Polymer localization in random media versus traffic jams

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Does a slow bond in driven stochastic flow always create a macroscopic traffic jam?

Does an attractive line defect always localize a directed polymer fluctuating in a quenched random landscape?

Numerical evidence strongly suggests neither is true. What are the properties of the intermediate still delocalized phase?
ABSTRACT

One basic example of driven stochastic processes is the so-called ASEP process where particles hop in a preferred direction along a one dimensional chain of discrete sites. This process also is a realization of the Burgers equation, is equivalent to so-called KPZ type interface growth, and describes the equilibrium properties of a directed polymer on a 2 dimensional lattice in the presence of quenched random landscape.

Its scaling properties in the stationary state and of temporal fluctuations are well known: the stationary state lacks correlations (the forwardly hopping particles are placed at random) but density fluctuations dissipate as a power law with dynamic exponent $z=3/2$. This is a rare example of an exactly soluble non-trivial process in non-equilibrium statistical physics.

Interestingly, this simple scaling behavior is highly unstable.
ABSTRACT - continued

One example of instability is the introduction of a so-called slow bond where the hopping probability is locally reduced.

As expected, a macroscopic traffic jam sets-in. In the directed polymer interpretation, the slow bond represents an attractive line defect parallel to the polymer, and the traffic jam that the polymer is bound to this line defect.

Over the years the numerical results have been ambiguous about when exactly simple localization sets-in. Some years ago we presented evidence, based on numerical results and analytical arguments, for the existence of an intermediate phase in a range of weak slow bond defect strengths, where the traffic jam density does not decay with distance to a constant, but decays to zero as a power law.

In this talk I review the connections between the various interpretations of the process, and focus on what the power law traffic jam means for localization of the quenched random polymer.
OUTLINE

1. The asymmetric exclusion process (ASEP) for driven stochastic flow and its body-centered solid-on-solid (BCSOS) KPZ type interface growth representation.
2. Summary of the slow bond results by Meesoon Ha, MdN, and Jussi Timonen (PRE 2003).
4. A discrete lattice directed polymer model in a quenched random landscape
5. Its interpretation in terms of KPZ growth and driven stochastic flow.
7. Numerical results of a different T=0 DPRM by Lee & Kim (PRE 2009)
8. The nature of the intermediate phase: Power law shaped traffic jams.
9. What type of (de-) localization in the DPRM?
1. Review of ASEP and BCSOS growth

- Driven stochastic flow through 1D channels
- 2D polymer localization
- 1+1D KPZ surface growth
- Facet ridge end points in 3D equilibrium crystals
BCSOS (brick laying) model for a growing interface

Vertical alternating rectangular 2x1 building blocks
All nearest neighbor heights differ by one unit

growth rule: Select one column site at random. Add (remove) one brick with probability $p$ ($q$) if this column is a local valley (hill).

growth at slopes is inhibited (KPZ parameter $\lambda < 0$)
in 1 dimension: exactly soluble by the Bethe Ansatz when periodic boundary conditions. The stationary state is exactly known by matrix product method for a larger set of boundary conditions
KPZ Langevin equation

If the local geometry of the interface is the only relevant degree of freedom, then the large scale properties of its evolution must be governed by (universality):

$$\frac{d}{dt} h(\vec{r}, t) = v_0 + \nu \nabla^2 h + \frac{1}{2} \lambda (\nabla h)^2 + \eta$$

the KPZ equation with uncorrelated noise

$$\langle \eta_{r_2, t_2} \eta_{r_1, t_1} \rangle = 2 \Gamma \delta_{r_1, r_2} \delta_{t_1, t_2}$$

The growth rate $v_0$ is modified by the local curvature of the surface (the $\nu$-term) and its local slope (the $\lambda$-term), and random fluctuations in the paper (density, flocking, potassium nitrate concentration).
the Burgers equation

For randomly stirred vortex free (curl free) fluids is equivalent to the KPZ equation (at $\lambda \equiv 1$).

$$\frac{d\vec{v}}{dt} + \lambda \vec{v} \cdot \vec{\nabla} \vec{v} = \nu \nabla^2 \vec{v} + \vec{f}(\vec{r}, t)$$

with velocity $\vec{v}(r) = -\vec{\nabla} h(r)$, viscosity $\nu$, and random force $\vec{f} = -\vec{\nabla} \eta$.

The non-linear $\lambda$ term arises here logically as part of the total derivative of the velocity.

In 1+1D, velocity is a scalar and we can reinterpret it as a particle density. The Burgers equation then describes a stochastic driven flow. The asymmetric exclusion process is the canonical example of this.
asymmetric exclusion process (ASEP)

The BCSOS interface model (KPZ growth) is equivalent to a driven flow of particles with hard core repulsive interactions:

Interpret the $S_{\mathbf{n}}^z = -1$ down-steps as particles and the $S_{\mathbf{n}}^z = +1$ up-steps as empty sites.
stationary state, fluctuations, and group velocity

The stationary ASEP state for periodic boundary conditions is disordered, random, without any correlations, but fluctuations scale in time as $l \sim t^{1/z}$ with the KPZ dynamic exponent $z = \frac{3}{2}$, and move with group velocity $v_g = 1 - 2\rho$ (tilt of KPZ surface).
boundary induced phase transitions

Phase transitions take place in open road set-ups with reservoirs on both ends; (exact matrix formulation results of the stationary state by, e.g., Derrida et al.)

In the maximum current (MC) phase the road controls the density, but in the low (high) density phase the input (output) reservoir ($\alpha$ or $\beta$) controls the bulk density.
Reservoirs create macroscopic traffic jams only beyond critical strengths. These are the reservoir controlled L and H phases.”

In the “maximum current phase” reservoirs create only queues with an exponential or power law shaped density profile

\[ \rho(x) \approx \Delta - Ax^{-\nu} \]

with \( \nu = 1/2 \) and zero global density offset, \( \Delta = 0 \)

Stationary state exactly soluble by matrix product method. That product structure can only generate \( \nu = 1/2 \) (such profiles lack long range correlations).
Do slow bonds always create traffic jams for all defect strengths or also only beyond a threshold?

Reservoirs do not transmit information, slow/fast bond can. Behind the slow bond a traffic jam develops. The issue is whether that queue is finite or infinite in length, with or without a density off-set, i.e., takes control over the bulk phase or not.

\[ p' = r p \]

what values does \( \nu \) take?
earlier work

Mean field theory predicts an infinite queue for all $r < 1$, and no queue for fast bonds, $r > 1$ (Wolf and Tang, ’90). Kandel and Mukamel (1992) suggested (for a slightly different model) the presence of a queuing transition at a $r_c < 1$ but their simulation data were inconclusive.

Janowsky and Lebowitz (1992-94) draw pictures as if $r_c = 1$, but their focus was on the shock wave fluctuations in the faceted phase far away from the slow bond.

Schütz (’93) determined the exact stationary state for periodic boundary conditions and parallel updating (the matrix method) and found $r_c = 1$. This does not contradict our results, because in parallel updating stochastic noise is weaker than in random sequential updating.
Propagating front in flameless combustion of paper
Jussi Timonen (Finland)

experimental realization of KPZ growth
detailed confirmation of KPZ scaling
Defects induced faceting

Vary the potassium nitrate concentration in a narrow band to illustrate the presence of the non-linear term.
mean field theory predicts faceting for enhanced concentration and a logarithmic non-faceted profile for reduced concentration (if $\lambda > 0$).

Faceted surfaces grow faster:
★ from experiment; drawn line from ASEP (numerical).

What is the real profile? Is there a faceting transition?
2. Resume of our numerical results for ASEP
Meesoon Ha – Timonen – MdN  (PRE 2003)

faceted density
profiles
open edges,
slow bond
in the middle
particle-hole
symmetry
order parameter

Various aspects of the profile are linked to the current:

\[ j = \alpha p \langle (1 - \rho_1) \rangle = p \langle \rho_x (1 - \rho_{x+1}) \rangle = p \rho_b (1 - \rho_b) \]

In the bulk (flat part) the stationary is uncorrelated.

The profile at the edges is exponential (as in open bc case without slow bond)

\[ \rho(x) \sim e^{-x/\xi} \quad ; \quad \xi \sim \Delta_b^{-\frac{1}{2}} \quad ; \quad \rho_b = \frac{1}{2} (1 + \Delta_b) \]
the order parameter

The numerical data for the current, the first site density, and the plateau value $\Delta_b$ (faceting angle), agree very well.

$\rho' = r \rho$
An exponential essential-singularity type infinite-order transition with $r_c = 1$, as suggested by the directed polymer renormalization studies does not fit our numerical data.
the critical point

Assume the order parameter vanishes as a powerlaw

\[ \Delta_c \sim |r - r_c|^{\beta}. \]

From straight line fits in log-log plot:

\[ r_c = 0.80 \pm 0.02 \]

\[ \beta = 1.5 \pm 0.01 \]
Conclusion: macroscopic queuing only appears beyond a threshold defect strength and we know the critical exponents of this queuing phase transition quite well – but there are skeptics.

Finite size scaling of $\Delta_b \sim N_s^{-x_\Delta}$ at $r = 0.80$, $x_\Delta = 0.370(5)$

Data collapse of the FSS scaling function $\Delta_b(N_s, \epsilon) = N^{-x_\Delta} S(N_s^{y_\epsilon} \epsilon)$ with $x_\Delta = \beta y$. 
KPZ type growth is equivalent to a directed polymer in 2 dimensions subject to a random potential, by the transformation, $W = \exp\left(\frac{\lambda}{2\nu} h\right)$,

$$\frac{dh}{dt} = \nu \nabla^2 h - \lambda (\nabla h)^2 + \eta \rightarrow \frac{dW}{dt} = \nu \nabla^2 W - \eta W$$
Directed polymer localization

The slow bond becomes a columnar defect with short ranged attraction. Queuing translates to how this potential localizes the polymer.

In the absence of the attractive line defect, the polymer spreads as

$$\sqrt{\langle x^2 \rangle} \sim t^{1/z}$$

with $z = 3/2$ (instead of $z = 2$ without quenched noise).

The directed polymer community was focused on the slow bond issue in the mid 1990-ties (e.g., Tang, Balents, Kinzelbach, Hwa, Straley, Lassig). The driving force behind these studies are applications of such directed polymers in terms of flux tubes in type-II dirty superconductors.
Those field theoretical results are far from rigorous. Our ASEP results suggest a richer structure than anticipated by them, and might not even contradict the field theory results, depending on the details of our intermediated phase in the polymer formulation.

Directed polymer localization

In the absence of the attractive line defect, the polymer spreads as

$$\sqrt{\langle x^2 \rangle} \sim t^{1/z}$$

with $z = 3/2$ (instead of $z = 2$ without quenched noise).

Above a critical dimension $D_c$ it should be localized for all $r < 1$; Power counting in the KPZ equation and associated field-theoretical renormalization studies suggest $D_c = 1$.  

Those field theoretical results are far from rigorous. Our ASEP results suggest a richer structure than anticipated by them, and might not even contradict the field theory results, depending on the details of our intermediated phase in the polymer formulation.
Schrodinger equation of a single particle in a quenched random external field

\[ W = \exp\left(\frac{\lambda}{2\nu} h \right) \]

\[ \frac{dh}{dt} = \nu \nabla^2 h - \lambda (\nabla h)^2 + \eta \quad \rightarrow \quad \frac{dW}{dt} = \nu \nabla^2 W - \eta W \]

Recall from elementary QM: in absence of the random field, a delta function potential always has one bound state while a delta function potential at the edge in half space has a transition to no bound state.

The second case relates to reservoir queuing in ASEP and the second case to our low bond issue. In the random potential the particle diffuses faster (with \( z=3/2 \) instead of \( z=2 \)). Does that allow it to escape for weak potentials?
Population dynamics in 1+1 dimensions

\[ W = \exp\left(\frac{\lambda}{2\nu} h\right) \]

\[ \frac{dh}{dt} = \nu \nabla^2 h - \lambda (\nabla h)^2 + \eta \quad \rightarrow \quad \frac{dW}{dt} = \nu \nabla^2 W - \eta W \]

Interpret \( W(x) \) as a population density.
\( \eta \) represents a quenched random fitness field (births and deaths).

A population starting at home, at \( x=0 \), spreads as

\[ \sqrt{\langle x^2 \rangle} \sim t^{1/z} \]

with \( z = 3/2 \) (instead of \( z = 2 \) without quenched noise).

But it stays localized if “home is hospitable all the time”, but (if we are right) only so if “home” is friendly beyond a threshold value.
quenched averages

From the directed polymer perspective the disorder average is quenched and must thus be taken at the free energy level.

\[ F(x)_t = -\ln(Z(x)_t) \]

These \( F(x)_t \) are constrained free energies, where the polymer is specified to go through points \((x, t)\) and \((0, 0)\). \( F(x)_t \) is the non-averaged height profile \( h(x)_t \) of the KPZ interface. Define also the unconstrained free energy

\[ \overline{F}_t = -\log(\sum_x Z(x)_t) \]

which is distinct from \( \overline{F}(x)_t \) summed over \( x \). The time derivative of the latter represents the average growth rate

\[ v_g = \sum_x \left[ F(x)_t - \overline{F}(x)_{t-1} \right] \]

of the interface and the average current in the driven stochastic flow representation.

localization length

The quenched averaged probability distribution

\[ P(x)_t = \left[ \frac{Z(x)_t}{\sum_x Z(x)_t} \right] = \frac{Z(x)_t}{\sum_x Z(x)_t} e^{F_t} \]

represents the probability that a polymer pinned at site \((0, 0)\) visits site \((x, t)\). The polymer is localized when \( P(x)_t \) does not diverge for large \( t \), and settles down in a stationary state form that decays sufficiently fast with \( |x| \).

\[ M^{(n)} = \sum_x x^n P(x)_t = \left[ \frac{\sum_x x^n Z(x)_t}{\sum_x Z(x)_t} \right] \]

time averages in simulations

Define \( F_\tau = -\sum_x \ln[Z(x)_\tau] \). The \( \tau \) average

\[ P(x)_{stat} = \frac{1}{N_\tau} \sum_\tau Z(x, \tau) e^{F_\tau} \]

is equal to the stationary state quenched probability distribution. The time average of \( F(x)_\tau = -\ln[Z(x)_\tau] \) yields the KPZ stationary state interface profile

\[ \overline{F}(x)_{stat} = -\frac{1}{N_\tau} \sum_\tau [F(x)_\tau - F_\tau] \]
We need to run simulations in the directed polymer representations to deal with these averaging issues.

We lack of a direct BCSOS –ASEP version of the Cole-Hopf transformation

(in BCSOS: exponentiation of discrete variables and no clean separation of noise and the local topology.)

Next best approach is to run simulations on a different discrete space-time regularization of the process.
4. directed polymer on a lattice

Consider a directed polymer following the bonds of a 2 dimensional \((y, \tau)\) square lattice. \(y\) and \(\tau\) are both spatial coordinates.

The polymer is forbidden to bend back in \(\tau\). It is allowed to wander only over one lattice constant in \(x\) between slice \(\tau\) and \(\tau + 1\). It is forced to go through the origin, \(y = 0\), in slice \(\tau = 0\) and point \(x\) in slice \(\tau\).

\[
Z(x)_t = \sum_{\{y(\tau)\}} e^{-E_p}
\]

\[
E_p (y(\tau); \eta(y, \tau)) = E_h N_h + E_v t + \sum_{\tau=0}^{t} V(y)_{\tau}
\]

is the partition function over all paths \(y(\tau)\) of the polymer between the origin \((0,0)\) and point \((x, t)\), for one specific realization of the quenched random landscape.
\[ V(y) = V_r \eta(y) - V_d \delta(y) \]
represents the landscape in which this polymer fluctuates. This has two components: the attractive line defect delta function potential \( V_d \) at \( y = 0 \); and the quenched random potential \( V_r \), with \( \eta \) a random number uncorrelated in space and time

\[ \langle \eta(y, t) \eta(y', t') \rangle = 2D \delta(y, y') \delta(t, t') \]
drawn from a uniform distribution in the interval \(-1 < \eta \leq 1\), such that \( D = 1/6 \).
transfer matrix

The above partition function obeys the following transfer matrix type recursion relation

\[ Z(x)_{t+1} = e^{-V(x)_t}[Z(x)_t + zZ(x+1)_t + zZ(x-1)_t] \]

with \( z = \exp(-E_h) \).

In the equivalent quantum mechanical interpretation, each polymer configuration is a world line of a single quantum mechanical particle in one dimensional space \( x \) and Euclidean time \( t \). \( Z(x)_t \) its (not yet properly normalized) wave function and the transfer matrix is its equation of motion.
continuum limit

The continuum limit for this process is the conventional (Euclidean time) Schrödinger equation for a single particle in a quenched random field. This follows from

\[ Z(x)_{t+1} = e^{-V(x)_t}[(1 + 2z)Z(x)_t + zZ(x + 1)_t - 2zZ(x)_t + zZ(x - 1)_t] \]

by dividing by \( Z(x)_t \) and taking the logarithm

\[ \ln \left(1 + \frac{1}{Z} \frac{\partial Z}{\partial t}\right) = -V(x)_t + \ln \left(1 + \nu \frac{1}{Z} \frac{\partial^2 Z}{\partial x^2}\right) \]

with \( \frac{\partial Z}{\partial t} = Z(x)_{t+1} - Z(x)_t \) and \( \frac{\partial^2 Z}{\partial x^2} = Z(x + 1)_t - 2Z(x)_t + Z(x - 1)_t \) discrete derivatives; and with \( \nu = z/(1 + 2z) \) and a zero point shift in the random potential \( V(x)_t \rightarrow V(x)_t - \ln(1 + 2z) \).

This leads for small \( \nu \) and small \( V(x)_t \) to

\[ \frac{\partial Z}{\partial t} = \nu \frac{\partial^2 Z}{\partial x^2} - V(x)_t Z(x) \]

We retain the discretized formulation however and simulate the process outside the continuum limit domain.
Cole-Hopf transformation

The connection with KPZ growth follows from the famous Cole-Hopf transformation, \( Z = e^{-h} \). \( h(x)_t \) is the free energy of the directed polymer in a specific quenched random field environment. In KPZ language \( h(x)_t \) represents the height profile of an interface (with respect to an (arbitrary) horizontal reference line). Divide the equation of motion by \( Z(x)_t \) and take the logarithm on both sides.

\[
\frac{\partial h(x)}{\partial t} = J(x)_t
\]

with \( \partial h(x)/\partial t = h(x)_{t+1} - h(x)_t \) the discrete time derivative and with current

\[
J(x)_t = V_r \eta(x)_t - V_d \delta(x) + E_v + J_D
\]

\[
J_D = -\ln \left[ 1 + ze^{\rho(x+\frac{1}{2})_t} + ze^{-\rho(x-\frac{1}{2})_t} \right]
\]

The discrete derivative \( \rho(x + \frac{1}{2})_t = -h(x+1)_t + h(x)_t \) represents the local interface slope (apart for a minus sign, introduced for reasons explained below).
continuum limit

Our process belongs to the KPZ universality class, i.e., under coarse graining it renormalizes towards the KPZ equation. The continuum limit is a sloppy way of performing such a renormalization transformation.

\[ J_D \simeq -\log[1 + 2z + z(\frac{\partial \rho}{\partial x} + \rho^2) + \cdots] \]
\[ \simeq -j_0 - \frac{z}{1 + 2z} \left[ \frac{\partial \rho}{\partial x} + \rho^2 + \cdots \right] \]

with \( j_0 = \log(1 + 2z) \). This reduces to the conventional continuum form of the KPZ equation

\[ \frac{\partial h(x)}{\partial t} = E_v - j_0 + V_r \eta(x)_t - V_d \delta(x) + \nu \frac{\partial^2 h}{\partial x^2} + \lambda \left( \frac{\partial h}{\partial x} \right)^2 + \cdots \]

with \( \nu = -\lambda = z/(1 + 2z) \). \( \lambda \) is negative!

We run our MC simulations at \( V_r = 10, z = 1, \) and \( E_v = 0 \).

\[ g = \sqrt{\lambda^2 V_r^2 D/\nu^3} \simeq 10. \]

Naive continuum limit estimates for \( g \) can be deceptive
Burgers Equation and ASEP

The equivalence to the Burgers equation is achieved by reinterpreting the local slope of the interface profile \( h(x)_t \) as a velocity field \( \rho = -\partial h/\partial x \), and by taking the spatial derivative

\[
\frac{\partial}{\partial t} \rho = -\frac{\partial}{\partial x} J(x)
\]

or explicitly in discrete formulation

\[
\frac{\partial}{\partial t} \rho(x + \frac{1}{2}) = -J(x + 1)_t + J(x)
\]

The velocity field can be reinterpreted as a particle density (but only in 1+1 dimensions), and turns the Burgers equation into a process of driven stochastic particle transport along a line, similar to the asymmetric exclusion process (ASEP).

\( J(x)_t \) represents the “particle” current though bond \( x \). The particles are driven and flow on average from “left-to-right”, provided \( E_v \) is larger than the definite negative \( J_D \) term. The local defect potential \( V_d \) represents a slow bond in this driven stochastic flow interpretation and gives rise to a traffic jam.
5. Results of our simulations for this model

- test of KPZ scaling at zero defect
- critical point from scaling of KPZ current
- localization length scaling in directed polymer language
- probability distribution profiles and directed flow profiles
a. KPZ scaling in absence of slow bond.

\[
G_\rho(x) = \langle \rho(x_0 + x) \rho(x_0) \rangle
\]

In our process the steps are unfortunately strongly anti-correlated within a distance of order \( x \sim 5 \).

The 1+1 dimensional continuum KPZ equation, and also the ASEP have stationary states that lack any correlations between the steps. In both cases, \( G_\rho(x) \) is zero for all \( x \neq 0 \).
temporal scaling of the moments

The slope of the curves for Fig.4 increases with system size, due to the finite size scaling effect. Simple fits yield: $2\beta = 0.625 \pm 0.05$ for $L = 16385$, $2\beta = 0.613 \pm 0.05$ for $L = 8193$, $2\beta = 0.600 \pm 0.05$ for $L = 4097$, and $2\beta = 0.585 \pm 0.05$ for $L = 2049$.

Empirically, the straight line fits for $\beta$ obey roughly

$$\beta(L) \approx \frac{1}{3} \left[ 1 - 0.052 \left( \frac{1000}{L} \right)^{0.32} \right]$$
\[ M_n = \langle [h(x)] - \langle h \rangle \rangle^n \]

The even moments obey the following scaling relations
\[ M_n(t, L, u_\nu) = b^{n\alpha} M_n(b^{-z} t, b^{-1} L, b^{y_\nu} u_{nu}) \]

with \( b \) the arbitrary scale factor, \( \alpha = 1/2 \) the stationary state roughness exponent and \( z = 3/2 \) the dynamic scaling exponent (\( \beta = \alpha/z = 1/3 \)). \( u_\nu \) is the leading irrelevant scaling field and \( y_\nu \) its scaling exponent. This leading irrelevant operator is the curvature term in the KPZ equation \( \partial^2 h/\partial x^2 \). The operator content of 1+1 dimensional KPZ is known to be simple, and therefore the value of \( y_\nu \) follows from power counting \( \alpha - z = -y_\nu + \alpha - 2 \rightarrow y_\nu = -1/2 \).

\[ M_2 \simeq 2.5L \left[ 1 + 4L^{-1/2} + \cdots \right] \]
\[ M_4 \simeq 3 \left(2.5L\right)^2 \left[ 1 + 7L^{-1/2} + \cdots \right] \]
\[ M_3 \simeq 30 \ L^{1/2} \]
The odd moments are zero in the so-called fixed point (the KPZ equation) by symmetry, but our model (as most processes) lacks that particle-hole type symmetry. The leading skewness operator has exponent $y_{sk} = -1$ (Neergaard and MdN). The odd moments therefore scale as

$$M_n(t, L, u_v) = b^{n\alpha + y_{sk}} M_n(b^{-\xi}t, b^{-1}L, b^{y_{nu}} u_{nu})$$

$$M_2 \approx 2.5L \left[ 1 + 4L^{-1/2} + \cdots \right]$$

$$M_4 \approx 3 \left(2.5L\right)^2 \left[ 1 + 7L^{-1/2} + \cdots \right]$$

$$M_3 \approx 30 \ L^{1/2}$$
b. critical point from KPZ current

[Graph showing current versus defect strength]
If there is a critical point \( r_c \), then we expect the order parameter, \( \Delta^2 = \tilde{J} = J(r) - J(0) \) to obey the scaling form

\[
\Delta(\epsilon, L) = b^{-x}\Delta(b^y\epsilon, b^{-1}L)
\]

such that the order parameter \( \Delta \) vanishes towards \( \epsilon = r - r_c = 0 \) as \( J(\epsilon, L) - J_0(L) \sim \epsilon^{2\beta} \) (with \( \beta = x/y \)). We can both find \( r_c \) and the value of \( \beta \) by plotting \( J(r) - J_0 \) as function of \( r - r_c \) for various trial values for \( r_c \). At the true value of \( r_c \) this line should become a straight line in a log-log plot close to the critical point and its slope is equal to \( 2\beta \). At other trial values for \( r_c \), the curves should veer-off.

The best estimate for the critical point is \( r_c \simeq 0.055 \pm 0.005 \) \( \beta \simeq 1.58 \pm 0.1 \) and \( 2x = 0.82 \pm 0.1 \).

Finally, we check the global scaling function.

\[
\tilde{J}(\epsilon, L) \simeq b^{-2x}S(\epsilon L^y)
\]
L4097 critical point from current

log current difference

log(r-rc)
rc=0.055 x=0.4 y=0.32 current fss scaling function
c. Polymer localization length
\[ \langle x^2 \rangle^{-1} \sim \Delta(\epsilon, L)^2 = b^{-2x} \Delta(b^y \epsilon, b^{-1} L) \]
Locating the critical point, $\epsilon_c$

straightest line on the log-log plot for $\epsilon_c = 0.08 \pm 0.01$
Scaling of the order parameter at the critical point

$\epsilon_c = 0.08$

The straight line indicates $\Delta = 0.95 \pm 0.02$
DPRM data collapse, $1/\langle x^2 \rangle$

Data collapse is passable, but we would like to get closer to the critical point.
7. numerical results for a DP on a triangular lattice at T=0

FIG. 1. (a) $\Delta x(\epsilon,t)$ as a function of $t$ on a discrete triangular structure with $\epsilon=0.00$, 0.01, 0.02, 0.03, 0.035, 0.04, 0.05, 0.06, 0.07, 0.08, and 0.09 from top to bottom. The data points up to $t=10^5$ are for $\epsilon=0.035$. The dashed line has the slope of 2/3, corresponding to $1/z$ of the

FIG. 2. (a) $\Delta u$ as a function of $\epsilon-\epsilon_c$ for arbitrary values $\epsilon_c$ =0.011, 0.016, 0.021, 0.026, and 0.031 from right to left. The most straight line is obtained at $\epsilon_c =0.021$, where $\Delta u \sim (\epsilon-\epsilon_c)^\beta$ with $\beta=2.76$. (b) The plot of $\Delta u$ as a function of $L$ at $\epsilon=0.021$, where the dashed line shows the power-law behavior $\Delta u \sim L^{-\alpha}$ with $\alpha=0.82$. 
8. **Density profile power law tails:**

In all three phases, the density profile has a powerlaw shape near the slow bond.

The details depend on if or how information travels through the slow bond, how information spreads along the chain (the KPZ dynamic exponent \( z = \frac{3}{2} \)), and the group velocity of such fluctuations.
I predicted from a self consistent analytic argument that the directed stochastic flow density profile obeys the forms

$$\rho(y) = \frac{1}{2} + \Delta_b + A y^{-\nu}$$

with:

- $\nu = \frac{1}{2}$ in the faceted phase (when $\Delta_b \neq 0$);
- $\nu = \frac{1}{3}$ for weak slow bonds ($r_c < r < 1$), and
- $\nu = \frac{2}{3}$ for fast bonds ($r > 1$; never facets)

Our ASEP simulations showed agreement with this, but our new DP model do not, probably because of the stronger FSS effects in the latter.
faceted powerlaw profile

The log-log plot of the profile
\[ \Delta(y) \approx \Delta_b + Ay^{-\nu} \]
yields \( \nu \approx \frac{1}{2} \)
stationary state, fluctuations, and group velocity

The stationary ASEP state for periodic boundary conditions is disordered, random, without any correlations, but fluctuations scale in time as \( l \sim t^{1/z} \) with the KPZ dynamic exponent \( z = \frac{3}{2} \), and move with group velocity \( v_g = 1 - 2\rho \) (tilt of KPZ surface).
explanation: an uncorrelated passage process

The powerlaw density tail, $\rho \sim y^{-\nu}$, with exponent $\nu = 1/2$, reflects that the passages of particles through the slow bond (SB) are stochastic uncorrelated events.

The group velocity $v_g$ of fluctuations points away from both sides of the SB. The number of passage fluctuations in the system is therefore proportional to $t_{flight} \sim N_s$.

The passage process is biased. It favors vacancies. The passing probability of particles is reduced by a factor $r$.

This explains why an excess of particles (vacancies) builds up in front (behind) the SB. The total mass of the power law tail $\rho \sim y^{-1/2}$, scales in the macroscopic queued phase as in a random walk, $\delta N \sim N^{1/2}$. 
density profile at and above $r_c$

Above and at $r_c$ the slow bond does not create a macroscopic queue, but only a power law shaped density correction

$$\rho(y) \sim \frac{1}{2} + Ay^{-\nu}$$

$$\nu \sim \frac{1}{3}$$

$0 \leq y \leq N_s/2$ (up to the defect bond)
The group velocity is equal to \( v_g \sim \frac{1}{2} - \rho \sim y^{-\nu} \). The time of flight of a fluctuation, from the slow bond (SB) to the reservoir, scales as \( t_{\text{flight}} \sim \int dy \ y^\nu \sim N_s^{\nu+1} \).

The SB processes vacancies more efficiently than particles. Fluctuations still detach from the SB and passage through it remains uncorrelated, if \( \nu < \frac{1}{2} \).

The excess number of cars in the queue represents again \( t_{\text{flight}} \) uncorrelated events, \( \delta N \sim t_{\text{flight}}^{1/2} \). Assume this is distributed as a power law. Self consistency implies:

\[
\delta N \sim t_{\text{flight}}^{1/2} \sim N_s^{\nu+1} \quad \rightarrow (\nu+1)/2 = -\nu+1 \rightarrow \nu = \frac{1}{3}.
\]

No KPZ properties are used. This result is very general.
density profile exponent $\nu$

Power law fits to the density profile

$$\Delta(y) \sim \Delta_b + Ay^{-\nu}$$

$\nu$ as function of $r$ shows the three distinct phases.
directed polymer model

ASEP
9. A localized or delocalized polymer in the intermediate phase?

Our queued phase represents the strongly localized state. It exists only beyond a critical defect strength \( r_c < 1 \).

The power-law shaped profile that remains for weaker slow bonds, represents a form of weaker localization, a stretched exponential,

\[
\langle W \rangle \sim e^{\frac{\lambda}{2\nu} \langle h \rangle} \sim e^{-Cx^{1-\nu}} \quad \text{with} \quad \nu = 1/3.
\]

But exponentiation of the average is totally invalid.

Our numerical results for the DP model suggests a simple power law form for \( W \), but this remains unresolved until we find a better model (Cole-Hopf for ASEP-?)
c. Polymer localization length

naive guess power law shaped probability distribution with $\nu = 0.075 (5) (?)$
OUTLINE

1. The asymmetric exclusion process (ASEP) for driven stochastic flow and its body-centered solid-on-solid (BCSOS) KPZ type interface growth representation.
2. Summary of the slow bond results by Meesoon Ha, MdN, and Jussi Timonen (PRE 2003).
4. A discrete lattice directed polymer model in a quenched random landscape.
5. Its interpretation in terms of KPZ growth and driven stochastic flow.
7. Numerical results of a different $T=0$ DPRM by Lee & Kim (PRE 2009)
8. The nature of the intermediate phase: Power law shaped traffic jams.
9. What type of (de-) localization in the DPRM?
density profile for fast bonds

A fast bond, $r > 1$, never creates a macroscopic queue; only a power law shaped density correction

\[ \rho(y) \sim \frac{1}{2} - A y^{-\nu} \]

\[ \nu \sim \frac{2}{3} \]
explanation: $\nu = 1/z$ (KPZ dynamic exponent)

The fast bond (FB) depletion power law is caused by fluctuations traveling towards the FB (instead of away) and particles passing more efficiently than vacancies.

For $\nu > z - 1 = \frac{1}{2}$, the time of flight of the center of mass of a fluctuation $t_{\text{flight}} \sim N_s^{\nu+1}$ is longer than the time it takes that same fluctuation to spread over the entire system $t \sim N_s^z$. The exponent $\nu$ cannot be insensitive anymore to the value of $z$.

A fluctuation created at distance $y$ arrives at the FB after $t \sim y^z$, with reduced amplitude (spreading) $A \sim y^{-1/z}$.

If superposition concepts apply, the density deficit at the SB scales as $\delta N \sim \int dy \; y^{-1/z} \sim N_s^{1-1/z} \sim N_s^{-\nu+1}$. 
FIG. 4. Determination of the critical point and critical exponents. (a) Double logarithmic $\Delta_b \sim |\epsilon|^\beta$ type plots of the order parameter with $\epsilon = r_c - r$ at $N_s = 4096$ for various choices of $r_c$. The best straight line is found at $r_c = 0.80(2)$ and with slope $\beta = 1.46(4)$. (b) Double logarithmic plots of $\Delta_b \sim N_s^{-x_d}$ as a function of system size $N_s$ at $r_c = 0.80$. The slope (dashed line) yields $x_d = 0.370(5)$. For clarity we show only the data for $\Delta_b$ obtained from $J$ (squares) and $\Delta_1$ (crosses).

FIG. 5. Data collapse by Eq. (6), i.e., the scaling function of the order parameter using the values $r_c = 0.80$, $x_d = 0.370$, and $\beta = 1.46$ as found in Fig. 4.
FIG. 6. The same data as in Fig. 2 fitted to a scaling form of type $\Delta_b$ ($\Delta_b = \exp[-a(1-r)^b]$), represent a so-called essential-singularity characteristic for a possible infinite-order-type transition with $r_c = 1$. The curves fail to straighten out, indicating that this is a poor fit.
FIG. 2. The order parameter $\Delta_b$ vs the strength $r$ of the special bond at $N_s = 4096$, determined from three different datasets: the average current $J$ (squares), the density $\Delta_1$ at the first site near the reservoir edge (crosses), and three parameter power-law fits to the density profiles near the special bond (circles).

FIG. 3. The order parameter $\Delta_b$ vs the strength $r$ of the slow-bond in the vicinity of the critical point $r_c$, as obtained from the average current dataset.

$$\Delta_b(N_s, \epsilon) = b^{-x} \Delta_b(b^{-1}N_s, b^x \epsilon),$$
FIG. 8. The density profiles in the $r\to\infty$ model at $N_s = 4095$, implemented as a normal chain with uniform hopping probability $p = 1$, but one special double occupancy site in the middle. The dashed line, with slope $\nu = 0.64$, serves as guide to the eye; a slope $\nu = 2/3$ seems too steep.
FIG. 1. Schematic density profiles are shown: for the slow-bond (SB) (a) $r<r_c$ (queued phase) and (b) $r_c\leq r<1$ (nonqueued SB phase) and for the fast bond (FB) (c) $r>1$ (nonqueued FB phase).
The graph shows the relationship between current and $1/L$ with the equation $y = -3.1015x + 6.4643$. The coefficient of determination $R^2 = 1$, indicating a perfect linear fit.