Electrodynamics II: Assignment 8.
Four standard problems.
Not graded.

1. A loose end from lecture where we found plane-wave solutions in conducting media. The wave equation for **E** in Jackson is in the absence of transport currents. Show that if there's finite conductivity, the wave equation acquires a term $\sigma\mu \frac{\partial}{\partial t}\mathbf{E}$. Similarly, show the wave equation for **B** acquires a term $\sigma\mu \frac{\partial}{\partial t}\mathbf{B}$. And hence show that there are plane wave solutions with $k^2 = \varepsilon\mu\omega^2 + i\mu\sigma\omega$.

2. A fun but counter-intuitive problem. Consider a plane wave making an incident angle θ_i on a semi-infinite collissionless ionized gas. If the gas had a definite boundary and a uniform value of the ion density *N* throughout, this would be a simple problem where we could treat the gas as a dielectric with $n < 1$. However, these assumptions are rarely valid, and solving the resulting non-linear wave equations in which the conductivity is a function of position is difficult to deal with. Let's instead assume the index of refraction *n* varies a negligible amount over one wavelength and the ion density *N* slowly increases with increasing depth into the gas.

a. Qualitatively sketch the path in the gas of a plane-wave ray with incident angle θ_i.

b. At each point on the ray's trajectory, the ray makes an angle θ with the normal direction; find θ.

c. Amazingly, the ray "turns around" within the gas. Find the index of refraction $n_{\pi/2}$ at the "turn-around" point of the ray. As a special case, suppose the incident angle θ_i is normal incidence; find the index of refraction *n* and the phase velocity of the ray at the "turn-around" point.

3. In class I mentioned that losses can be accommodated by an imaginary part of the index of refraction. Consider a plane wave in a poor, nearly lossless, conductor incident in the normal direction on a plane conducting surface. Show that the transmitted and reflected amplitudes are the same as those in the non-conducting case except the index of refraction in the conductor can be considered complex. That is, show in the conductor that the amplitudes are $E_{0r} = \left(\frac{1-z}{1+z}\right) E_{0i}$ and $E_{0t} = \left(\frac{2}{1+z}\right) E_{0i}$ with $z=n_0/n_c$ (where n_0 is the index of refraction in the poor conductor, n_c the complex index of refraction in the conductor, and μ in both is μ_0).

4. The conductivity of air containing mobile electrons is given very approximately by $\sigma = -i(Ne^2/\omega m)$ where e is the electron charge and m the electron mass. Find the resulting propagation velocity and hence the index of refraction n. This is related to the previous problem for radio waves bouncing off the ionosphere.