading on steel response requires estimated maximum strain rate demands developed in structures subjected to earthquake loading. As previously discussed (Section 2.2.8) appropriate maximum strain rates for reinforced concrete bridges subjected to earthquake loading may be estimated on the basis of the dynamic properties of the bridge and the concrete and steel material parameters. For reinforcing steel, maximum strain rate may be estimate for both the elastic load range and the inelastic load range. For the case of unyielded reinforcing steel, maximum strain rate may be estimated assuming that maximum loading is applied during a time interval equal to $0.25T_{bridge}$. Here T_{bridge} defines the period of the bridge assuming some cracking of concrete under service level and environmental loading and typically ranges between 0.25 sec. and 1.0 sec. Assuming that maximum load corresponds to the yield strength of the reinforcing bar, this implies a maximum strain rate of 0.03 per second. For the case of yielded reinforcing steel, it is reasonable to assume that the maximum load corresponds to the steel developing ultimate strength at a strain demand of approximately 0.15. Further, it may be assumed that this load is applied during an interval equal to $0.25T_{eff}$. Here T_{eff} defines the effective period of the bridge with inelastic deformation and ranges between 0.5 sec. and 2.0 sec., assuming an average ductility demand of 4. On the basis of these assumptions it follows that the maximum strain rate for reinforcing steel is 1.2 per second.

In order to improved the accuracy of experimental results, laboratory testing of reinforcing steel typically is carried out using coupons that have been machined to a uniform diameter over a given range and then notched to localize the failure mechanism. This facilitates measurement of steel area. Ma *et al.* [1976] tested both unmachined and machined bars and noted that machined specimens had slightly higher yield strength and slightly higher initial hardening modulus. However, strength variations between the machined and unmachined bars were less than 5 percent. Thus, experimental testing of either unmachined or machined reinforcing bars is appropriate; however, accurate measurement of initial steel area is necessary.

3.2.2 Behavior of Reinforcing Steel Subject to Axial Loading

Experimental data define the response of reinforcing steel subjected to uniaxial, pseudo-static monotonic tensile loading, cyclic tensile loading and reversed cyclic loading This information is extended by monotonic and cyclic testing of reinforcement at variable load rates.

Figure 3.2 shows a typical engineering stress-strain history for reinforcing steel subjected to monotonically increasing strain demand. Important characteristics of this response include the following:

1. Initial response is linear-elastic for stress demand less than the initial yield strength.
2. For strain demand exceeding that corresponding to the initial yield strength, there is a slight drop in strength below the initial yield strength. Strength is maintained at this lower yield strength for moderate increase in strain demand. This range of response is referred to as the yield plateau or the Lüders plateau, and the material yield strength typically is defined to be the average strength for loading within this strain range.
3. Increasing strain demand results in increased strength. This strain-hardening regime is maintained to a peak strength that typically exceeds the yield strength by

thirty to sixty percent (Figure 3.3). The ratio of peak strength to nominal strength is a function of the steel specification, grade and batch composition.

4. At severe tensile strain demand, reinforcement begins to neck and strength is reduced.
5. At a maximum strain demand, the steel reinforcement fractures and load capacity is lost.

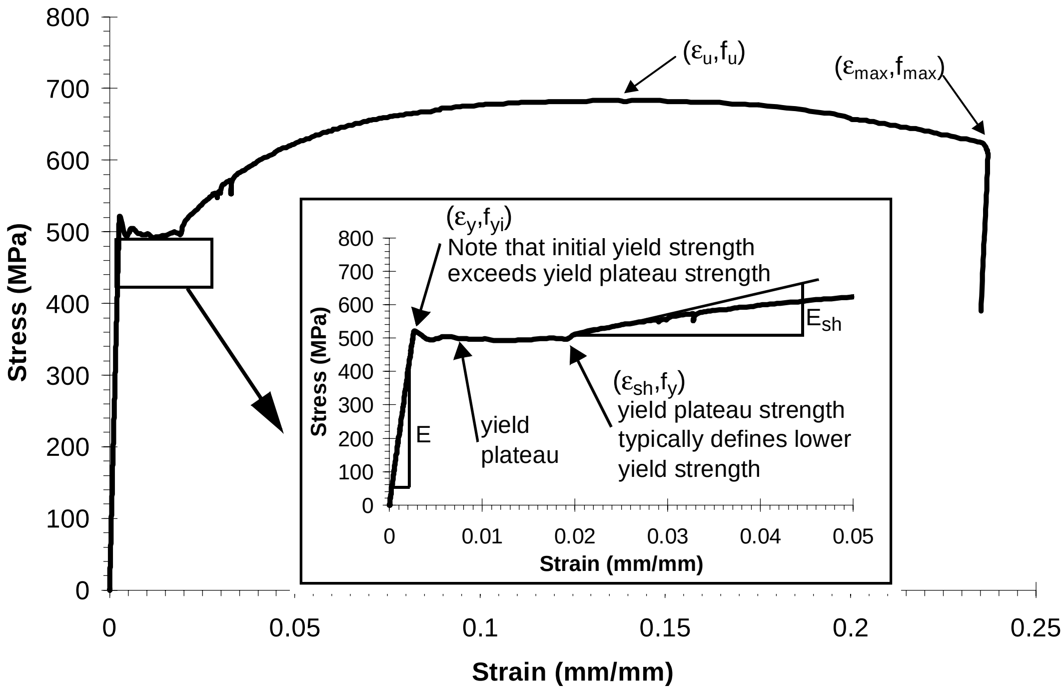

Figure 3.2: Tensile Monotonic Stress-Strain History for Typical Reinforcing Steel Bar (Data for A706 Grade 60 Reinforcement [Naito, 1999])

This monotonic steel response may be defined by a few material parameters as identified in Figure 3.2. These include the elastic modulus, E; the lower yield strength, f_y; the strain at which strain hardening initiates, ε_{sh}; the strain at which peak strength is achieved, ε_u; the peak strength, f_u; the strain at which fracture occurs, ε_{max}, and the capacity prior to bar fracture, f_{max}.

Figures 3.3 presents the behavior of samples of reinforcing steel subjected to monotonically increasing tensile strain demand. Some variation in response results from variability in steel specification and steel grade. However, these data show that two steel batches with the same specification and grade may have significantly different post-yield behavior (response of #4 and #6 A706 reinforcement as shown in Figure 3.3).

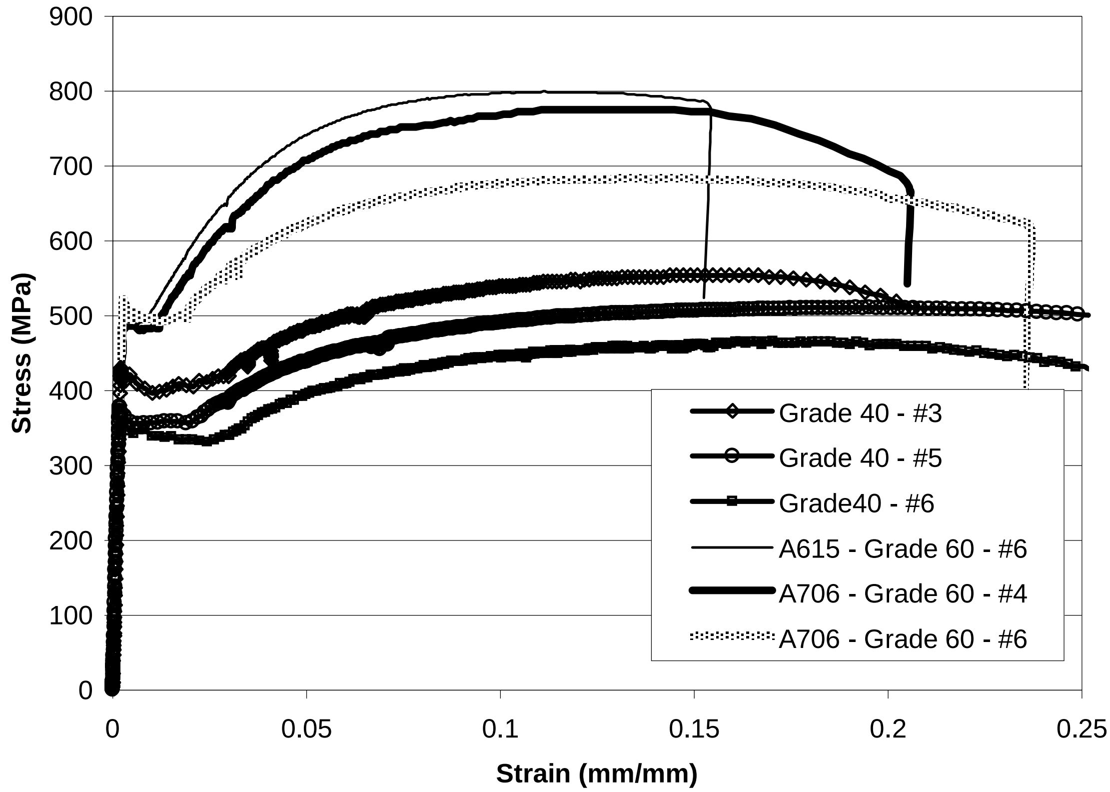

Figure 3.3: Stress-Strain Histories for Specifications and Grades of Reinforcing Steel Typically Used in the United States (Data from Lowes and Moehle, 1995; Mazzoni,

Figures 3.4, 3.5 and 3.6 present data for steel reinforcement subjected to reversed cyclic loading. The strain histories used in these material tests are representative of the strain history observed in actual structures. The extent of compressive stress/strain demand is a function of the load history. Figure 3.4 provides data for a system with limited compressive stress demand while Figure 3.5 provides data for steel subjected to moderate compressive strain demand. Figure 3.6 shows data for reinforcing steel subjected to a reversed cyclic strain demand with symmetric inelastic strain increments. While this is not representative of strain histories in actual systems, these data exhibit particular characteristics of response that are not obviously revealed by the previous data sets. Important characteristics of response shown in Figures 3.4, 3.5 and 3.6 include the following.

1. Upon unloading, steel exhibits a loss of linearity prior to achieving the yield strength in the opposite direction. This loss of linearity is referred to as the Bauschinger effect [Bauschinger, 1887]. Researchers have noted that this effect becomes more pronounced with increased strain demand [*e.g.*, Ma, 1976].
2. The initial tangent to the unloading stress-strain response is slightly less than the initial elastic stiffness. This characteristic has been observed by a number of researchers [*e.g.*, Bauschinger, 1887; Panthaki, 1991]
3. Reinforcing steel exhibits isotropic strain hardening, characterized by increasing strength under increasing inelastic strain demand. This is observed under cyclic as well as monotonic loading (Figures 3.3 through 3.6).
4. Reinforcing steel exhibits cyclic strain softening, defined by Ma *et al.* [1976] as reduced tangential stiffness under multiple cycles to particular strain limits (Figures 3.4, through 3.6). This loss of strength under cyclic loading to a prescribed strain limit is particularly evident in Figure 3.6.

These characteristics will be considered in development of a material model for this investigation.

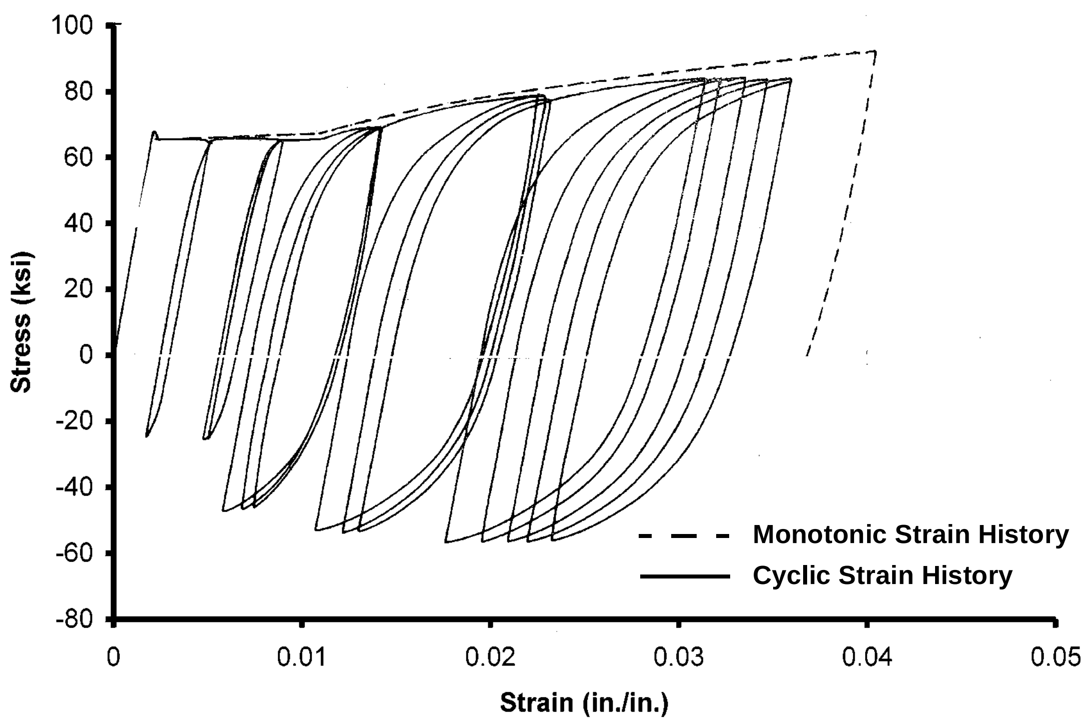

Figure 3.4: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Moderate Compressive Stress Demand (Experimental Data from Ma *et al.*, 1976)

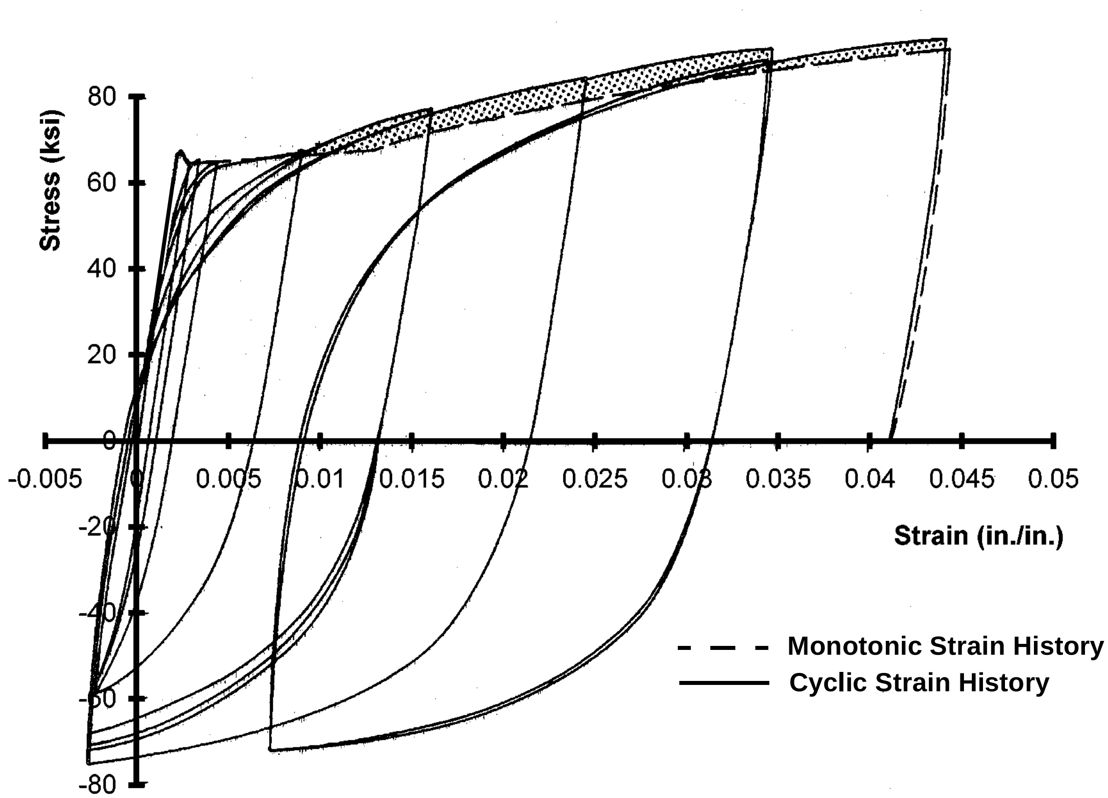

Figure 3.5: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Moderate Compressive Strain Demand (Experimental Data from Ma *et al.*, 1976)

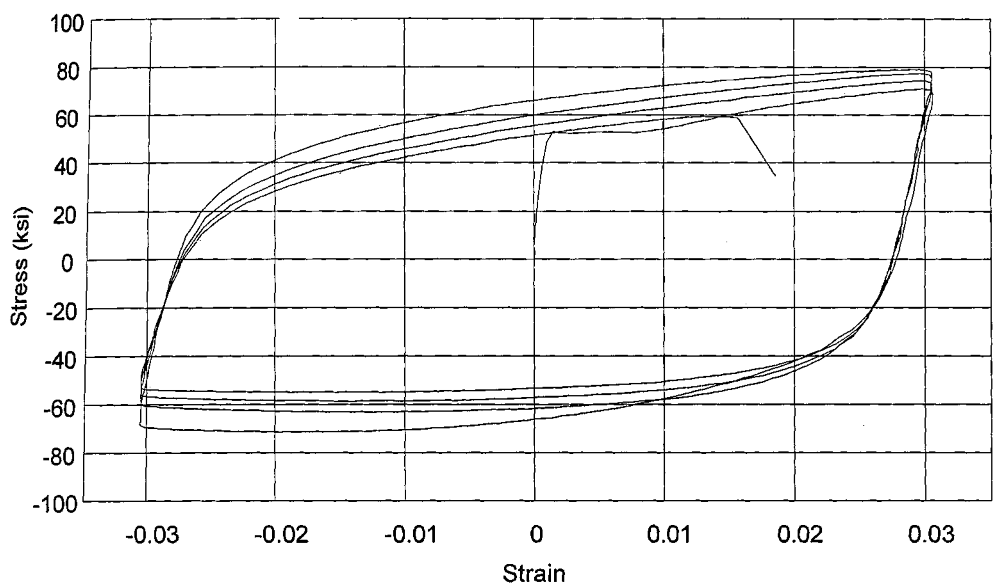

Figure 3.6: Response of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Compressive and Tensile Strain Demands (Experimental Data from Panthaki [1991] as presented by Chang and Mander [1994])

3.2.3 Effect of Load Rate

Experimental data collected during previous investigations define the effect of increased strain rate on steel material response. These data may be used to adjust steel strength for loading in the range of strain rates developed in structures subjected to earthquake loading. For reinforcing steel, maximum strain rates developed under earthquake loading are approximately 0.3 per second for elastic steel approaching yield strength and 1.2 per second for yielding steel developing ultimate strength. Figure 3.7 shows the ratio of dynamic to static tensile yield strength as a function of strain rate. These data are for "mild structural steel" [Manjoine, 1944] and A36 steel [Chang and Lee, 1987] as well as steel grades typically used for reinforcing steel [Cowell, 1965] and A615 steel [Mahin *et al.*, 1972]. These data show that for typical reinforcing steel, dynamic yield strength is approximately 10 percent larger than the static yield strength within the range of load rate that is appropriate for structures subjected to earthquake loading. Additionally, these data show that the increase in yield strength observed under dynamic loading is less significant for higher grade steels. Investigations conducted by Mahin *et al.* [1972] show that increased strain rate has a negligible effect for steel loaded beyond the initiation of strain hardening and for steel subjected to reversed cyclic loading. Similarly, data collected by Manjoine [1944] show a maximum increase in ultimate strength of mild structural steel of approximately 4 percent within the range of strain rates appropriate for this investigation. Results of several investigations indicate that increased strain rate has relatively little effect on elastic modulus [ACI Committee 439, 1969; Mahin *et al.*, 1972; CEB, 1988].

The experimental data show that dynamic loading has a relatively limited effect on the response of reinforcing steel. Increased strain rate will result in increased yield strength; however, the strength increase beyond that observed under pseudo-static load

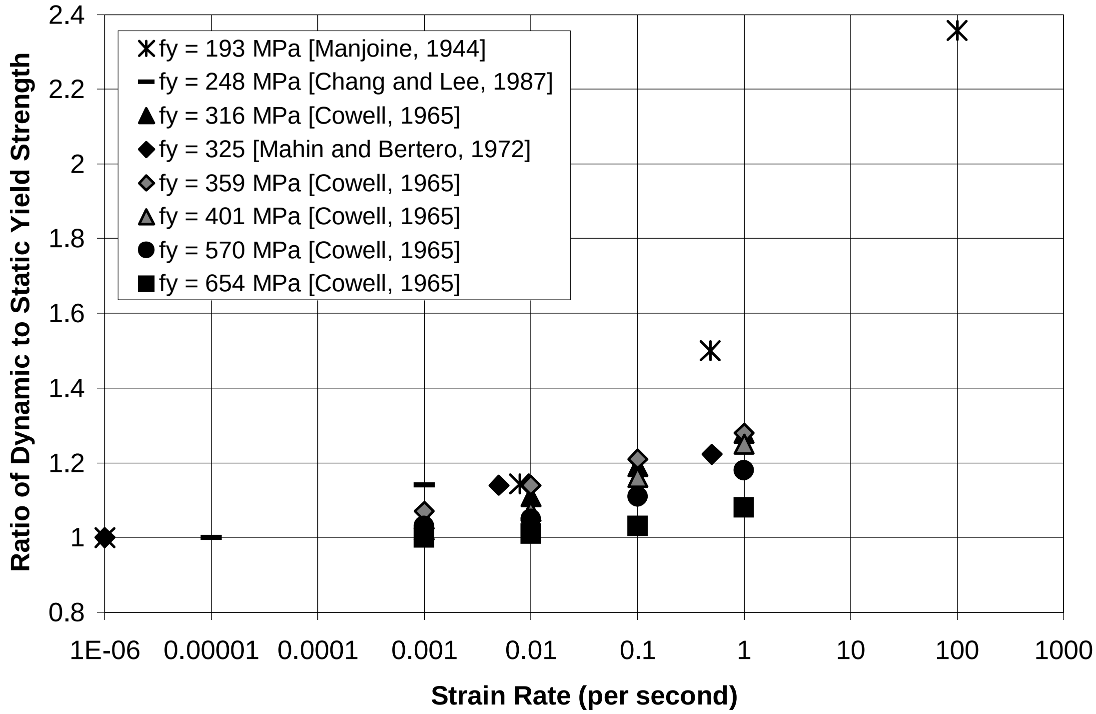

Figure 3.7: Effect of Strain Rate on Steel Yield Strength (Data from sources cited in the Figure)

conditions is at most ten percent within the range of steel grades and strain rates of interest to this investigation.

3.3 Steel Constitutive Models

An number of researchers have proposed models to characterize the response of reinforcing steel for use in analysis of reinforced concrete structures. Some of these models are developed on the basis of material constitutive theories; however, the majority of these are phenomenological models that characterize the macroscopic response on the basis of experimental data. The fundamental characteristics of the one-dimensional steel response are relatively simple. Thus, the appropriate material model not only predicts this response with a reasonable level of accuracy but is also calibrated to fit experimental data with relative ease and is computationally efficient.

3.3.1 Material Constitutive Theory Applied to Modeling Steel Behavior

Various theories have been proposed to characterize the response of reinforcing steel subjected to reversed cyclic loading on the basis of microscopic material response. A number of these models are identified and discussed by Cofie [1983]. The simplest and most computationally efficient model for predicting steel behavior is that developed on the basis of modern plasticity theory. The one dimensional behavior of reinforcing steel is representative of an *elastic-plastic* material. In particular, results of experimental testing show the accumulation of unrecoverable, plastic deformation and an unloading stiffness that is approximately equal to the initial elastic material stiffness. Additionally, steel exhibits isotropic strain hardening, characterized by increased strength under increased inelastic strain demand. Further, the premature yielding associated with the Bauschinger effect may be characterized by a plasticity model that incorporates kinematic strain hardening.

A one-dimensional constitutive relationship developed on the basis of plasticity theory and incorporating linear isotropic and kinematic strain hardening is defined by the following set of equations:

$$f(\sigma, \alpha_p) = (|\sigma - \beta| - q(\alpha_p)) \tag{3-1a}$$

$$\dot{\varepsilon}_p = \gamma \operatorname{sgn}(\sigma - \beta) \tag{3-1b}$$

$$\dot{\alpha}_p = \gamma \tag{3-1c}$$

$$\dot{\beta} = \gamma H \operatorname{sgn}(\sigma - \beta) \tag{3-1d}$$

$$q(\alpha_p) = \sigma_Y + K\alpha_p \tag{3-1e}$$

$$\gamma \geq 0 \qquad \dot{f} \leq 0 \qquad \gamma\dot{f} = 0 \tag{3-1f}$$

where all plasticity variables follow the previous definitions (Section 2.3.2), σ_Y is the yield strength, K and H, respectively, are the isotropic and kinematic hardening parameters and the function *sgn(.)* is defined to be 1.0 for a positive operand and -1.0 for a negative operand.

Integration of these equations and implementation in an incremental solution algorithm results in an explicit solution algorithm. Additionally, explicit equations govern the determination of the material parameters from experimental data. Figure 3.8 shows a typical stress-strain history as predicted by Equations (3-1a) through (3-1f). While this model can be calibrated to predict steel strength in the vicinity of cyclic peak strain demands, the model does not represent well the observed steel response along the path between points of peak strain demand (Figure 3.4). However, for some applications, the inaccuracy of this model is acceptable given the simplicity of the formulation and the ease with which it may be calibrated to best fit observed response.

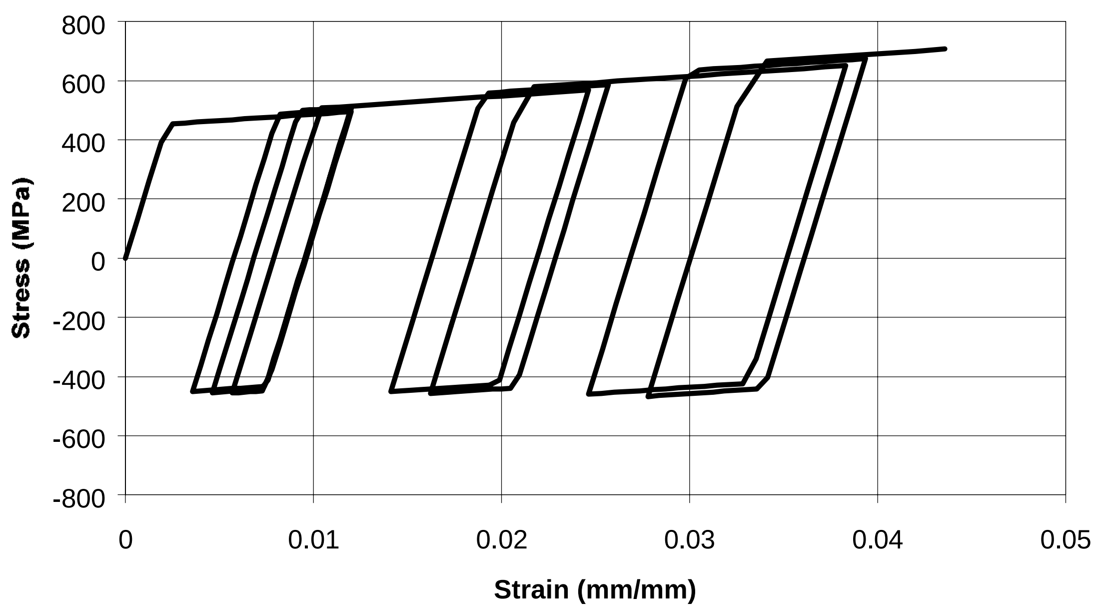

Figure 3.8: Steel Stress-Strain History as Predicted on the Basis of Plasticity Theory

The accuracy with which the plasticity model predicts the observed response may be improved through use of a more sophisticated hardening rule and additional hardening variables. However, this may not be the most efficient model for characterizing steel response.

3.3.2 Phenomenological Model

A more representative model for the response of reinforcing steel subjected to reversed cyclic loading can be achieved through the use of phenomenological models in which non-linear equations are calibrated on the basis of experimental data. One of the first models of this type is that proposed by Ramberg and Osgood [1943]. Various other models have followed. Recently, a number of models have been developed on the basis of work done by Menegotto and Pinto [1973]. Also recently, several models have been developed that characterize behavior of the basis of the natural steel stress and strain rather than the engineering stresses and strains.

3.3.2.1 Characterization of Steel Response Using the Ramberg-Osgood Equation.

The model proposed by Ramberg and Osgood [1943] uses a single non-linear equation to characterize the observed curvilinear response of reinforcing steel subjected to monotonic loading. This model defines the normalized strain to be a function of the normalized stress (stress and strain increments are normalized with respect to twice the yield value):

$$\varepsilon_{norm} = \beta\sigma_{norm}(1 + \alpha|\sigma_{norm}|^{n-1}) \tag{3-2}$$

The model may be extended for the case of reversed-cyclic loading by introduction into Equation (3-2) of the stress and strain at which load reversal occurs. While this model has been shown to predict the one-dimensional steel response with acceptable accuracy, the explicit dependence on the stress reduces the ease with which this model is implemented in a typical strain-driven finite element analysis program. Additionally, using the model to represent monotonic response does not provide for description of the yield plateau, a characteristic of response that may control system behavior. These issues are addressed by more recent models in which stress is defined to be an explicit function of the strain and the monotonic response is characterized more accurately.

3.3.2.2 Characterization of Steel Response Using the Menegotto-Pinto Equation

Menegotto and Pinto [1973] propose a model for characterizing reinforcing steel in which the response is defined by the following non-linear equation:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{1/R}} \tag{3-3}$$

where the effective strain and stress (ε^*, σ^*) are a function of the unload/reload interval, b is the ratio of the initial to final tangent stiffnesses and R is a parameter that defines the shape of the unloading curve. In this implementation, it is assumed that the reference curves (stress-strain curves that bound the cyclic response) as well as unloading and reloading response may be characterized by Equation (3.3). This implementation also neglects characterization of the yield plateau. In recent years, a number of researchers have proposed material models that use the Menegotto-Pinto equation to characterize the unloading-reloading response of reinforcing steel. One such model is that proposed by Stanton and McNiven [1979]. This model uses an approximate version of the Menegotto-Pinto equation (Equation 3-3) to improve computational efficiency and assumes that the reference curves for steel subjected to cyclic loading follows the monotonic envelope. A second model is that proposed by Filippou *et al.* [1983]. This model incorporates Equation (3-3) exactly to describe unloading response. The model follows from the assumption that the reference curve defining the cyclic stress-strain response is tri-linear. Isotropic, cyclic strain hardening is incorporated through shifting of the reference curve as a function of the plastic strain increment. Recently, Equation (3-3) has been incorporated into a sophisticated model proposed by Chang and Mander [1994]. This model assumes that the shape of the reference curve is defined by the monotonic stress-strain response. The model accounts for cyclic strain hardening through shifting of the reference curve as a function of strain history. Additionally, the model incorporates variability in initial unloading stiffness, cyclic strain softening and memory of multiple load-unload cycles. Each of these four models predicts with acceptable accuracy the observed cyclic response of reinforcing steel subjected to strain histories typical of those observed in reinforced concrete structures subjected to simulated earthquake loading. Of these models, that proposed by Chang and Mander [1994] provides quite accurate prediction of steel response while that proposed by Filippou *et al.* [1983] provides both reasonably accurate prediction of response and relatively simple implementation and calibration.

All of the previously discussed models assume a symmetric response for loading in compression and tension. However, data suggest that this may be not be an appropriate assumption (Figure 3.9). Data also show that the monotonic response in compression and tension are essentially the same if the response is characterized by the natural strains and stresses ($\bar{\varepsilon}$, $\bar{\sigma}$), defined as follows:

$$\bar{\varepsilon} = \ln(1+\varepsilon) \tag{3-4a}$$

$$\bar{\sigma} = \ln(1+\sigma) \tag{3-4b}$$

Recently two models have been proposed that define material response on the basis of the natural stress-strain history [Dodd and Restrepo-Posada, 1995; Balan *et al.*, 1998]. These models assume a shape for the cyclic reference curve as defined by the monotonic natural stress-strain history. These models predict various aspects of cyclic response including the Bauschinger effect, reduced elastic modulus, isotropic strain hardening, and cyclic strain softening. The models differ in the non-linear equations used to model individual characteristics of material response.

The symmetry of compression and tension response as characterized in the natural stress-strain system is conceptually pleasing; however, it is not obvious that this is necessary for modeling the response of reinforcing steel in reinforced concrete structures subjected to earthquake loading. For this steel, the load history typically is not symmetric

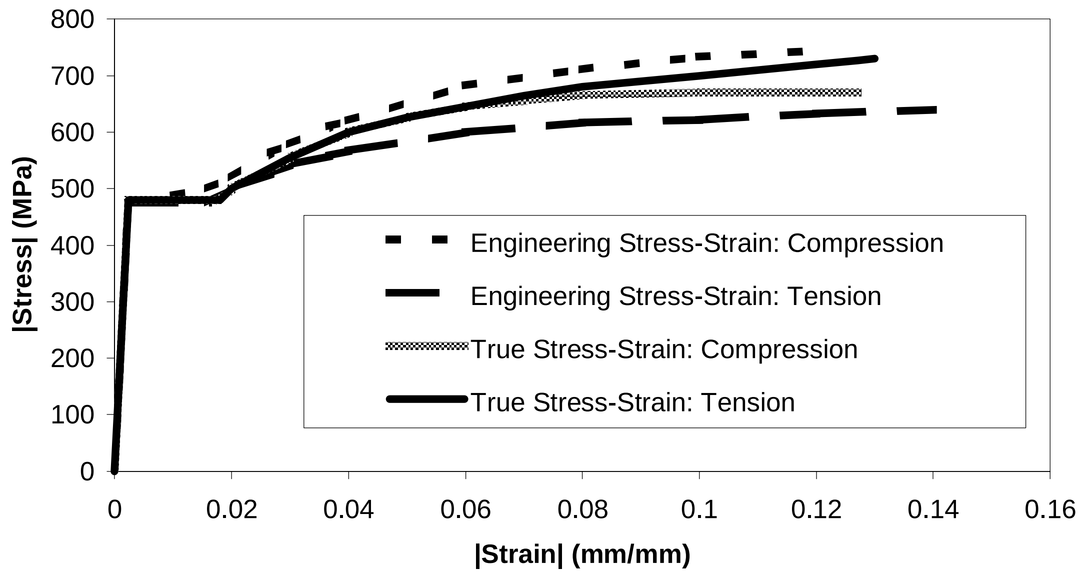

Figure 3.9: Engineering Versus Natural Stress-Strain History for Reinforcing Steel Subjected to Monotonic Compression and Tension Loading (Data from Dodd and Restrepo-Posada [1995])

with load histories showing significant tensile strain demand and limited compressive strain demand. For these cases, characterization of the model on the basis of the tensile monotonic response is perhaps appropriate. Additionally, it is not clear that the enhanced accuracy of these model justifies the additional complexity and computational effort.

3.4 Characterization of the Response of Reinforcing Steel

For this investigation, a material model is developed that defines those fundamental characteristics of steel behavior that control the response of reinforced concrete structures subjected to earthquake loading. This model follows from several previously proposed models and predicts the uniaxial steel material response as defined by the presented experimental data. Consideration of past research indicates that a macroscopic material model is most appropriate for prediction of steel response. Further, the results of past research show that steel behavior may be characterized with acceptable accuracy on the basis of engineering strains and stresses. Here, it is assumed that the observed moderate increase in steel yield strength as a function of increased strain rate may be neglected as this increased strength will only be observed during short periods of rapid loading, will not control development of a specific failure mechanism in the beam-column joint system and will facilitate correlation with observed laboratory tests in which loading is pseudo-static. Strain hardening may determine system strength and necessarily is incorporated into the model. Here strain hardening is characterized through the assumption that plastic deformation in both tension and compression results in nonlinear hardening as defined by the experimentally observed monotonic stress-strain response. Finally, experimental data show that the effect of cyclic strain softening is limited, and this characteristic of response is not incorporated into the material model.

The proposed model defines the response of reinforcing steel subjected to reversed cyclic loading on the basis of three characteristic stress-strain response curves: a reference curve, an unloading curve and a reloading curve. For this investigation, the monotonic stress-strain histories define the model reference curves. Experimental data show that the monotonic tensile stress-strain history bounds the tensile response of reinforcing steel subjected to reversed cyclic strain histories with severe tensile strain demands and moderate compressive demand. For reinforcing steel subjected to severe compressive and tensile strain demands, the monotonic stress-strain histories, shifted to account for accumulated plastic deformation provide a reasonably accurate bound for the observed response. Also for this investigation, unloading and reloading curves are defined using the Menegotto-Pinto equation and calibration parameters provided by previous researchers. Previous research shows that the Menegotto-Pinto equation represents well the unloading and reloading response of reinforcing steel subjected to cyclic loading. Additionally, previous research provides calibration parameters for this equation that are appropriate for modeling grades of reinforcing steel typically used in construction of reinforced concrete structures.

The reference curve is defined on the basis of the monotonic stress-strain history as determined from experimental testing. Here the equation proposed by Chang and Mander [1994] is used to describe the monotonic response in compression or tension:

$$\sigma = \frac{E(\varepsilon - \varepsilon_{shift})}{\left(1 + \left(\frac{E(\varepsilon - \varepsilon_{shift})}{\sigma_y}\right)^{10}\right)^{0.1}} + (\sigma_u - \sigma_y)\left(1 - \left|\frac{\varepsilon_u - (\varepsilon - \varepsilon_{shift})}{\varepsilon_u - \varepsilon_{sh}}\right|^{p}\right) \tag{3-5a}$$

where

$$p = E_{sh}\frac{\varepsilon_u - \varepsilon_{sh}}{\sigma_u - \sigma_y} \tag{3-5b}$$

and σ is the engineering stress, ε is the engineering strain, ε_{shift} is a function of the plastic deformation in either compression or tension and steel material parameters are as previously defined. The model variable, ε_{shift}, is defined, as proposed by Chang and Mander, to be a function of the strain at which strain hardening occurs in compression (or tension) and of the extreme strain experienced in compression (or tension). For the tensile reference curve, this parameter is defined as follows (here material parameters characterize the monotonic compressive response):

$$\varepsilon_{shift}^{tension} = k\left(\varepsilon_{sh} - \frac{f_y}{E}\right) + (1-k)\left(\varepsilon_{min} - \frac{f_{min}}{E}\right) + \varepsilon_{shift}^{compression} \tag{3-6}$$

where k is a weighting parameter defined as follows:

$$k = \exp\left(\frac{\varepsilon_{min}}{5000(\varepsilon_y)^2}\right) \tag{3-7}$$

It is important to note that the model variable ε_{shift} provides a measure of plastic deformation; this variable is analogous, but different in definition, to the plastic deformation defined in classical plasticity theory.

Unloading and reloading curves are defined using the Menegotto-Pinto equation. A curve is defined completely by the strain-stress point at which there is load reversal, $(\varepsilon_o, \sigma_o)$; the target strain-stress point on a reference curve, $(\varepsilon_t, \sigma_t)$, and the tangent to the curve at either end point. The target strain-stress point is taken equal to the extreme strain in the target direction, shifted to account for plastic deformation, and the stress is defined by the reference curve. Here it is assumed that the initial tangent to the curve upon a load reversal is equal to the initial elastic modulus. The model relationship proposed by Chang and Mander is used to predict the tangent to the stress-strain history as the material approaches the reference curve, E_t:

$$E_t = \frac{1}{\frac{1}{E} + \left(\frac{\varepsilon_{ex} - \varepsilon_y}{\varepsilon_{sh} - \varepsilon_y}\right)\left(\frac{1}{E_{sh}} - \frac{1}{E}\right)} \tag{3-8}$$

where ε_{ex} is the extreme strain previously achieved in tension or compression. Thus, for unloading or reloading, stress as a function of strain is defined as follows:

$$\sigma = \sigma_o + E(\varepsilon - \varepsilon_o)\left(b + \frac{(1-b)}{\left(1 + \left|A\frac{\varepsilon - \varepsilon_o}{\varepsilon_t - \varepsilon_o}\right|^R\right)^{1/R}}\right) \tag{3-9}$$

where A and b are model parameters that constitute a modification to the original Menegotto-Pinto equation. Chang and Mander derive the following relationship between the parameters:

$$A = \left(\left(\frac{E(1-b)}{E_{sc} - bE}\right)^R - 1\right)^{1/R} \tag{3-10}$$

where E_{sc} is the secant modulus to the unload-reload curve. The modification of these model parameters from the initial implementation by Menegotto and Pinto ensures the curve passes through both the initial and final stress-strain points and achieves the target slope at the final point. Chang and Mander propose a solution in which these models parameters are derived at each load reversal. However, here the requirement that the final slope be the target slope is relaxed allowing for explicit definition of Equation (3-9) and Equation (3-10). Thus, in this model b is defined as follows:

$$b = \frac{E_t}{E} \tag{3-11}$$

From Equations (3-9), (3-10) and (3-11), it follows that the steel stress and the algorithmic tangent are defined as follows:

$$\sigma = f_o + E(\varepsilon - \varepsilon_o)\left(b + \frac{1-b}{\left(1 + \left|\varepsilon - \varepsilon_o \frac{E - E_t}{E_{sc} - E_t}\right|^R\right)^{1/R}}\right) \tag{3-12a}$$

$$\frac{d\sigma}{d\varepsilon} = Eb + \frac{E(1-b)}{\left(1 + \left|\varepsilon - \varepsilon_o \frac{E - E_t}{E_{sc} - E_t}\right|^R\right)^{(1+1/R)}} \tag{3-12b}$$

3.5 Comparison of Material Model with Experimental Data

The proposed steel material model is implemented in the finite element program FEAP [Taylor, 1998; Zienkiewicz and Taylor, 1987 and 1991]. This implementation is used to analyze the response of a several-element mesh of reinforcing steel subjected to various load histories. The behavior of steel as predicted by the material model is compared with the experimentally observed response for a variety of load histories including monotonic tension, cyclic tensile loading, reversed cyclic loading with moderate compressive stress demand and reversed cyclic loading with severe strain demand in both tension and compression.

Figure 3.10 shows the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile strain demand and moderate compressive stress demand. Differences between the two histories primarily result from differences in the prescribed strain histories. The proposed model represents well the observed response. Figure 3.11 shows the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile and moderate compressive strain demand. The proposed material model represents well the fundamental characteristics of the observed response. However, for this strain history, in which cyclic strain increments are relatively large in both tension and compression, the model under represents the observed strain hardening and does not represent the reload curves accurately. Given that this inaccuracy is relatively small and that this strain history is extreme for an actual system, the errors in model are acceptable. Figure 3.12 presents the behavior of reinforcing steel subjected to reversed cyclic loading with severe tensile and compressive strain demands. Here, the proposed model represents well the fundamental characteristics of the response. However, the model does not represent the loss in strength that is observed following cycling to a fixed strain. Since this loss of strength is minimal and most significant for the case of multiple cycles to a fixed strain in which there is no accumulation of additional plastic strain and thus no additional material strain hardening, this inaccuracy is not significant.

3.6 Conclusions

The material model proposed for use in this investigation represents with acceptable accuracy the behavior of reinforcing steel within the range of loading that is appropriate for reinforced concrete bridges subjected to earthquake loading. The model employes a macromodel framework to describe the engineering stress-strain history of reinforcement subjected to reversed cyclic loading. The model uses nonlinear equations and calibration factors established by others and is readily calibrated to represent the response of typical reinforcing steel on the basis of parameters established through monotonic stress-strain histories. This model is appropriate for predicting the response of typical reinforcing steels subjected to variable reversed cyclic loading including load histories with severe tensile strain demand and moderate to extreme compressive stress and strain demands.

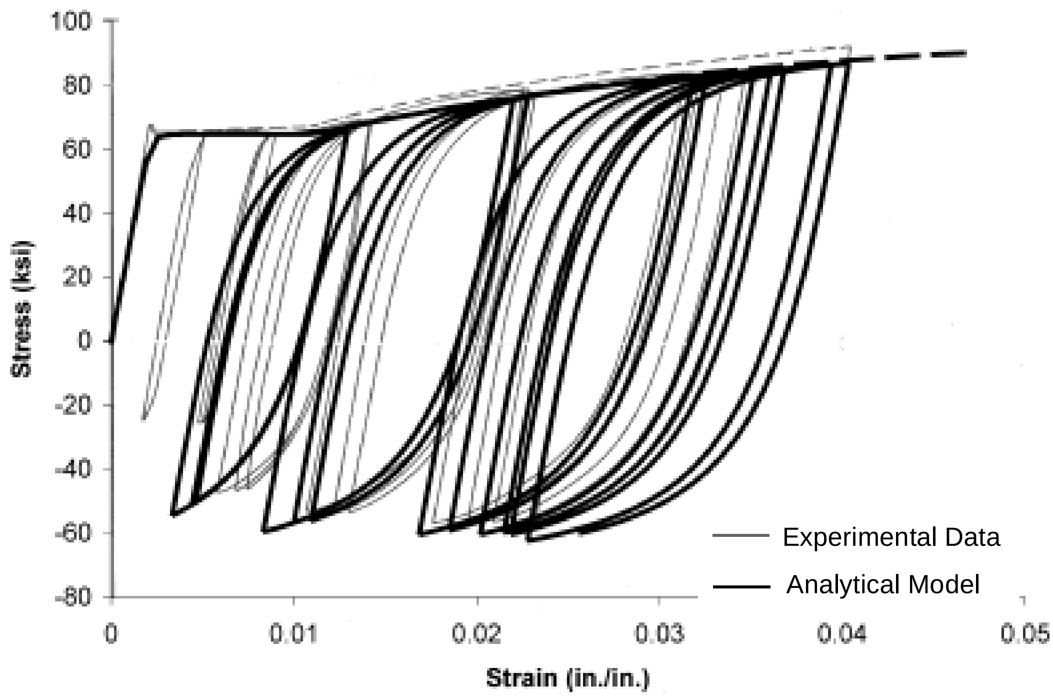

Figure 3.10: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile Strain Demands and Moderate Compressive Stress Demands as Predicted and as Observed (Data as Presented in Figure 3.4)

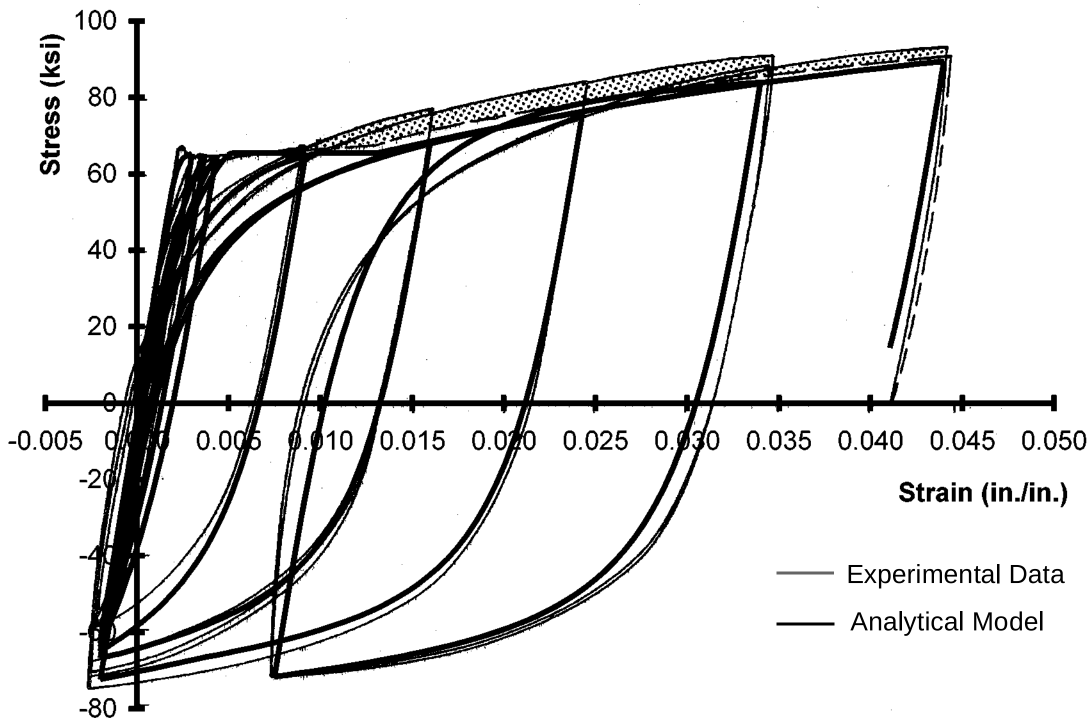

Figure 3.11: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile Strain Demands and Moderate Compressive Strain Demands as Predicted and as Observed (Data as Presented in Figure 3.5)

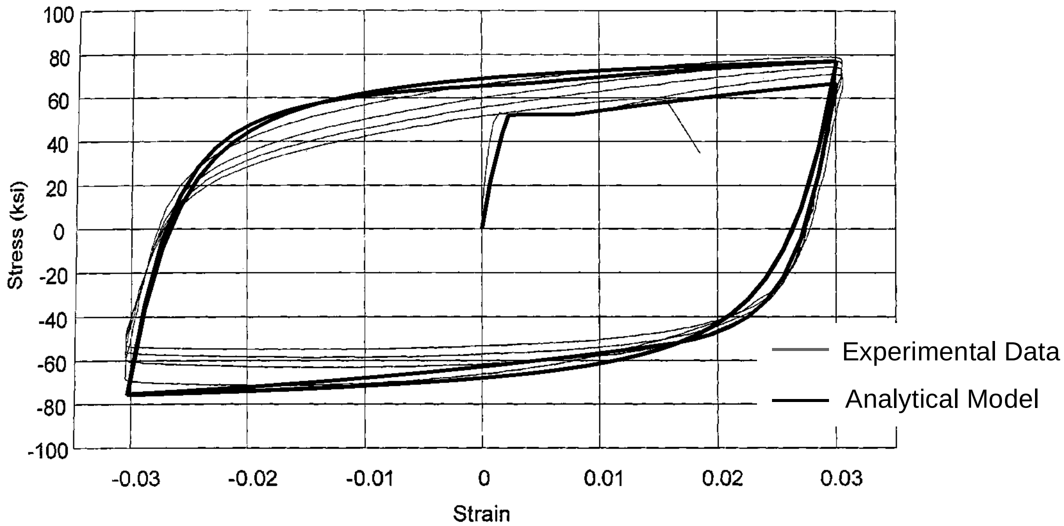

Figure 3.12: Behavior of Reinforcing Steel Subjected to Reversed Cyclic Loading with Severe Tensile and Compressive Strain Demands as Predicted and as Observed (Data as Presented in Figure 3.6)