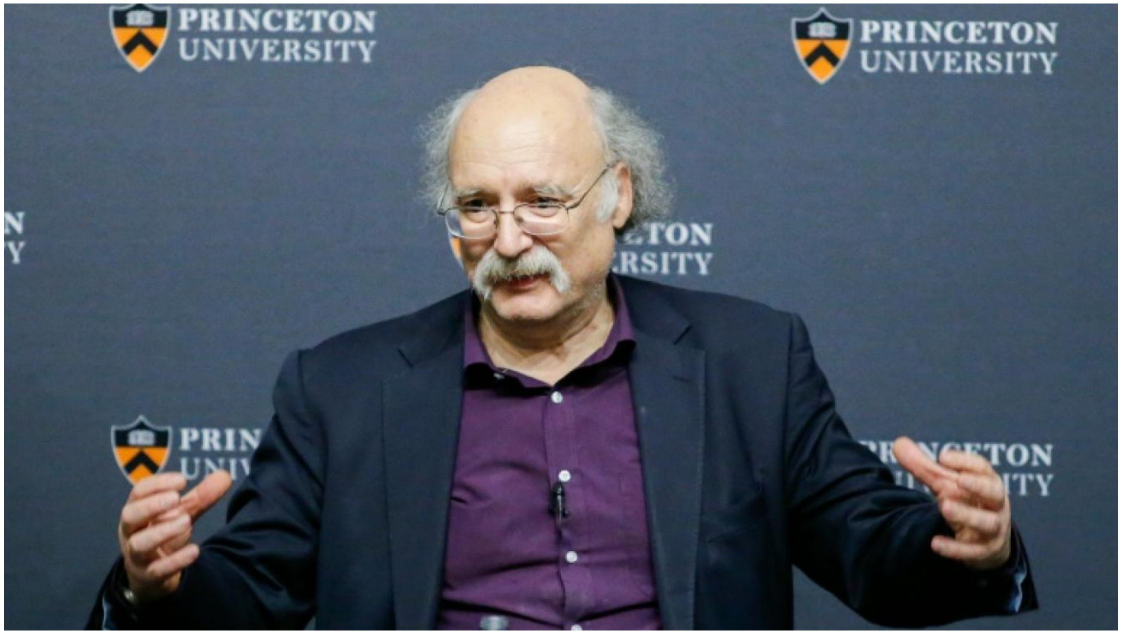
David J. Thouless

Born: 1934, Bearsden,
United Kingdom

University of Washington

Prize motivation:

"for theoretical discoveries of topological phase transitions and topological phases of matter"



F. Duncan M. Haldane

Born: 1951, London,

Princeton University

J. Michael Kosterlitz

Born: 1942, Aberdeen,

Brown University,

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

This year's Laureates opened the door on an unknown world where matter can assume strange states. They have used advanced mathematical methods to study unusual phases, or states, of matter, such as superconductors, superfluids or thin magnetic films. Thanks to their pioneering work, the hunt is now on for new and exotic phases of matter. Many people are hopeful of future applications in both materials science and electronics.

OUTLINE

1. Overview of David Thouless' contributions and the Nobel prize and current impact

2. Classic local order-parameter critical phenomena and topology.

3. Kosterlitz-Thouless Phase Transitions

4. Integer Quantum Hall Effect

5. The TKNN topological invariant

speaker:

Marcel den Nijs, UW

statistical physics, surface science, (non-) equilibrium critical phenomena, quantum phase transitions, neural science (ECOG)

Physics Colloquium, Dept of Physics, UW, October 10 2016


The Nobel Prize in Physics 2016
Topology – what is it?
KUNGL. VETENSKAPS AKADEMIEN
Topologogy is a field of mathematics that describes properties that are stable and only change in integer steps: 1, 2, 3…
The number of holes is a topological invariant that is always an integer, but never anything in between.
n = 2
n = 0
n = 1

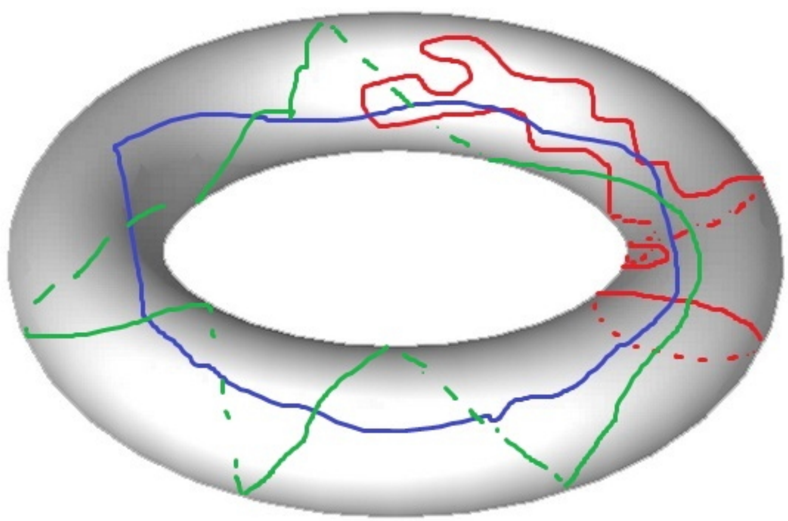
Topology and genus of smooth objects. (from Wikipedia)

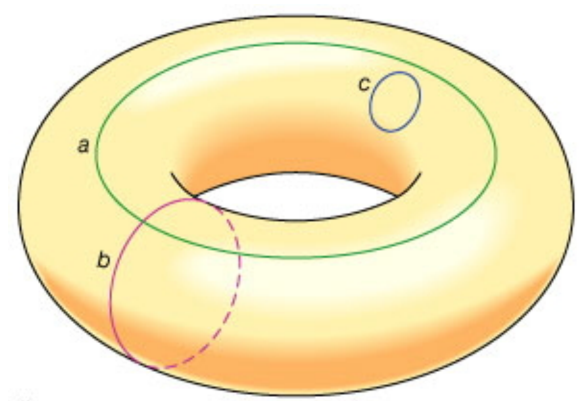
The energy of a cell lipid layered cell membrane depends on curvature, not on surface tension. Its total energy varies (almost) only with the genus. So our cells could rather easily shape shift from “bagels” into “coffee mugs” (by controlling their volume).

Red blood cells are an example of genus-0 shape shifters.

genus 0 genus 1 genus 2

Topology and homotopy of a torus (from Wikipedia)





Closed loops that go “around” the torus can not be removed by deforming them. They are topologically trapped. Such loops have specific (2-component) topological winding numbers.

Local variables can not measure the winding number. (Unless you do something fancy - like coloring-in the areas on both sides of the loops).

The pretzel has 3 holes and in topology is classified as an object of genus-3.

But when you look closer you see many more tiny worm holes.

While at the atomic scale the pretzel consists of ordinary atoms for which the rules of Physics (quantum mechanics mostly) are well understood.

Question: So what is the big deal?

Answer: Most of the microscopic properties and details play no role whatsoever in the processes at the coarse grained macroscopic scale. They are not expressed and become invisible.

Our type of condensed matter physics and statistical physics/field theory concerns the novel phenomena and laws of nature that emerge at the coarse grained scale (smoothened level).

Those new phenomena and laws play a crucial role in emergent technologies involving, e.g., topological insulators, graphene, nanotubes and quantum computation.

This year's Physics Nobel Prize honors the shift in paradigm from atomic type meso-scopic thinking (like in terms of local order parameters) into topological based concepts (vortices, dislocations, topological invariants...)

In particular, it honors the foundations for this paradigm shift laid 30+ years ago.

The prize was probably awarded this year because this type of topological driven thinking is evolving rapidly right now, both theoretically and experimentally, driven by the nano-technology and information revolutions whirling around us.

Two example of this are the current research efforts on topological insulators and quantum computing.

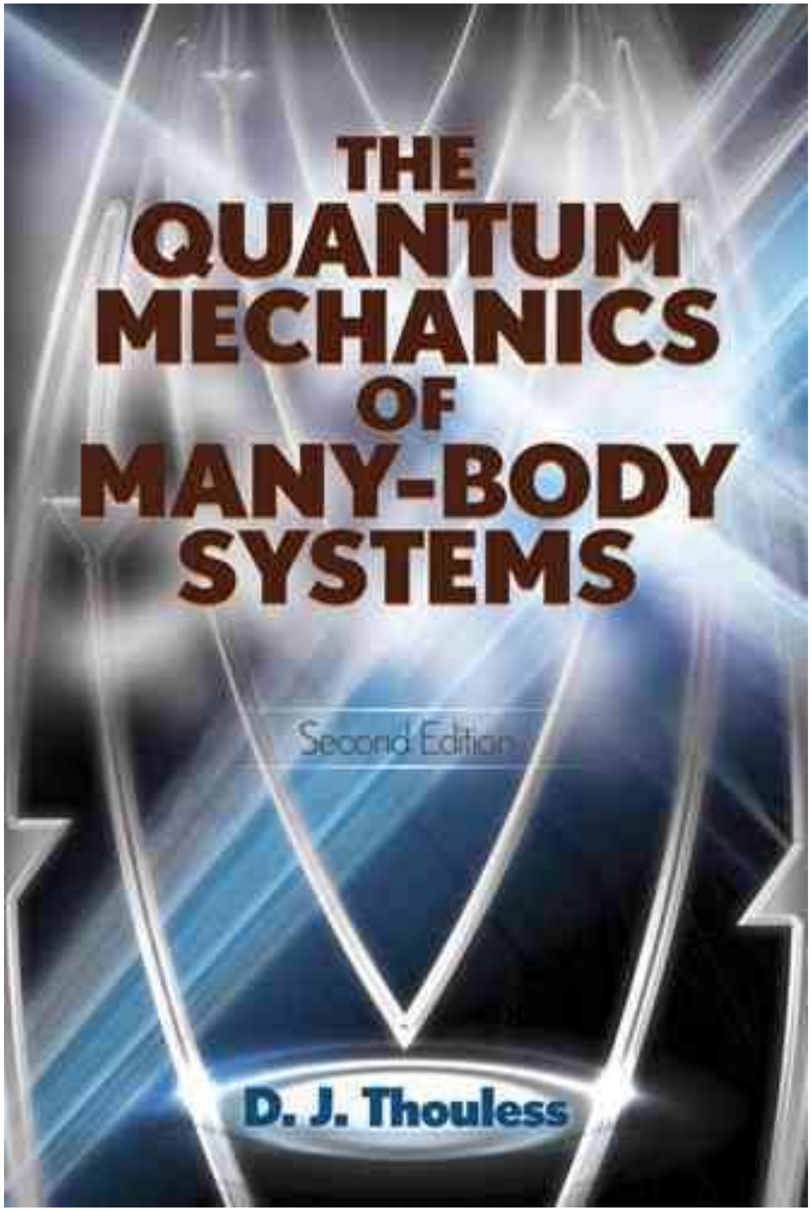
Here at the UW, we are close to being well positioned to play once more a major role. We are in the middle of rejuvenating the condensed matter efforts on the experimental side; with the recent hires of Xiaodong Xu, Paul Wiggins, Kai-Mei Fu, Arka Majundar, Jiun-Haw-Chu, ... All hold joint appointments with other department across campus. It is essential to rebuild the condensed matter theory side as well.

David Thouless' 1/2 part of the 2016 Physics Nobel Prize is based on two seminal contributions:

1. **Kosterlitz-Thouless Phase Transitions** (1973)
   explaining (only one of many more applications) the superfluid phase transition in Helium-4 films on two dimensional surfaces.

2. **The TKNN topological invariant** (1982)
   (**T**houless - **K**ohmoto - **N**ightingale - den **N**ijs)
   explaining the integer quantum Hall effect.

The KT-transition contribution belongs to his Birmingham days.
We discovered the TKNN invariant here at the UW
(in the old Physics building – now Mary Gates Hall).



David is known also for many other important contributions to Physics, e.g., to the theory of localization of many body quantum systems in the presence of disorder; the "Thouless Energy".

Early in his career he wrote a famous book on Many Body Quantum Mechanics, first published in 1966, second edition in 2014. It is even translated in Russian.

David Thouless

Undergraduate, Cambridge, 1955
PhD, Cornell, 1958 (Hans Bethe)

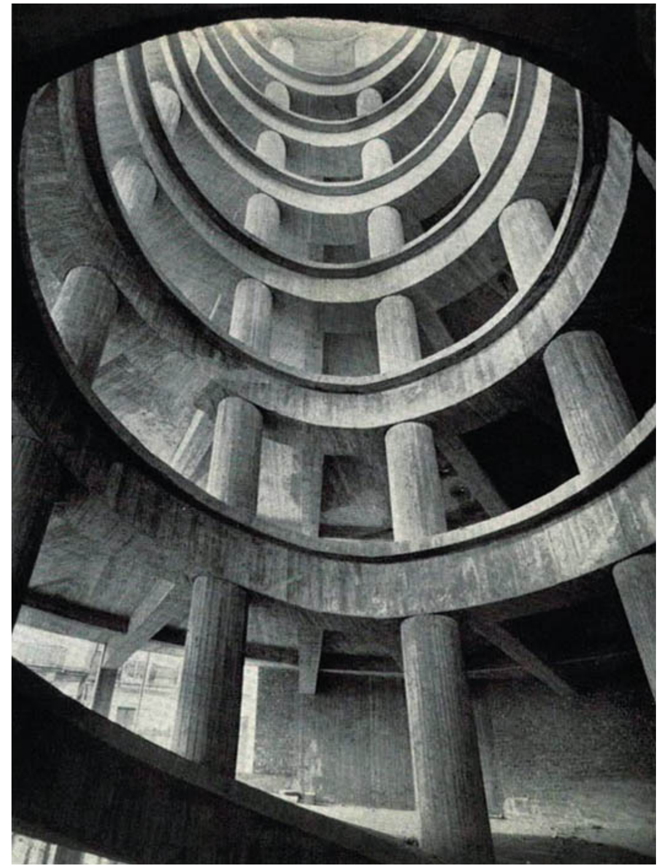
Professor of Mathematical Physics, University of Birmingham, 1965-78.
Professor of Applied Science, Yale University, 1979-80.
Royal Society Research Professor, University of Cambridge, 1983-5.
Professor of Physics, University of Washington, 1980 to 2003; Edwin Uehling Distinguished Scholar, 1988-98; Emeritus Professor 2003-.

David Thouless

Maxwell Prize of the Institute of Physics, 1973.
Fellow of the Royal Society, 1979.
Holweck Prize of the Institute of Physics and Societé Française de Physique, 1980.
Fritz London Award for Low Temperature Physics, 1984.
Fellow of the American Physical Society, 1987.
Wolf Prize for Physics, 1990.
Paul Dirac Medal of the Institute of Physics, 1993.
Member of the National Academy of Sciences, 1995.
Lars Onsager Award for Statistical Physics of the Am. Phys. Soc., 2000.

The topological set-up most relevant for today is that of a spiral.

Imagine a very wide spiral parking garage.

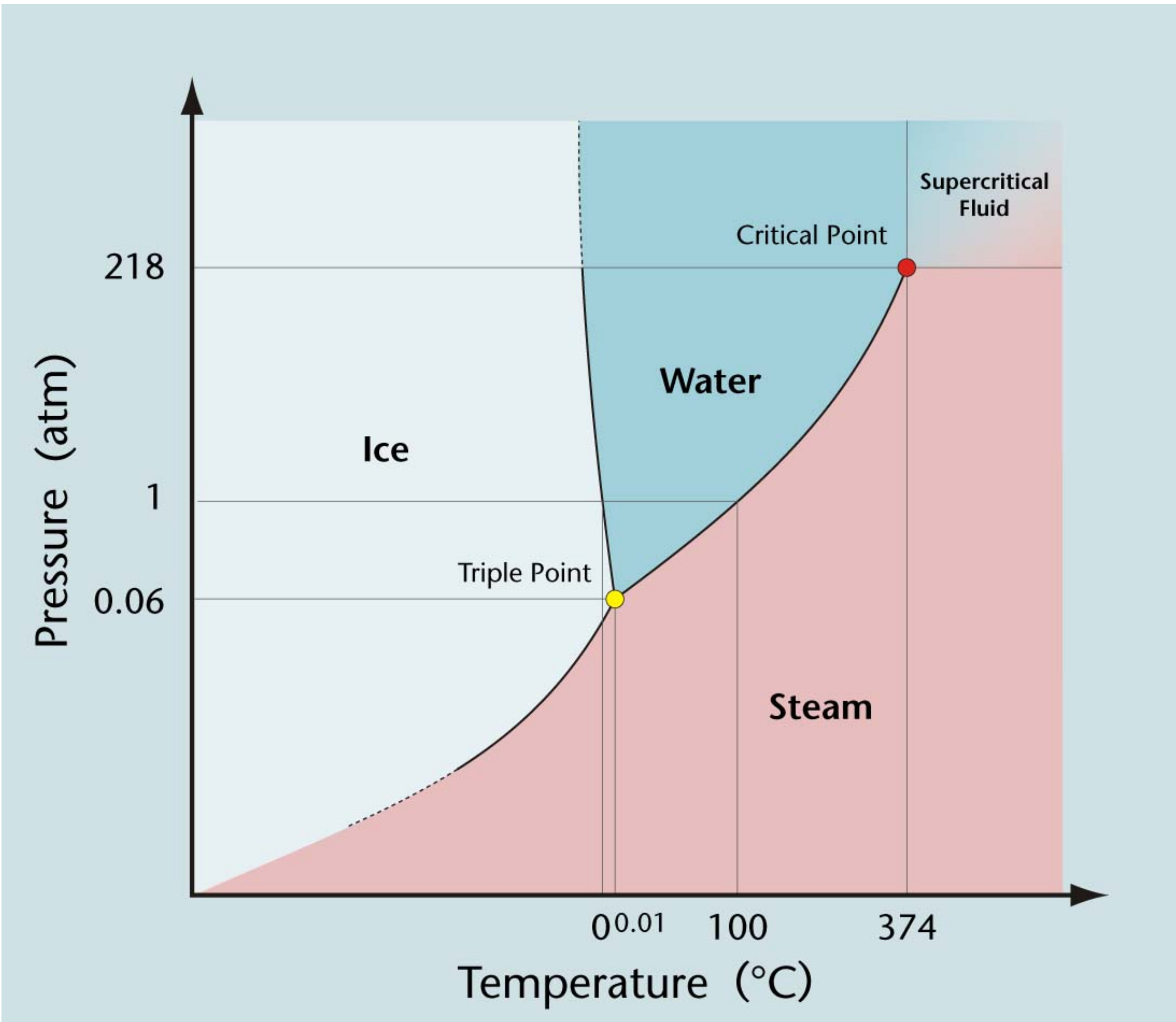


The topological defect sits in the center, but is invisible locally unless you are close to it.

If you are too far from the center you do not even "know/see" you live on a spiral.
That is also why I can not find better pictures.

Every closed contour type walk that does not go around the center leads you back to where you started. Paths that go around the defect make you end-up one floor higher or lower. This height difference is the "winding number."

2. Classic local order-parameter critical phenomena and topology.



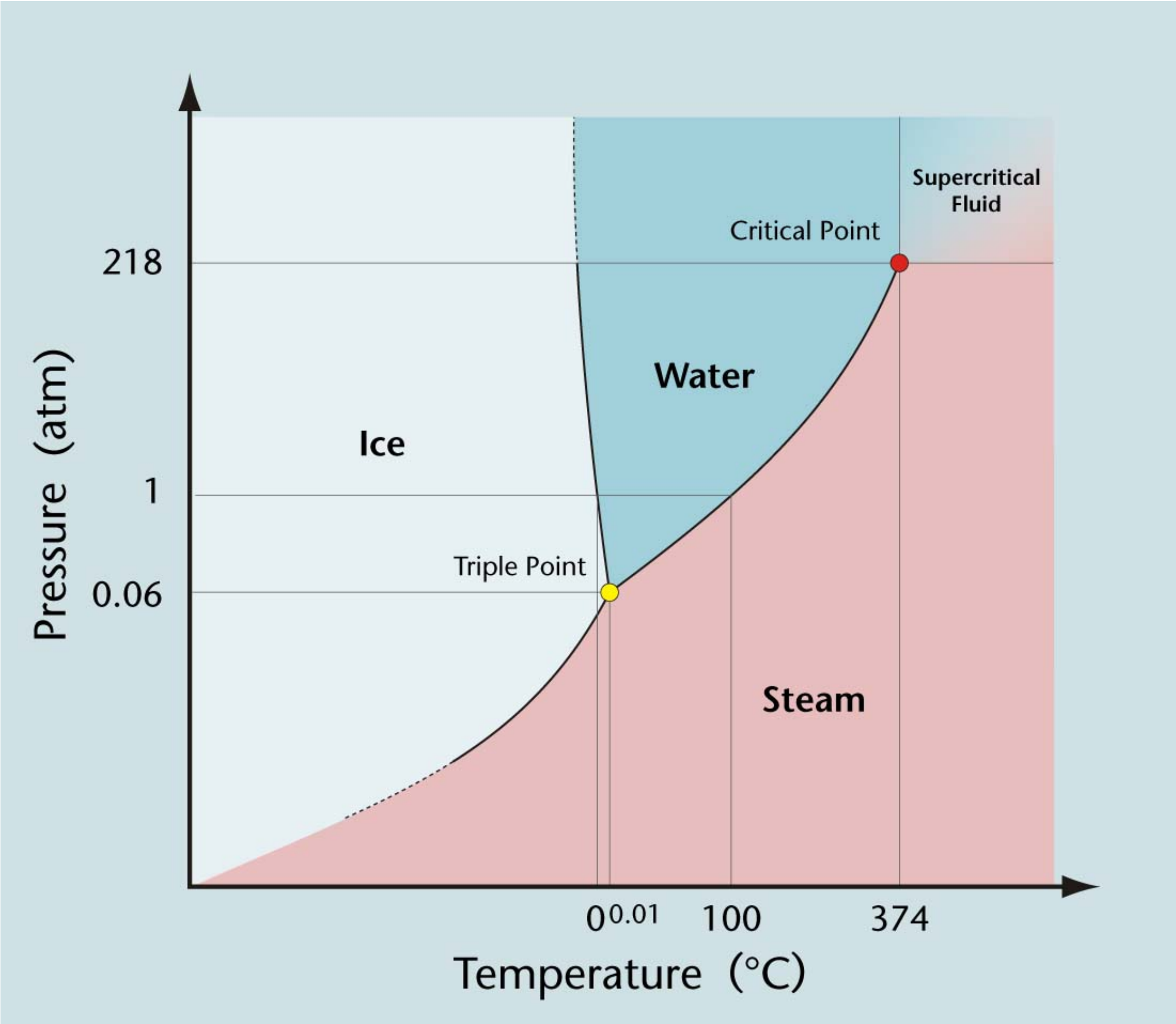
At boiling, water is equally “happy” in the vapor as liquid phase (phase coexistence, Gibbs free energies are equal).

local order parameter:

a local measurement of density tells you in which phase you are.

Understood since 1900 (van der Waals)

2. Classic local order-parameter critical phenomena and topology.

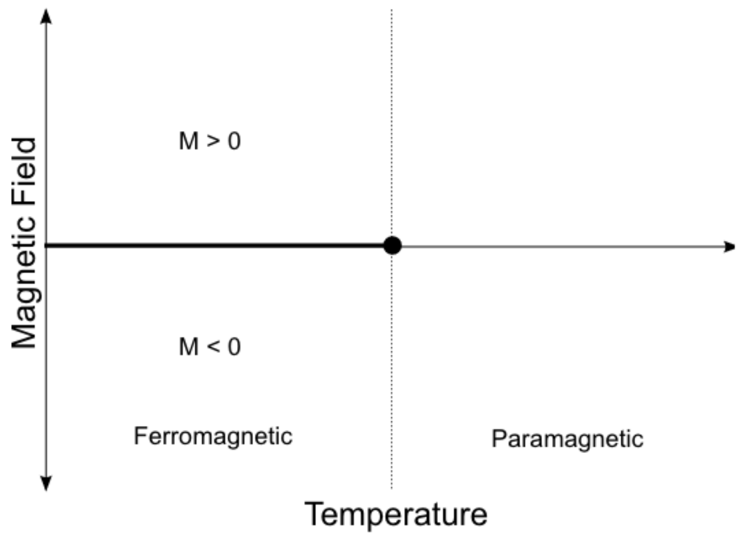


Spontaneous broken ergodicity.

The vapor does not turn into a liquid by random thermal fluctuations.

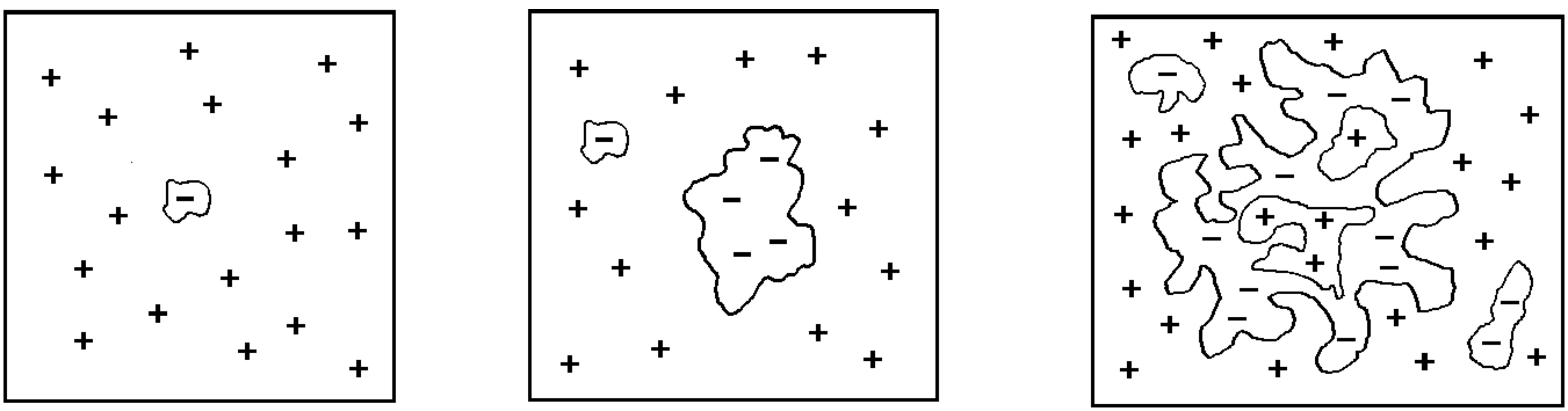
The Free energy barrier scales with system size L as
$\Delta F \sim L^{(D-1)}$
with D the spatial dimension.

Peierls, van Hove, Bloch (1930-ties):
no equilibrium phase transitions in 1D



In an easy axis Ferro magnet (Ising model) the same type of phase transition occurs, but the two phases can then be easily transformed into each other by a global spin flip: the spin–up versus spin-down coexisting phases.

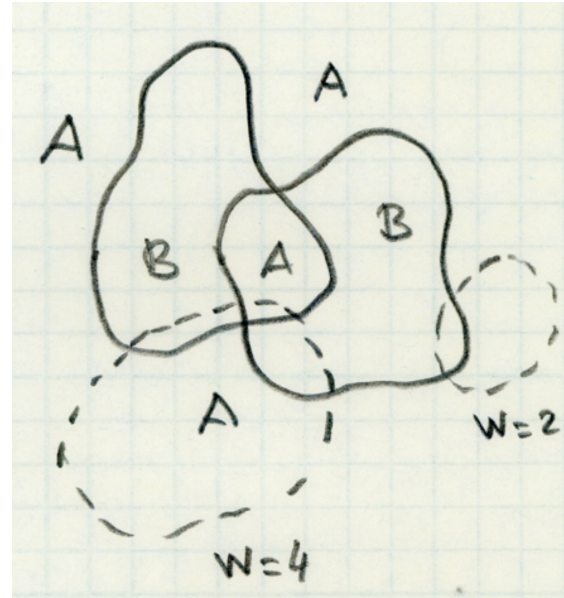
Here the spontaneous magnetization acts as the local order parameter.



$T<<T_c$ $\quad$ $T<T_c$ $\quad$ $T \leq T_c$

In the post Kosterlitz-Thouless era we tend to reformulate this topologically. This started with the next generation – including myself. We focus on the domain walls:

Every closed walk across the surface crosses an even number of domain walls.



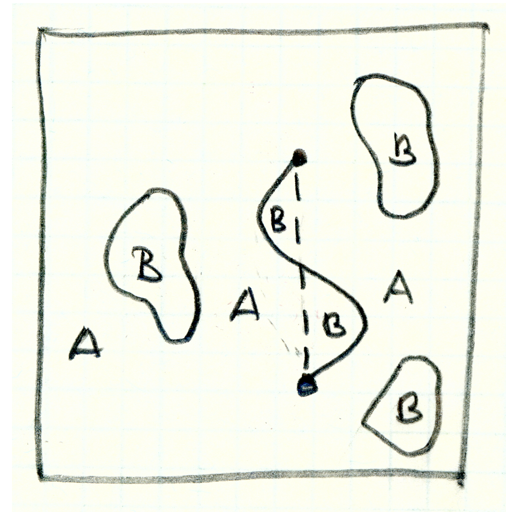
For periodic boundary conditions in one direction:
Every closed path crosses an even number of domain walls

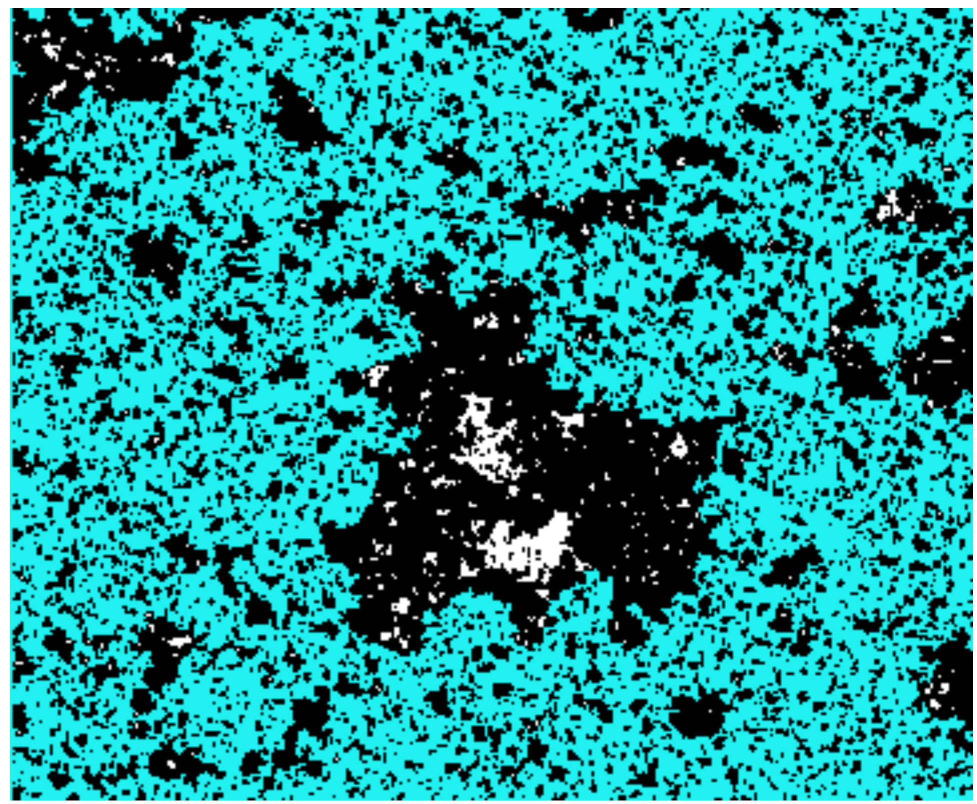
For anti-periodic boundary conditions, the Mobius strip:
Every closed path that winds around the strip an odd (even) number of times, crosses an odd (even) number of domain walls; because when you walk around once you end up at the opposite side of the strip and interpret spin-up as spin-down. A topological defect is imposed by the boundary condition.

Iimpose an open domain wall into the system at a distance R apart (which requires a branch cut). The free energy of this topological dislocation like defect scales as:

- **below Tc**: linear with distance R and proportional to the surface tension (confinement, dislocation pairing).
- **at Tc**: logarithmically (scale free fractal fluctuations).
- **above Tc**: does not depend on R (de-confinement, free dislocations).

In these Ising universality class phase transitions, the topological charge of the domain walls remains somewhat hidden. (They act similar to being their own “anti-particles”.)

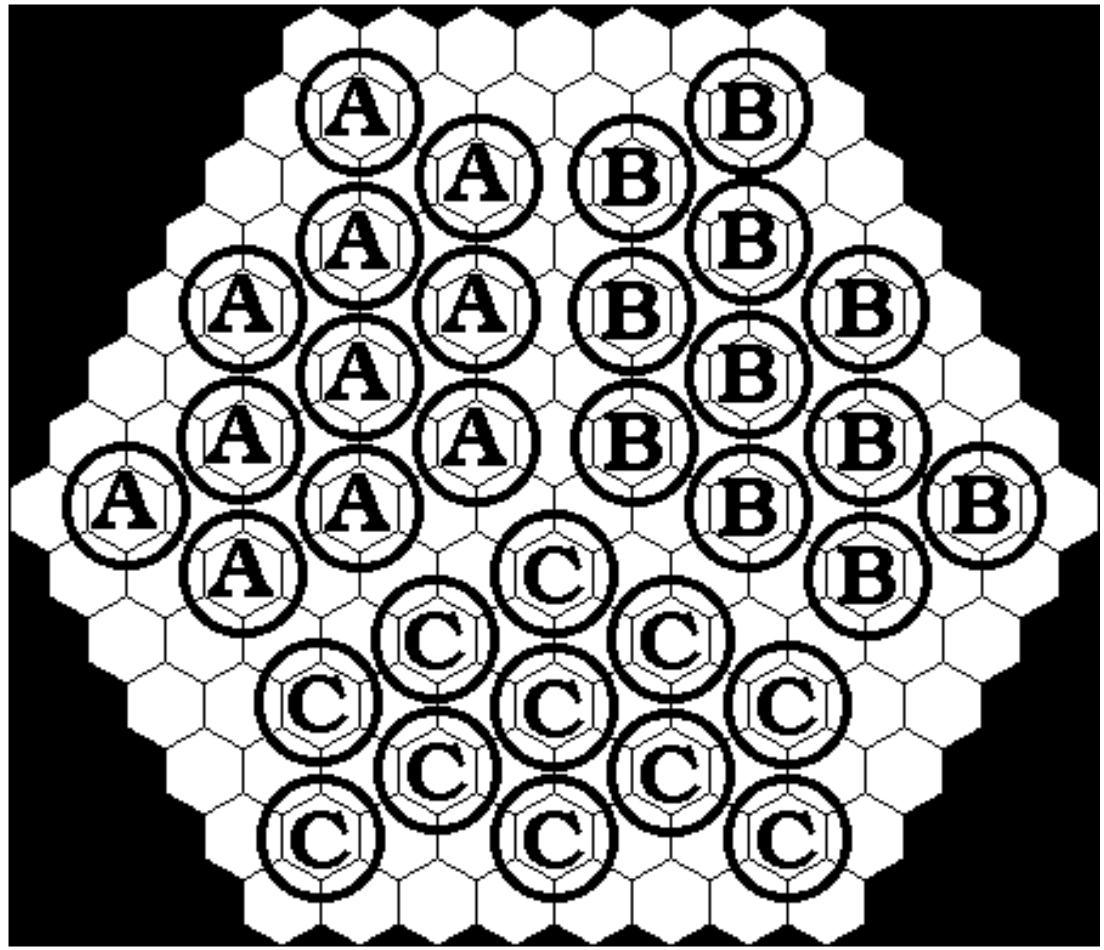


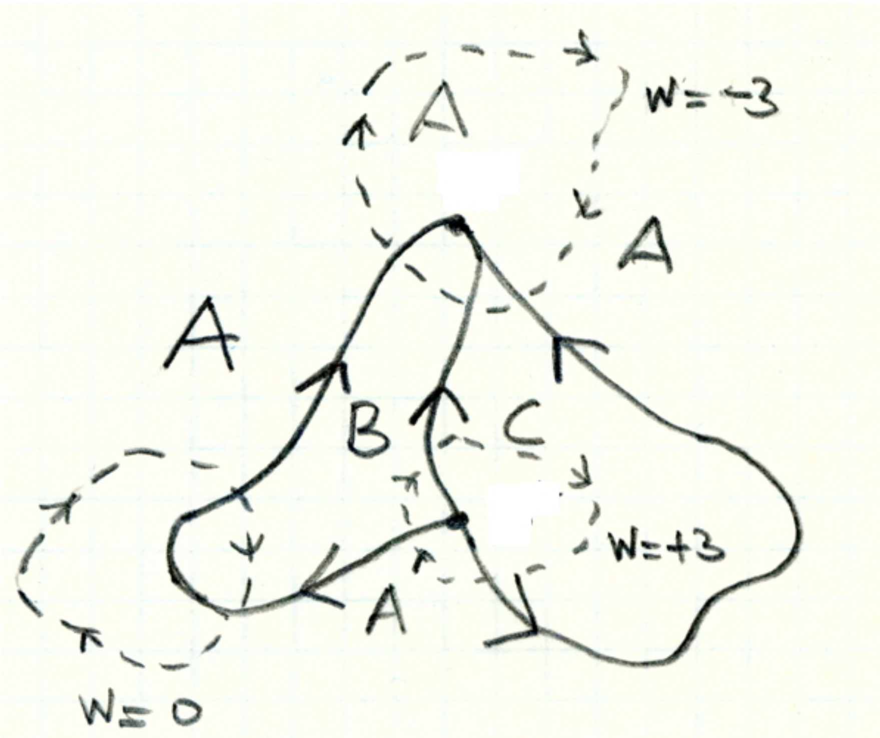


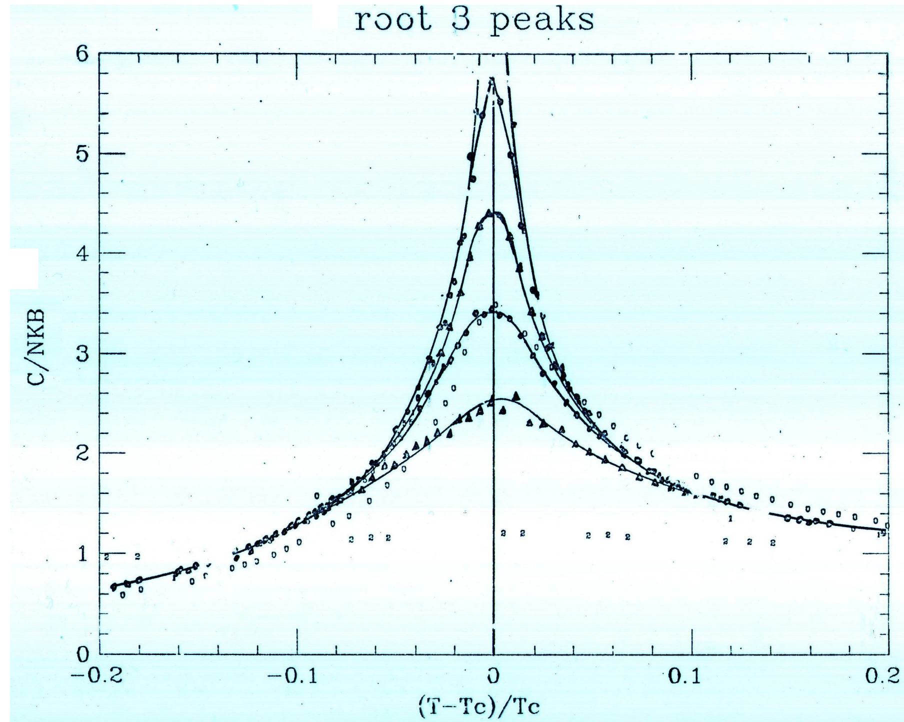
Starting in the mid 1970-ties this department became famous for the experiments by Greg Dash, Oscar Vilches, and Sam Fain, of physisorbed single layers of noble elements like Helium and Krypton on graphite surfaces.

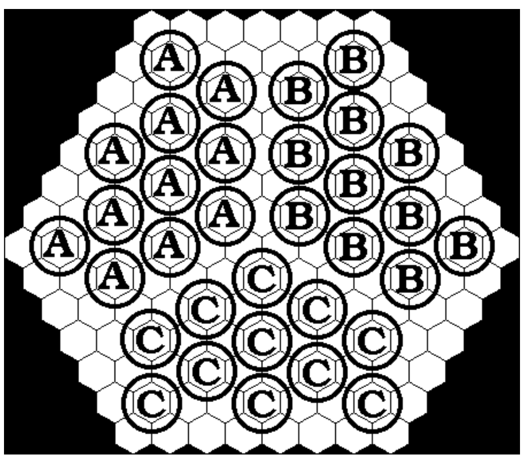
This provides a nice example of an application where the domain walls have a modulo 3 topological chiral charge. (Expressed by directional arrows.)

The adsorbed atoms form three coexisting commensurate phases, where they sit in the A, or the B, or the C positions. (nearest neighbor sites are blocked by size and/or zero point motion).



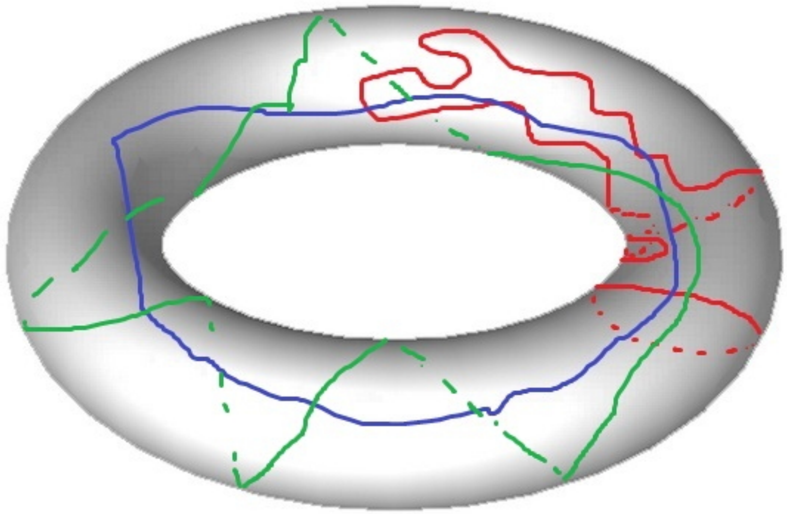


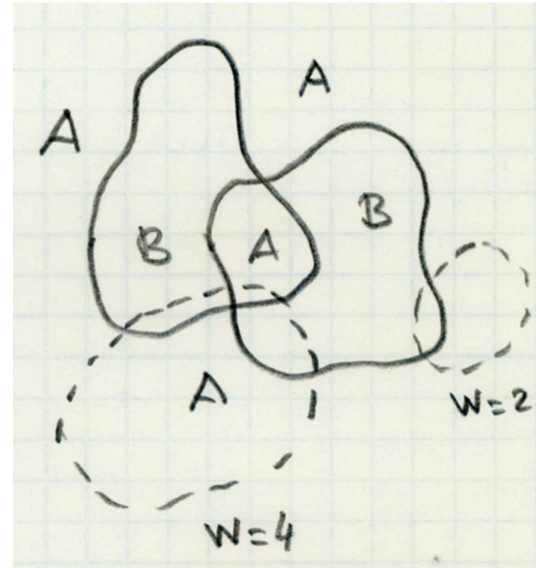




The specific heat divergent at criticality;

experiments: Vilches&Dash starting in the mid 1970-ties.
theory: C~ $|T\text{-}Tc|^{-1/3}$ (den Nijs 1979).





In the post Kosterlitz–Thouless (1970-ties) era, the classification of phase transition in term of the local order parameter is often replaced by listing the topological charges of domain wall excitations, their merging/creation rules, and how they can be topologically trapped when on a torus.

This point of view has proven to be very productive. We often make progress in Physics by reformulating and restating what we know already from a different perspective.

The KT transition research was instrumental for developing this topological perspective of phase transitions.

3. Kosterlitz-Thouless Phase Transitions

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Ordering, metastability and phase transitions in two-dimensional systems

J M Kosterlitz and D J Thouless
Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT, UK

Received 13 November 1972

Abstract. A new definition of order called topological order is proposed for two-dimensional systems in which no long-range order of the conventional type exists. The possibility of a phase transition characterized by a change in the response of the system to an external perturbation is discussed in the context of a mean field type of approximation. The critical behaviour found in this model displays very weak singularities. The application of these ideas to the *xy* model of magnetism, the solid–liquid transition, and the neutral superfluid are discussed. This type of phase transition cannot occur in a superconductor nor in a Heisenberg ferromagnet, for reasons that are given.

CC/NUMBER 44
NOVEMBER 4, 1991

This Week's Citation Classic®

Kosterlitz J M & Thouless D J. Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys.—C—Solid State Phys.* 6:1181-203, 1973.
[Department of Mathematical Physics, University of Birmingham, England]

This paper showed how a type of order more robust than the conventional sort of long-range order can exist in two-dimensional systems. When the temperature is raised, the ordered state is destroyed by the dissociation of pairs of defects such as vortices or dislocations, and a new type of phase transition is produced. [The *SCI*® indicates that this paper has been cited in more than 1,930 publications, making it the most-cited paper in this journal.]

Comments by David himself about his work in 1991, when this paper was cited 1930 times.

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Defect-Driven Phase Transitions

David Thouless
Department of Physics FM-15
University of Washington
Seattle, WA 98195

Arguments against the existence in two-dimensional systems of the type of long-range order associated with superfluidity in helium, freezing of solids, or magnetization of isotropic magnets had been given in the 1930s by R.E. Peierls and L.D. Landau, and had been made rigorous in the 1960s by P.C. Hohenberg, and by N.D. Mermin and H. Wagner. Despite this, there were many theoretical and experimental indications that some sort of ordered state existed at low temperatures for two-dimensional systems such as helium films.

The idea that superfluidity might be destroyed by the spontaneous formation of quantized vortices, or solids melted by the formation of dislocations, is an old one, but it never looked convincing for bulk systems. In two dimensions, the statistical mechanics of such defects is particularly simple, as the position of a defect is given by a point, whereas it is specified by a path in three dimensions. We used the close analogy between the behavior of a collection of defects and that of a two-dimensional gas of positive and negative electric charges. At low temperatures, all the charges are bound in pairs of "molecules," and at some critical temperature, the largest of these molecules dissociated to form a conducting plasma. This is the nature of the phase transition that we argued should exist in superfluid films, in two-dimensional solids, and in two-dimensional magnets with a preferred plane of magnetization.

I stumbled across this idea while preparing a course of lectures on superfluidity and superconductivity, but I was prepared to expect the peculiar nature of the phase transition by work I had done on a one-dimensional problem two years earlier. I recruited J. Mike Kosterlitz, who was then a postdoctoral particle theorist and is now at Brown University, to work on this problem. This paper was the result of our collaboration. Before we had published it, we became aware that Berezinskii,[1] in the Soviet Union, had anticipated some of our ideas, although our use of the renormalization group[2] gave us insights that he missed.

We got many of the interesting features of the transition right, but there were important modifications that were made by other people. It was pointed out that, in addition to the spontaneous formation of dislocations in a solid, associated with the start of viscous flow, there was a further transition of the same sort possible in which disclinations would appear, and orientational order lost. We had argued that this sort of transition should not occur for superconductivity but had forgotten that the penetration depth in a thin film is very large.

The initial reaction of the physics community to this work was polite interest rather than enthusiasm, and it was several years before an experimental test was made. In 1978, D.J. Bishop and J.D. Reppy[3] published the results of their experiment on the superfluid density of a helium film, and correlated their results with other experiments to show the constancy of the ratio of superfluid density to transition temperature; this is a characteristic of the theory which was convincingly demonstrated by D.R. Nelson and Kosterlitz[4] in 1977. The dramatic experimental results started the boom in work on this theory, and on its experimental manifestations. Two recent reviews of the theory and its applications have been written by P. Minnhagen[5] and K.J. Strandburg.[6] The layered copper oxide superconductors have helped to maintain activity in this subject.

1. **Berezinskii V L.** Destruction of long-range order in one-dimensional and two-dimensional systems possessing a continuous symmetry group. II. Quantum systems. *Zh. Eksp. Teor. Fiz. SSSR* 61:1144-56, 1971. (Cited 110 times.)
2. **Kosterlitz J M.** The critical properties of the two-dimensional *xy* model. *J. Phys.—C—Solid State Phys.* 7:1046-60, 1974. (Cited 915 times.)
3. **Bishop D J & Reppy J D.** Study of the superfluid transition in two-dimensional ^{4}He films. *Phys. Rev. Lett.* 40:1527-30, 1978. (Cited 175 times.)
4. **Nelson D R & Kosterlitz J M.** Universal jump in the superfluid density of two-dimensional superfluids. *Phys. Rev. Lett.* 39:1201-5, 1977. (Cited 285 times.)
5. **Minnhagen P.** The two-dimensional Coulomb gas, vortex unbinding and superfluid-superconducting films. *Rev. Mod. Phys.* 59:1001-66, 1987.
6. **Strandburg K J.** Two-dimensional melting. *Rev. Mod. Phys.* 60:161-307, 1988.

Received October 2, 1990

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David Thouless
Department of Physics FM-15
University of Washington
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For continuous type order parameters the free energy barrier for coexisting phases scales as

$$\Delta F \sim L^{(D-2)}$$

implying no phase transitions for those systems in D=1 and D=2

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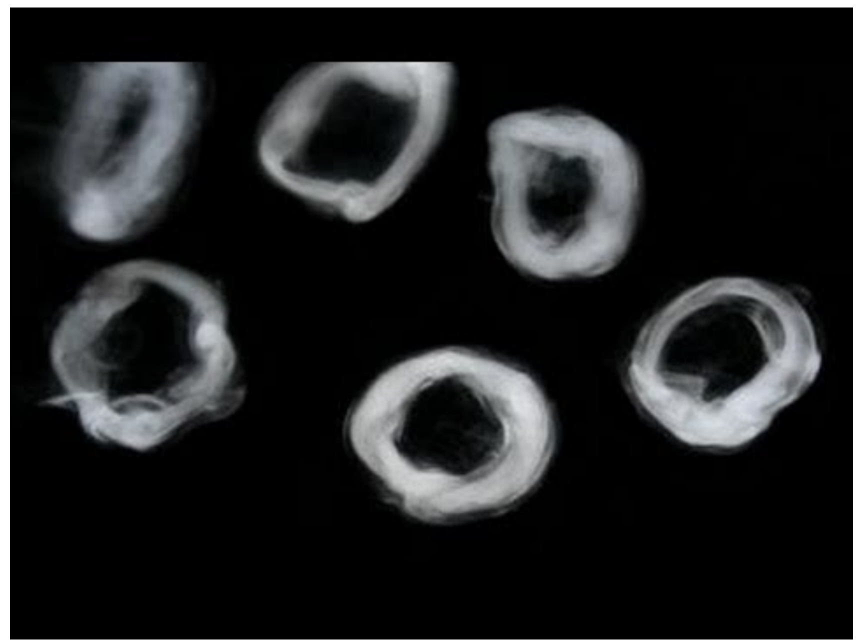
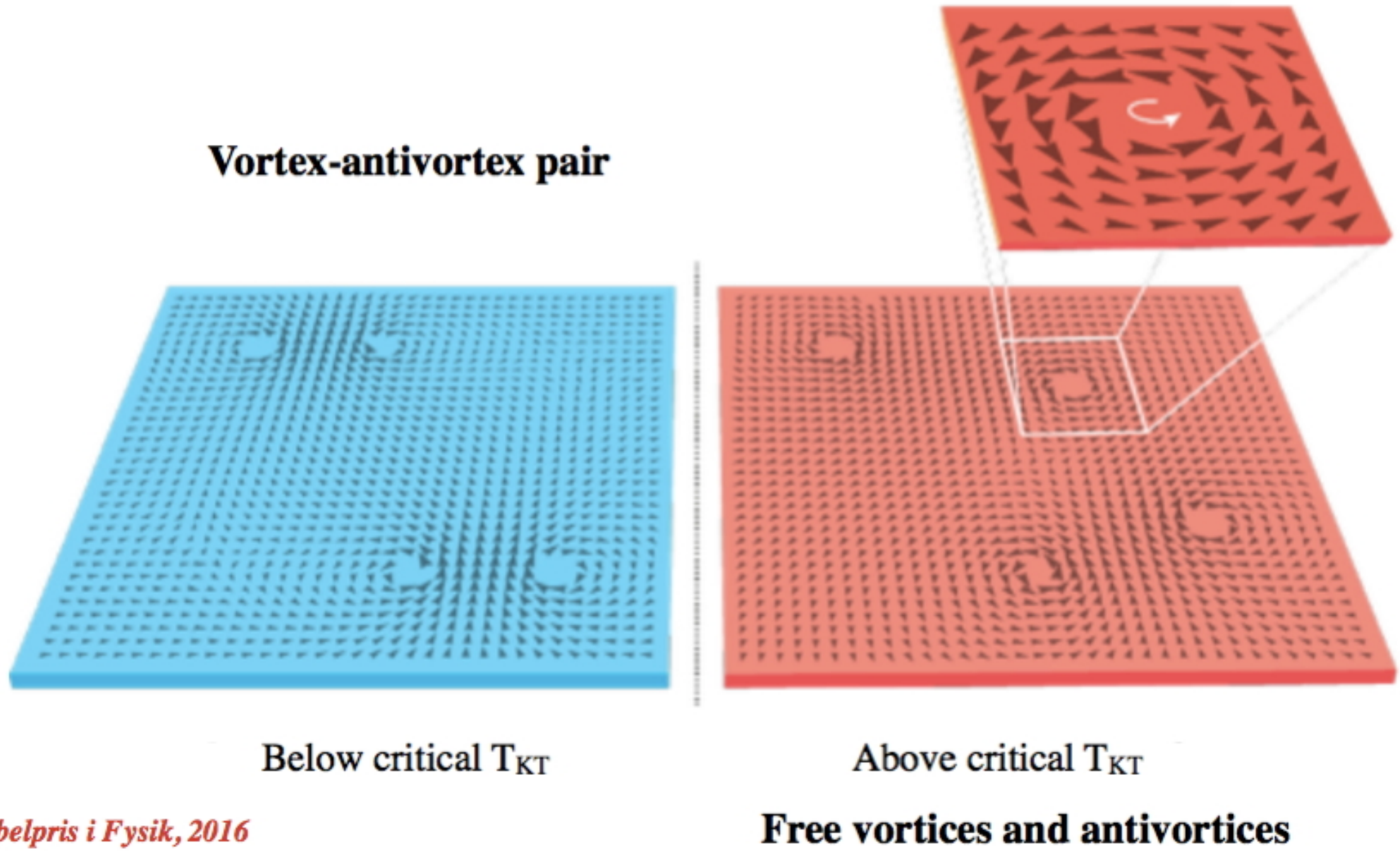
in 3D vortices are
like smoke rings





Nobelpris i Fysik, 2016
T. H. Hansson

The whirlpools appear in pairs and trap superfluid in their circular motion such that at the macro scale the superfluidity vanishes at T_{KT}.

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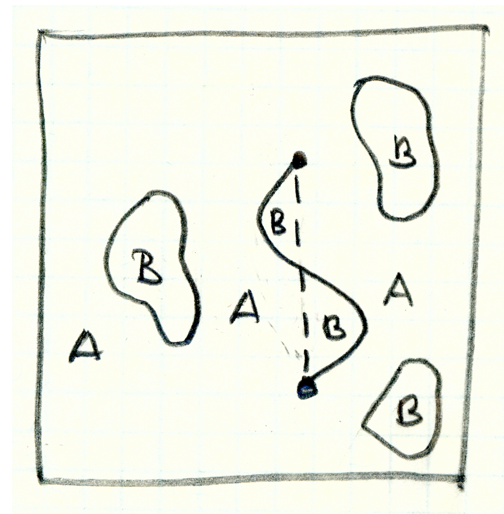
Those "electric charges" are the cores of the vortices and also the dislocations in the topological reinterpretation of the above mentioned conventional phase transitions.

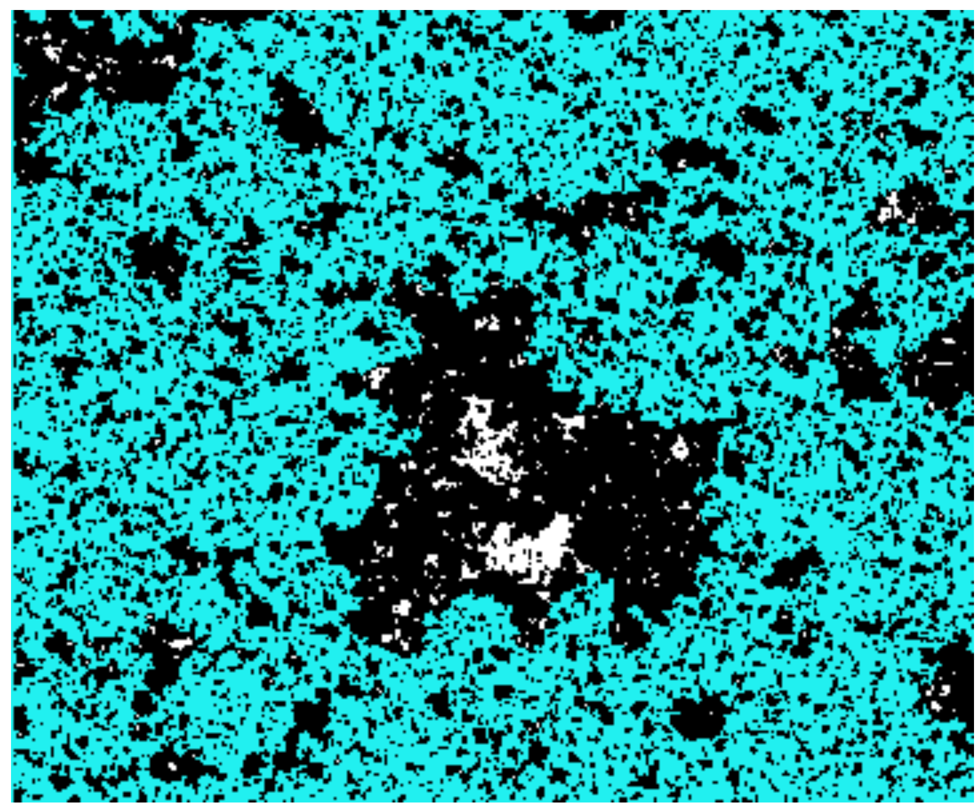
The core difference between continuous and discrete degrees of freedom is that the linear bound (confined) dislocation phase can not appear for the continuous degrees of freedom, because of $\Delta F \sim L^{(D-2)}$. The low temperature phase is a critical phase.

Recall the previous viewgraph for Ising type order:



If we impose an open domain wall into the system into the system at a distance R apart (which requires a branch cut), then the free energy of this topological defect scales:

- linear with R below Tc and proportional to the surface tension (confinement).
- logarithmically at Tc. (scale free fractal fluctuations)
- does not depend on R above Tc (de-confinement).



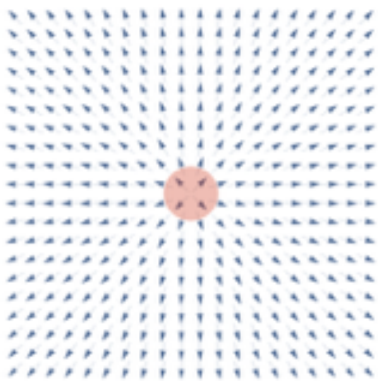
In Ising universality class phase transitions, the topological charge of the domain walls remains somewhat hidden. (They act like being their own "anti-particles".)

Kosterlitz' and Thouless' thermodynamical argument for a phase transition driven by "vortex liberation"

$$\Psi(r) = \sqrt{\rho}e^{i\phi(r)}$$

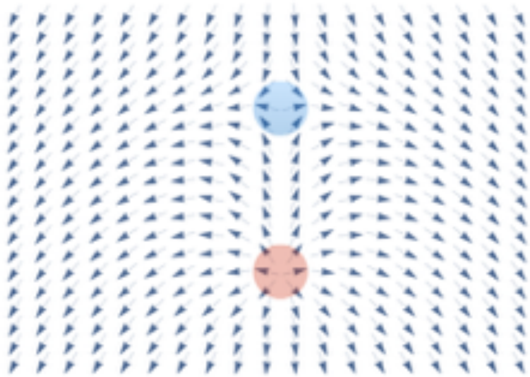
$$\vec{J} = \frac{h}{m}\rho\nabla\phi$$

single vortex



$$E_v = J\pi \ln \frac{L}{a}$$

vortex - antivortex pair



$$E_{pair} = J2\pi \ln \frac{r}{a}$$

Free energy for a single vortex

$$F = E - TS = J\pi \ln\left(\frac{L}{a}\right) - T\ln\left(\frac{L^2}{a^2}\right)$$

The entropy balances the energy for: $T_{KT} = \frac{J\pi}{2}$

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Teaching graduate courses is good!!!

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[Department of Mathematical Physics, University of Birmingham, England]

Before we had published it, we became aware that Berezinskii,[1] in the Soviet Union, had anticipated some of our ideas, although our use of the renormalization group[2] gave us insights that he missed.

1. **Berezinskii V L.** Destruction of long-range order in one-dimensional and two-dimensional systems possessing a continuous symmetry group. II. Quantum systems. *Zh. Eksp. Teor. Fiz. SSSR* 61:1144-56, 1971. (Cited 110 times.)
2. **Kosterlitz J M.** The critical properties of the two-dimensional *xy* model. *J. Phys.—C—Solid State Phys.* 7:1046-60, 1974. (Cited 915 times.)
3. **Bishop D J & Reppy J D.** Study of the superfluid transition in two-dimensional ^{4}He films. *Phys. Rev. Lett.* 40:1527-30, 1978. (Cited 175 times.)
4. **Nelson D R & Kosterlitz J M.** Universal jump in the superfluid density of two-dimensional superfluids. *Phys. Rev. Lett.* 39:1201-5, 1977. (Cited 285 times.)
5. **Minnhagen P.** The two-dimensional Coulomb gas, vortex unbinding and superfluid-superconducting films. *Rev. Mod. Phys.* 59:1001-66, 1987.
6. **Strandburg K J.** Two-dimensional melting. *Rev. Mod. Phys.* 60:161-307, 1988.

This is why this is also often called the "BKT transition"

CC/NUMBER 44
NOVEMBER 4, 1991

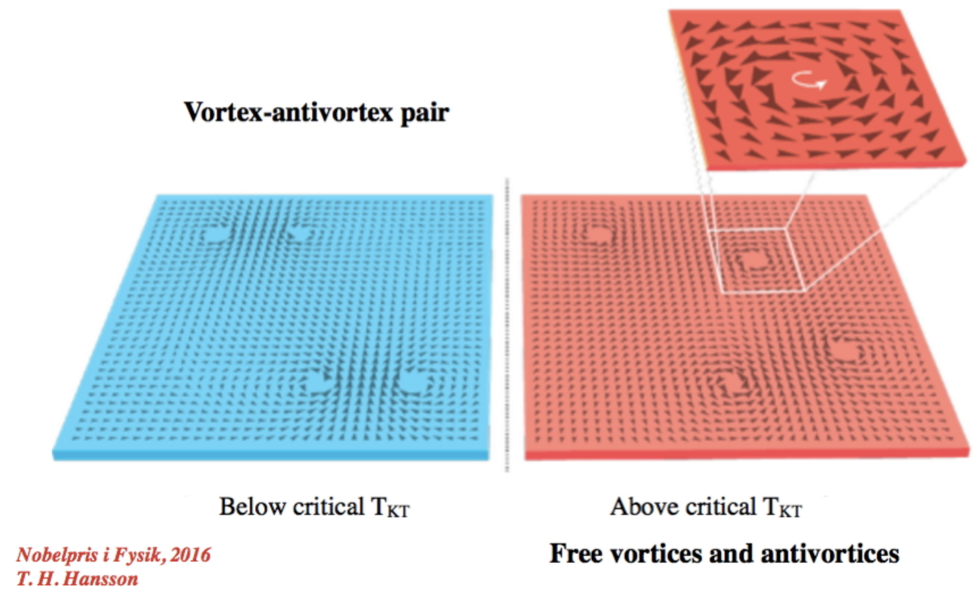
This Week's Citation Classic®

Kosterlitz J M & Thouless D J. Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys.—C—Solid State Phys.* 6:1181-203, 1973.
[Department of Mathematical Physics, University of Birmingham, England]

The initial reaction of the physics community to this work was polite interest rather than enthusiasm, and it was several years before an experimental test was made. In 1978, D.J. Bishop and J.D. Reppy[3] published the results of their experiment on the superfluid density of a helium film, and correlated their results with other experiments to show the constancy of the ratio of superfluid density to transition temperature; this is a characteristic of the theory which was convincingly demonstrated by D.R. Nelson and Kosterlitz[4] in 1977.

We got many of the interesting features of the transition right, but there were important modifications that were made by other people. It was pointed out that, in addition to the spontaneous formation of dislocations in a solid, associated with the start of viscous flow, there was a further transition of the same sort possible in which disclinations would appear, and orientational order lost. We had argued that this sort of transition should not occur for superconductivity but had forgotten that the penetration depth in a thin film is very large.

In classical hydrodynamics (think about hurricanes) the density and velocity fields are independent. In superfluids they are linked by the amplitude and gradient of the phase of a single complex scalar field order parameter.



The whirlpool vorticity (its strength; its winding number) is therefore quantized.

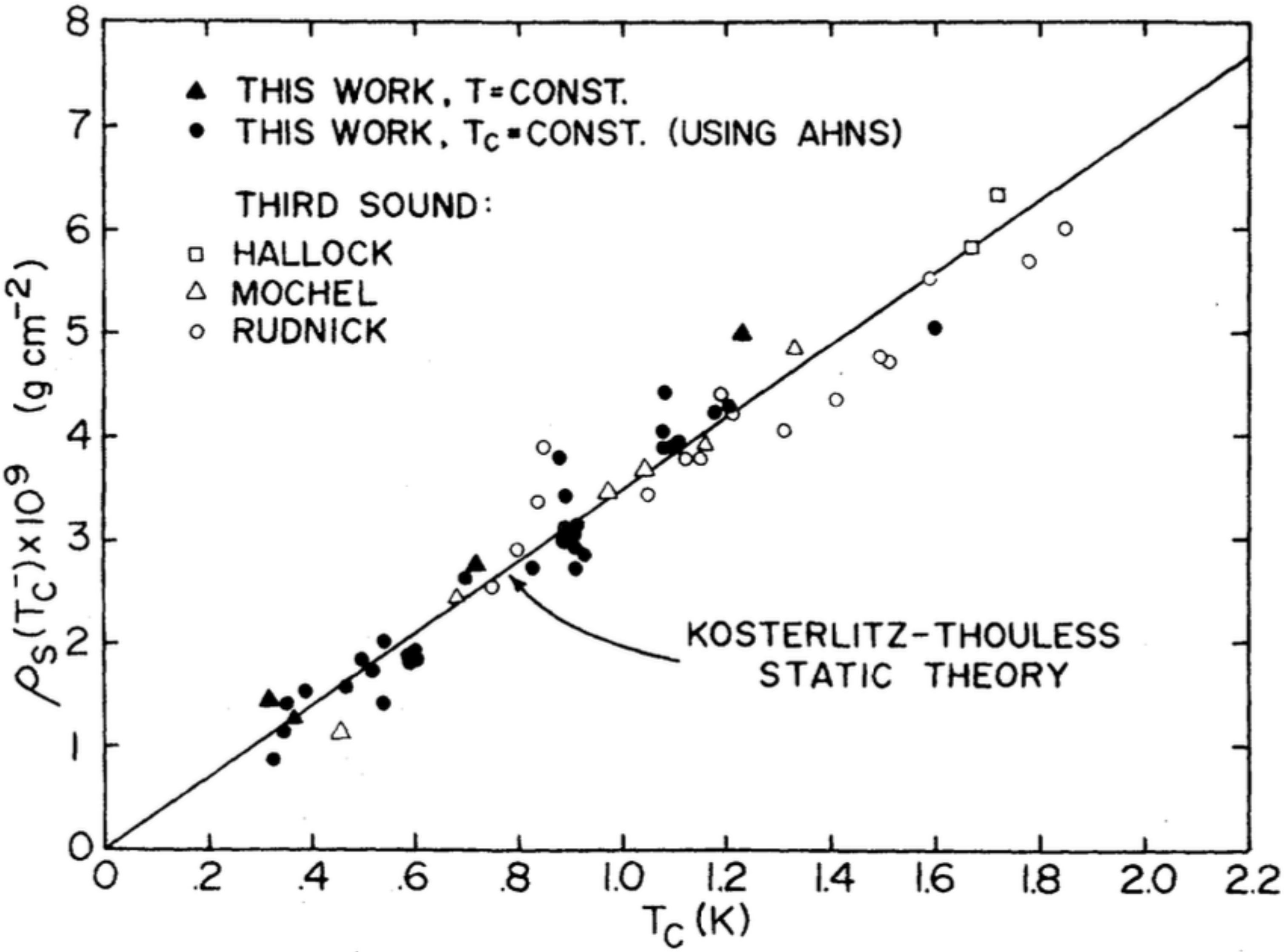
The whirl pools trap superfluid in their circular motion such that at the macro scale the superfluidity vanishes at T_{KT}.
The macroscopic superfluid density jumps to zero with a universal jump.

The torsion oscillator experiment of Bishop and Reppy confirmed these predictions in detail, in particular the actual value of the jump predicted by Nelson and Kosterlitz.

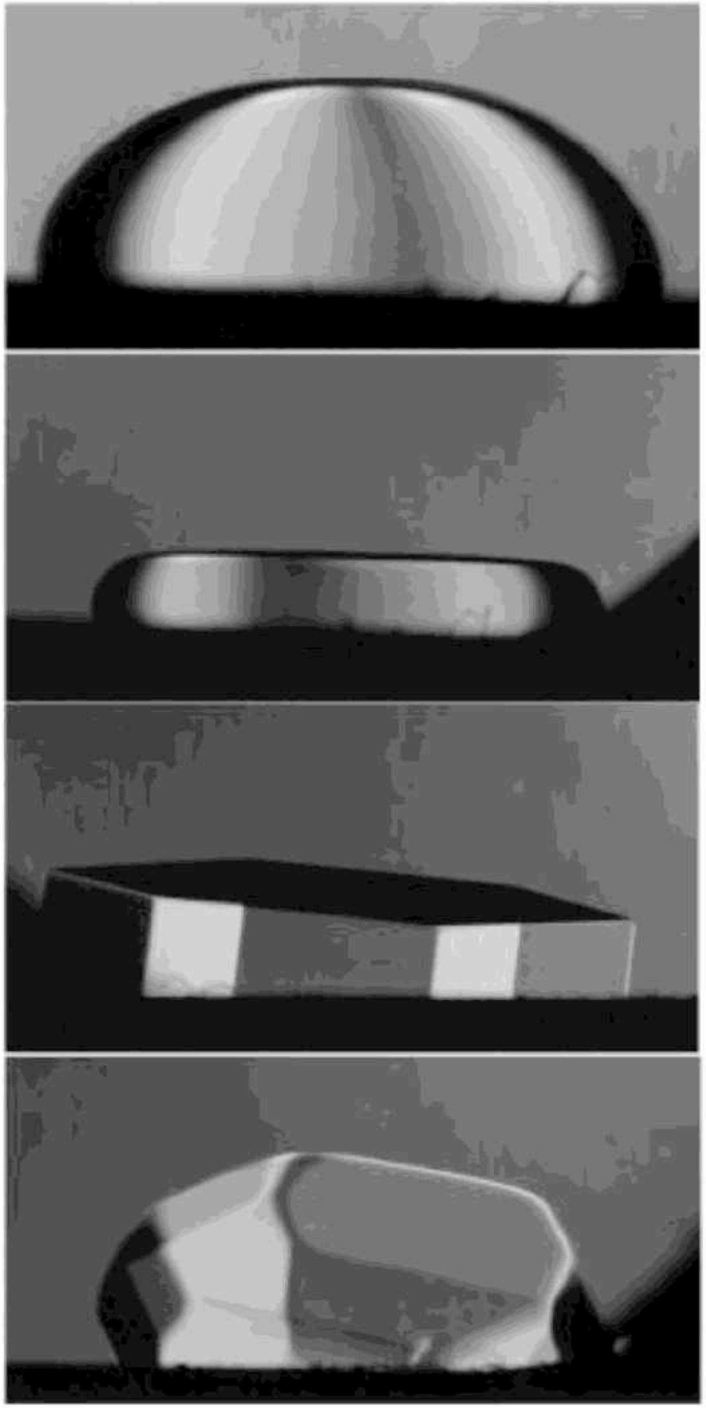
Study of the Superfluid Transition in Two-Dimensional ^{4}He Films

D. J. Bishop and J. D. Reppy
*Laboratory of Atomic and Solid State Physics, and Materials Science Center,
Cornell University, Ithaca, New York 14853*
(Received 20 April 1978)

We have studied the superfluid transition of a thin two-dimensional helium film adsorbed on an oscillating substrate. The superfluid mass and dissipation when analyzed in terms of the dynamic theory of Ambegaokar, Halperin, Nelson, and Siggia support the Kosterlitz-Thouless picture of the phase transition in a two-dimensional superfluid. The value for the jump in the superfluid density at the transition given by Kosterlitz and Thouless, $\rho_s(T_c^-) = 8\pi k_B(m/h)^2 T_c$, is in good agreement with estimates from experiment.



THIS WORK, T=CONST.
THIS WORK, T_C=CONST. (USING AHNS)
THIRD SOUND:
HALLOCK
MOCHEL
RUDNICK
KOSTERLITZ-THOULESS STATIC THEORY
$\rho_S(T_C^-)\times 10^9$ $(g\,cm^{-2})$
T_C (K)



The rounding of crystaline equilibrium crystal surfaces with temperature is a second example of a direct application of KT transitions.

At the KT transition (in its so-called dual representation) a flat facet shrinks to zero and its curvature jumps to a universal value.

The picture shows the evolution of a helium-4 crystal (from the Balibar group in Paris).

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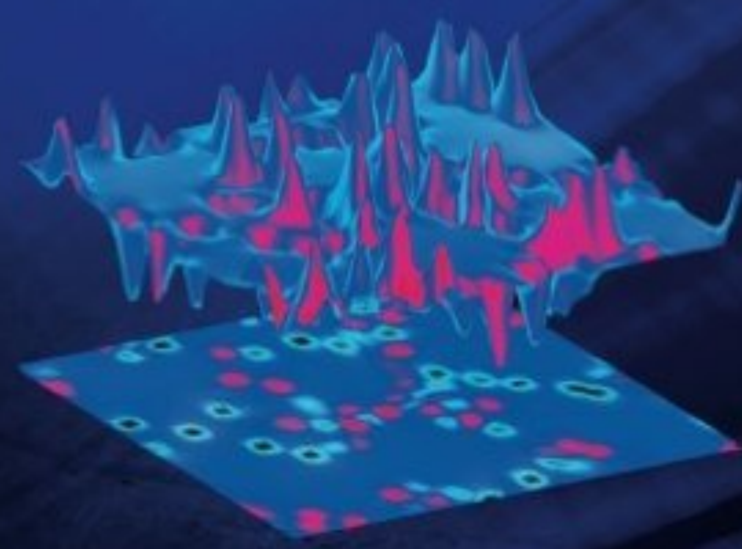
The dramatic experimental results started the boom in work on this theory, and on its experimental manifestations. Two recent reviews of the theory and its applications have been written by P. Minnhagen[5] and K.J. Strandburg.[6] The layered copper oxide superconductors have helped to maintain activity in this subject.

40 YEARS OF BEREZINSKII–KOSTERLITZ–THOULESS THEORY



Jorge V José

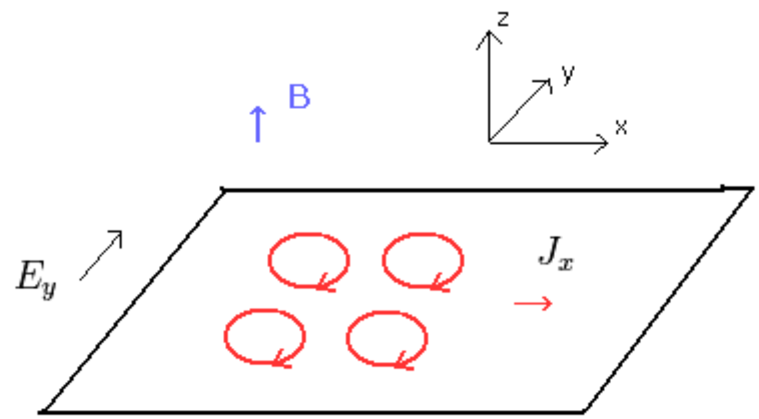
Editor

World Scientific

4. The Integer Quantum Hall Effect"

Consider an ideal electron gas confined into a plane.

Add a very strong perpendicular magnetic field.
In classical mechanics, the electrons start moving in circles.



Add an electric field in the x-direction.
In classical mechanics, the electrons start spiraling sideways in the perpendicular direction, creating the Hall current $J_{H.}$

In quantum mechanics the circular orbit are quantized. giving rise to so-called Landau energy levels. It is only possible to pack $N= L^2/(l_B)^2$ electrons on the surface in the lowest Landau level, with l_B the magnetic length.

$$\mathcal{H} = \frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 + \left(\frac{\hbar}{i}\frac{\partial}{\partial y} - eBx\right)^2\right] + e\mathcal{E}x$$

$\vec{A} = (0, eBx)$ Landau Gauge magnetic length $l = \sqrt{h/eB}$

the Hall conductance of a fully filled Landau level is quantized

$$\sigma_H = \frac{J_H}{\Delta V} = \frac{J_H}{L_x \mathcal{E}} = \frac{e^2}{h}$$

Until 1980 nobody expected this fine-structure like quantum number in the ideal electron gas calculation to be stable against reality, such as adding random potentials. Localized electrons are trapped and that should reduce the Hall current.

Worse: In 2D localization theory, disorder localizes all electrons in the absence of a magnetic field. (The Landau level broadens into a band.) Later it was proven in field theory that one delocalized state remains in the presence of the magnetic field.

Then in 1980 experimentalists upset the theorists.

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing, G. Dorda, and M. Pepper
Phys. Rev. Lett. **45**, 494 – Published 11 August 1980

ABSTRACT

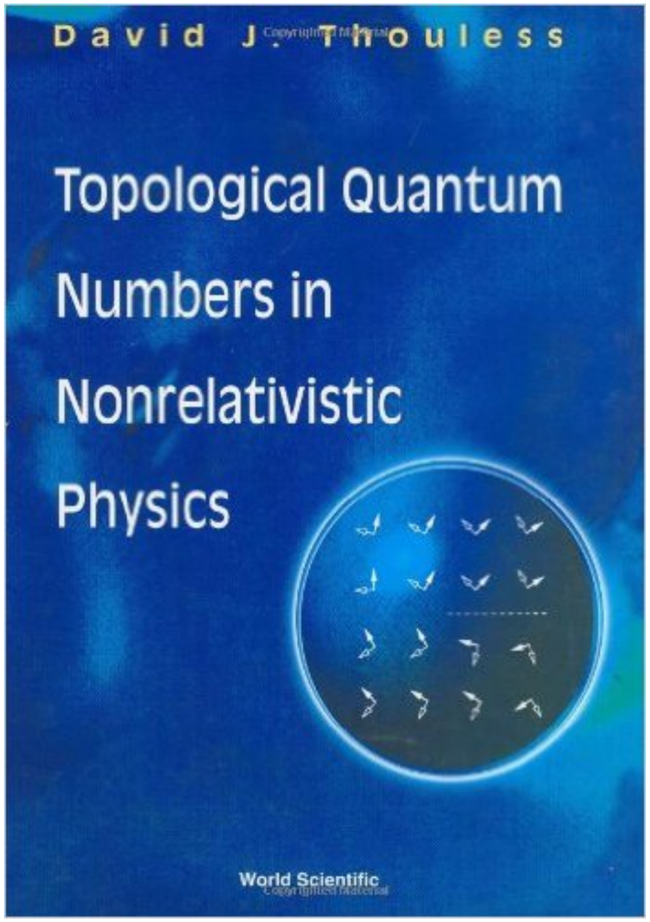
Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

Received 30 May 1980

The integer quantum Hall nowadays defines the unit for electric resistance, the Ohm; related to the fine-structure constant as

$$R_H = h/e^2 = 25813\ \Omega \qquad \frac{1}{\alpha} = \frac{h}{e^2}\frac{2}{\mu_0 c} = 137.035968(23)$$

von Klitzing Nobel prize 1985



Preface

Topological quantum numbers crept up on the physics community before the community was aware of them. I did not think in these terms until I started working on the topological aspects of long range order in the early 1970s, although I had been working on aspects of superfluidity that are now regarded as topological for several years before that. I should have known earlier of the importance of topology, as I was then a colleague of Tony Skyrme, whose pioneering work on topological quantum numbers is now so well known. It was around this time that there began to be a wide awarenesss of the importance of topology both amongst elementary particle theorists and field theorists, and amongst people who worked on superfluids and liquid crystals. The issue was brought sharply into focus for me in 1980, when Hans Dehmelt asked me about how the quantum Hall effect could possibly be used to determine the fine-structure constant when so little was known about the details of the devices used and so little understood about the theory.

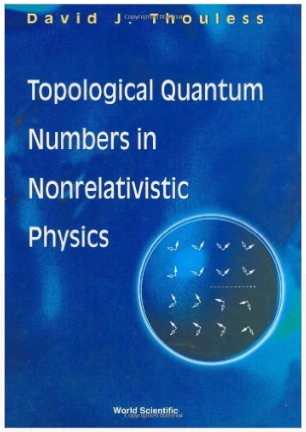
Dehmelt's question is one of the unifying themes of this book, particularly in Chapters 2 to 5 and in Chapter 7. The answer is not entirely simple, since, although topological quantum numbers can provide a correspondence between countable integer quantities and physical observables, this correspondence is not usually exact, and corrections may be more or less important.

A second theme, provided by the work on liquid crystals, and on the A phase of superfluid ^3He, is the use of topological quantum numbers to classify defects, in situations where the relevant group is finite, rather than isomorphic to the infinite group of integers.

The third theme, covered in the last chapter, is the importance of topological concepts in the theory of phase transitions in two dimensions.

I have tried in this book to give enough background material to make it accessible to people whose knowledge of quantum mechanics and statistical mechanics is at the level expected in the second year of a U.S. graduate program in physics. For Chapters 6 and 8 a little knowledge of the theory of finite groups is also necessary. I have not assumed any previous knowledge of topology.

Thouless, D. J. (1998). Topological Quantum Numbers in Nonrelativistic Physics. World Scientific. ISBN 981-02-2900-3.



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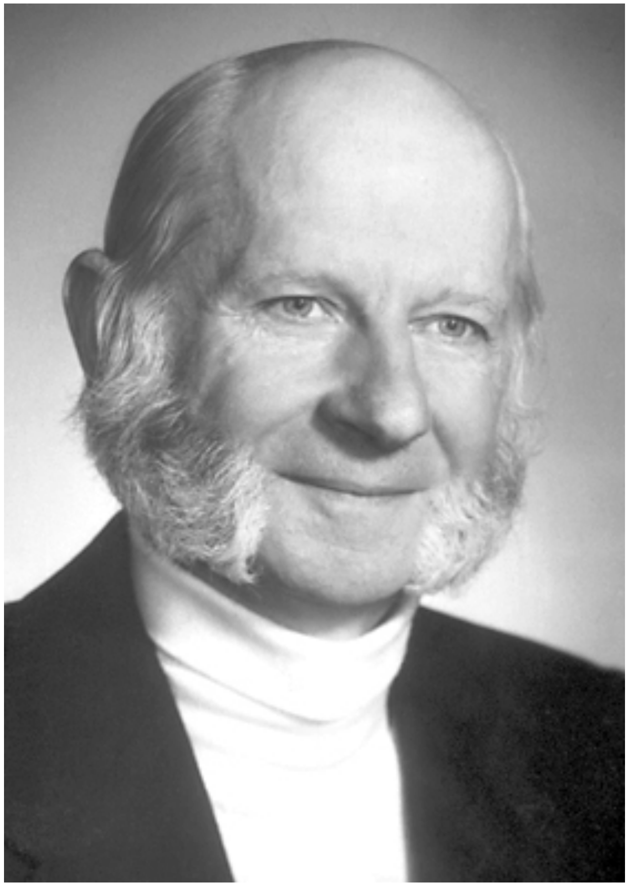

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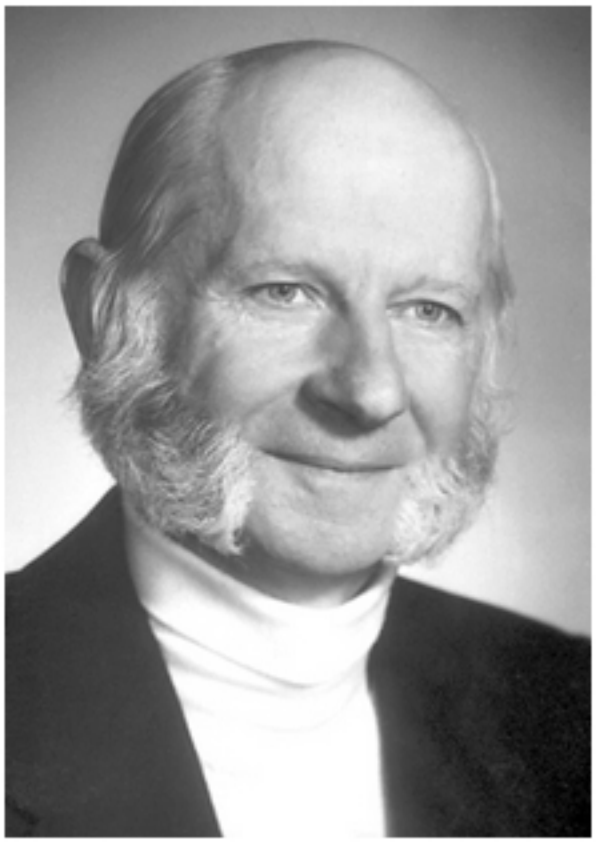
The Nobel Prize in Physics 1989

Norman F. Ramsey, Hans G. Dehmelt, Wolfgang Paul

Hans G. Dehmelt - Facts



Hans G. Dehmelt

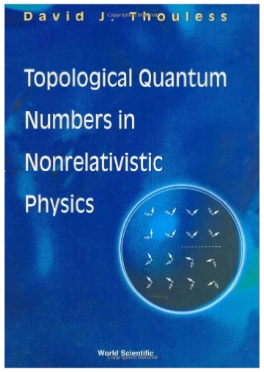
Born: 9 September 1922, Görlitz, Prussia (now Germany)

Affiliation at the time of the award: University of Washington, Seattle, WA, USA

Prize motivation: "for the development of the ion trap technique"

Field: atomic physics

Prize share: 1/4



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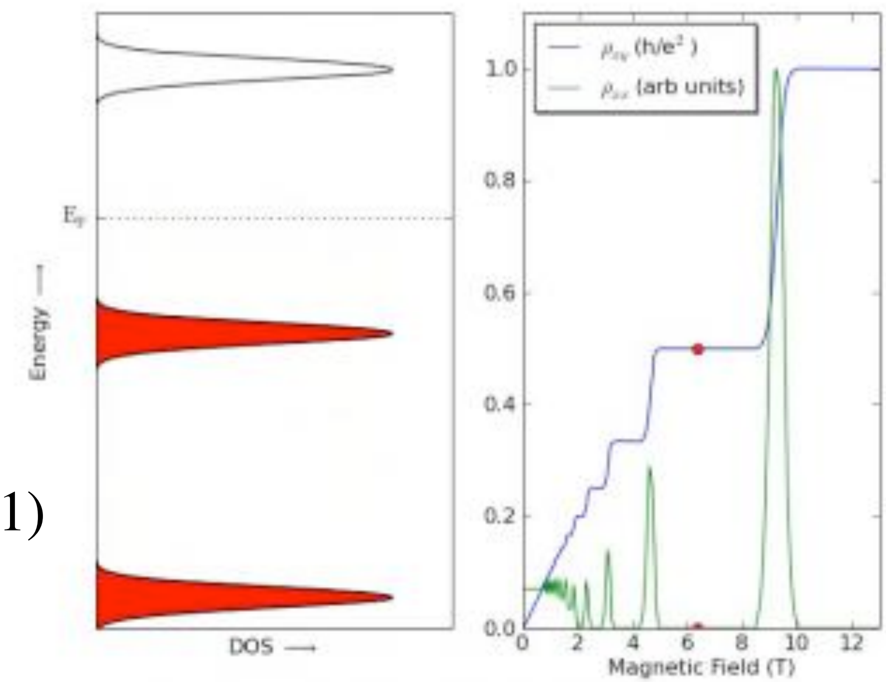
The Quantum Hall effect

(von Klitzing, 80, NP 85)

- **2d elektron gas**
- **Clean samples, low temperature, high magnetic fields**
- **Quantized Hall conductance on the plateaux**
- **Longitudinal conductance = 0 on the plateaux**

Similar to a band insulator, but

Why is the quantization so exact??



- **Gauge invariance** (Laughlin -81)
- **Topology** (Thouless et.al. -82)

Thouless: The topology explains !

Using linear response theory, the Hall conductance can be related to the single particle wave functions in the filled bands:

$$\sigma_H = \frac{e^2}{2\pi h} \sum_n \int_{BZ} d^2k \, \mathcal{B}(\vec{k}, n)$$

DJ Thouless, M. Kohmoto, M.P Nightingale, and M. Den Nijs. PRL, **49**, 405, 1982.

$$\mathcal{B}(\vec{k}, n) = \partial_{k_x} \mathcal{A}_y(\vec{k}, n) - \partial_{k_y} \mathcal{A}_x(\vec{k}, n)$$

Berry field

$$\mathcal{A}_j(\vec{k}, n) = i \langle u_{\vec{k},n} | \partial_{k_j} | u_{\vec{k},n} \rangle$$

Berry potential

The Brillouinzone is a closed surface, so the integral over a magnetic field must, according to Dirac, be quantized!

$$\frac{1}{2\pi} \int_{Bz} \mathcal{B}(\vec{k}, n) = C_1(n)$$

is the first Chern number, which is a topological invariant

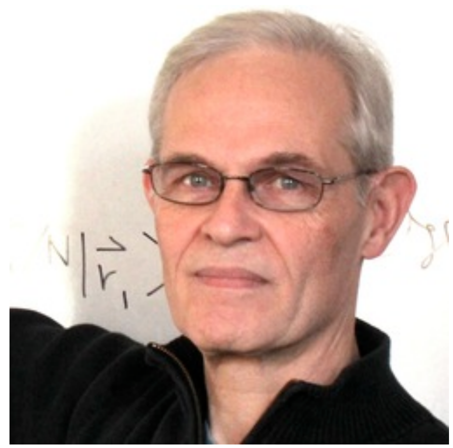
Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)

The Hall conductance of a two-dimensional electron gas has been studied in a uniform magnetic field and a periodic substrate potential U. The Kubo formula is written in a form that makes apparent the quantization when the Fermi energy lies in a gap. Explicit expressions have been obtained for the Hall conductance for both large and small $U/\hbar\omega_c$.

PACS numbers: 72.15.Gd, 72.20. Mg, 73.90.+b






Kubo-Greenwood Formula.

The Hall conductance formula

$$\sigma_H = \frac{e}{m} \frac{1}{L_x L_y} \sum_k \langle \hat{p}_y \rangle$$

is approximated in linear transport theory by the $T = 0$ Kubo formula

$$\sigma_H = i \frac{e^2 \hbar}{m^2} \frac{1}{L_x L_y} \sum_\alpha \sum_\beta \frac{\langle \alpha | \hat{p}_y | \beta \rangle \langle \beta | \hat{p}_x | \alpha \rangle - \langle \alpha | \hat{p}_x | \beta \rangle \langle \beta | \hat{p}_y | \alpha \rangle}{(E_\alpha - E_\beta)^2}$$

with $|\alpha\rangle$ and $|\beta\rangle$ single particle eigenstates of the full system $\hat{H}_0$ in the absence of the driving electric field. α and β are the single particle states. The derivation follows from basic perturbation theory in small $\lambda = e\mathcal{E}$ with $\hat{V} = \hat{x}$, and by using the standard identity

$$\langle \alpha | \hat{x} | \beta \rangle = \frac{i\hbar}{m} \frac{\langle \alpha | \hat{p}_x | \beta \rangle}{E_\beta - E_\alpha}$$

which allows us rewrite this also as

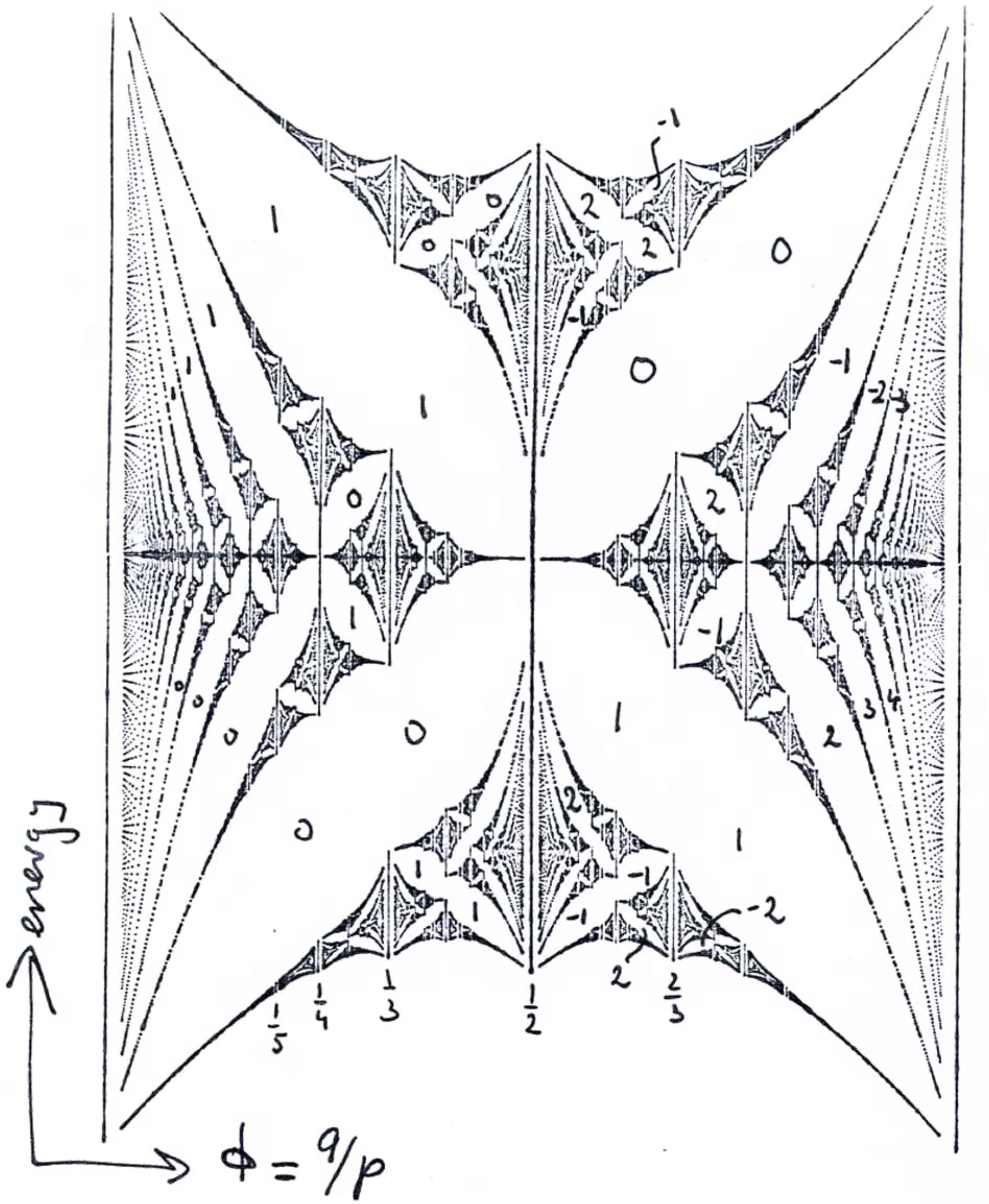
$$\sigma_H = i \frac{e^2}{\hbar} \frac{1}{L_x L_y} \sum_\alpha \langle \alpha | [\hat{x}, \hat{y}] | \alpha \rangle$$

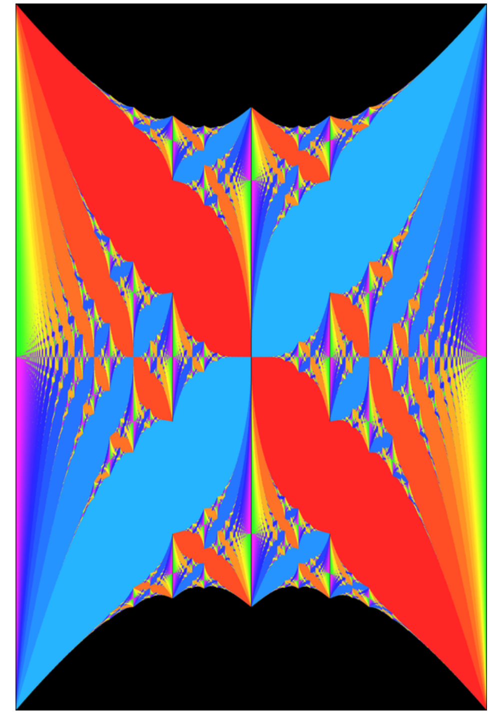
x and y do not commute anymore in this setup since $\hat{p}_y = \frac{\hbar}{i}[\frac{\partial}{\partial y} - eBx]$. Time reversal symmetry breaking rears it head, as well as the Berry phase.

In retrospect there was an elephant in the room that nobody spotted, not in the ideal electron version, nor the generalization with a random potential. But David kept kicking the Kubo formula and decided to generalize it by adding a periodic potential.

$$\hat{\mathcal{H}} = \frac{1}{2m} \left[\left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 + \left(\frac{\hbar}{i} \frac{\partial}{\partial y} - eBx \right)^2 \right] + V_x \cos(2\pi \frac{x}{a}) + V_y \cos(2\pi \frac{y}{b})$$

fully aware that his leads to a famous self-similar band structure known as the Hofstadter butterfly.


energy
$\phi = q/p$
$\frac{1}{5}$
$\frac{1}{4}$
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{2}{3}$



The eigenvalue problem for this is know as Harper's equation, introduced by Peierls as PhD topic to his student Harper, and was studied extensively in the 1970-ties, in particular by Hofstädter, Azbel, and Aubry, because of the self-similarity and fractal properties of its band structure.

The incommensurability factor $\Phi = l^2/ab$ is equal to the ratio of the magnetic length square $l^2 = h/eB$ with the area of the unit cell of the periodic potential. The ratio locks-in to rational numbers when the length scales are commensurate, denoted as $\Phi = q/p$. The Landau level splits into p bands.

Mahito Kohmoto (David's postdoc) and David started this, soon Peter Nightingale (postdoc with Michael Schick and sharing the office with Mahito) started the numerical calculations, and finally I (postdoc with Eberhard Riedel) joined the effort too. I knew Mahito very well from my postdoc days in Chicago with Leo Kadanoff, where Mahito was one of his graduate students. Peter is Dutch like I, and is one of the inventors of finite size scaling methods is critical phenomena.

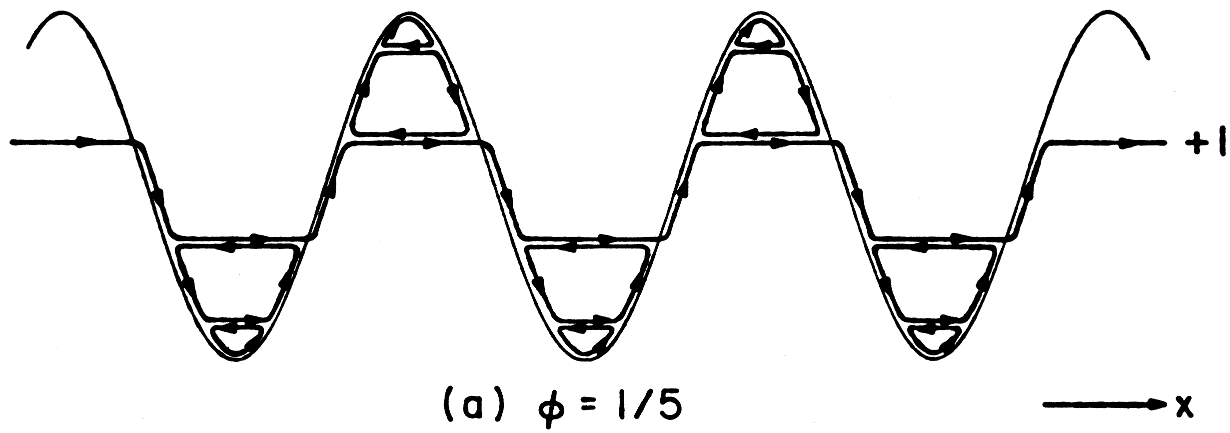
We had great fun discovering the self-similar values of the Hall conductance quanta in the various gaps. Then, but not until we had almost finished the paper, the elephant came in view.

The gaps sizes and the Hall conductance can be classified by the solutions of the Diophantine equation

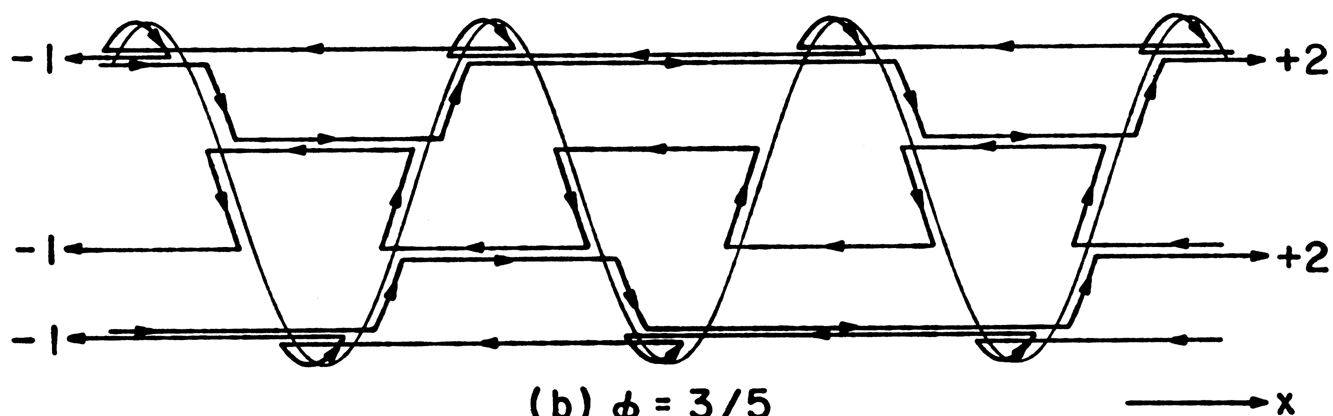
$$m = qs_m + pt_m$$

Find the smallest integers s_m and t_m that satisfy this equation . $m = 1, 2, \cdots, p$ is the sub band label. s_m orders the magnitudes of the gaps, the smaller $|s_M|$ the larger the gap between bands m and $m+1$. The Hall conductance for when the Fermi surface resides inside that sub gap is equal to

$$\sigma_H^m = \frac{e^2}{h} t_m$$



(a) $\phi = 1/5$



(b) $\phi = 3/5$

The periodic potential naturally leads to Bloch wave functions

$$\Psi(x,y) = e^{iK_x x + iK_y y} u(x,y)$$

The original Hamiltonian transforms into an operator acting on the $u(x,y)$

$$\hat{\mathcal{H}} = \frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{\partial}{\partial x} + \hbar K_x\right)^2 + \left(\frac{\hbar}{i}\frac{\partial}{\partial y} + \hbar K_y - eBx\right)^2\right] + V_x \cos(2\pi\frac{x}{a}) + V_y \cos(2\pi\frac{y}{b})$$

The expectation values can be written as derivatives of the Bloch wave functions, just as in conventional solid state physics.

$$\langle\alpha|\vec{p}|\alpha\rangle = \frac{m}{\hbar}\langle\alpha|\frac{\partial\hat{H}}{\partial\vec{K}}|\alpha\rangle$$

$$\sigma_H = i\frac{e^2}{2\pi h}\iint d\alpha\ \left[\langle\frac{\partial\alpha}{\partial K_y}|\frac{\partial\alpha}{\partial K_x}\rangle - \langle\frac{\partial\alpha}{\partial K_x}|\frac{\partial\alpha}{\partial K_y}\rangle\right]$$

The non-commutative nature of $\hat{x}$ and $\hat{y}$ resides now in the crystal momenta derivatives.

The elephant in the room was suddenly exposed when we naturally rewrote this as a directed contour integral along the edge of the "Brillouin zone"

$$\sigma_H = \frac{e^2}{h}\frac{1}{4\pi i}\oint d\alpha\left[\langle\alpha|\frac{\partial\alpha}{\partial K_{||}}\rangle - \langle\frac{\partial\alpha}{\partial K_{||}}|\alpha\rangle\right]$$

The contour integral is directed. This elephant also holds in the absence of the periodic potential (as long as the Landau level is fully filled and there is an energy gap).

$$\sigma_H = \frac{e^2}{h}\frac{1}{4\pi i}\oint d\alpha \Big[\langle \alpha | \frac{\partial \alpha}{\partial K_{||}} \rangle - \langle \frac{\partial \alpha}{\partial K_{||}} | \alpha \rangle \Big]$$

Nowadays, after Berry's paper in 1984, we refer to a form like

$$\vec{\mathcal{A}} = \langle \frac{\partial \alpha}{\partial K_y} | \frac{\partial \alpha}{\partial K_x} \rangle - \langle \frac{\partial \alpha}{\partial K_x} | \frac{\partial \alpha}{\partial K_y} \rangle$$

as a Berry curvature and to

$$\vec{\mathcal{B}} = \vec{\nabla} \times \vec{\mathcal{A}} = \langle \alpha | \frac{\partial \alpha}{\partial K_{||}} \rangle - \langle \frac{\partial \alpha}{\partial K_{||}} | \alpha \rangle$$

as a Berry phase. These $\vec{\mathcal{A}}$ and $\vec{\mathcal{B}}$ symbols resemble the vector potential and magnetic field in the Aharonov- Bohm effect, which is a simpler and older example of a Berry phase. The result of the contour integration must be a multiple of 4π provided the edge of the Brillouin zone is gapped and the adiabatic theorem applies.

$$n = \frac{1}{4\pi i}\oint d\alpha \Big[\langle \alpha | \frac{\partial \alpha}{\partial K_{||}} \rangle - \langle \frac{\partial \alpha}{\partial K_{||}} | \alpha \rangle \Big]$$

is a Chern number, the so-called "first Chern class of a U(1) principle fiber bundle on a torus" (Avron-Seiler-Simon PRL 51(1983) 51). (Fibers = magnetic Bloch functions, torus = magnetic Brillouin zone.)

TKNN story teaches us that:

1. Dig-in when your ”Kubo formula” does not seem to do what it should do.
2. Hope there is an elephant in the room to explain it.
3. Be not afraid to study esoteric looking model generalizations in fundamental research.
4. The art of it is to have the insight, or luck, to find a view point where the elephant looks familiar and natural.
5. Spotting the elephant is then easy and just “a small step”.
6. Have fun.

The genius of David Thouless to choose the periodic potential generalization, not the random potential one, was the essential step.

When I was young, studying phase transition in low dimensions was often still frowned upon.

Ignore those people!

Be yourself!

"Why aren't you wearing 3-D glasses!?! Didn't you read on the bulletin board that we'd all be working in 3-D this week!?!"

David and Margaret are both retired faculty at the UW (Physics and Virology/Pathology)

They moved back to Cambridge now, but did so only last month.

on the phone last week:

me: “David how does it feel to be a Nobel laureate (finally)?

David: “It feels odd”

"My father was moved and honored to learn of the Nobel Prize, and he was delighted to hear that he would share it with Mike Kosterlitz and Duncan Haldane. He is grateful to all his friends and colleagues around the world who have sent congratulations and made such lovely comments about his contributions to physics.”

(Prof. Michael Thouless, University of Michigan).