Electrodynamics II: Assignment 7
Due February 28 at 11:00 am in class
(don't submit it to the instructor's mailbox).

3. Dirac's angular momentum decomposition. A volume contains plane waves propagating in vacuum. Show the total angular momentum in the volume is $\iiint \epsilon_0 E_i(\mathbf{r} \times \nabla)\, A_i\, dv + \iiint \epsilon_0 \mathbf{E} \times \mathbf{A}\, dv$. Because the second term doesn't contain **r**, it's independent of the choice of origin and in a sense is the intrinsic spin of the electromagnetic wave. I find this a strange interpretation. Nonetheless, you often find the second term defined as the spin angular momentum

2. A "paraxial beam" is a plane-wave-like beam. However, instead of the plane-wave front extending to infinity, the front is confined to a finite cross section where the transverse size of the beam is many wavelengths. An example of this is a laser beam. Such beams aren't a topic of this course, but Jackson problem 7.28 gives the electric and magnetic fields for a circularly-polarized paraxial beam propagating in the z-direction. The first electric-field term is what you expect for the infinite-extent plane wave. Additional terms arise where the fields slowly fall off at the edge of the beam (hence the spatial derivatives). Notice how the additional terms represent fields that are, counter-intuitively, along the propagation direction. Use this result to show that the angular momentum per length contained in this circular-cross-section beam projected onto the z-axis is $L_z = \pm\frac{U}{\omega}$ where U is the energy in the beam per length. I find this a peculiar result. Jackson problem 7.29 apparently intends you to do this by evaluating first the angular momentum density and finding it only depends on the additional paraxial terms. You then integrate the angular momentum density and energy density within all space. The surprising outcome is that the angular momentum density explicitly requires non-zero paraxial terms, yet the total angular momentum does not. Show also that the other components of the total angular momentum vanish,

3. This coming Wednesday in lecture we'll discuss total internal reflection. An isotropic point light source is embedded within a semi-infinite dielectric with index of refraction n. Find the fraction of the point-source power escaping the dielectric into air.

4. Energy flow with total internal reflection. Also this Wednesday we'll discuss the evanescent surface wave present with total internal reflection. Find the time-averaged Poynting vector of the transmitted evanescent wave parallel and perpendicular to the interface.