Electrodynamics II: Assignment 6
Due February 21 at 11:00 am in class
(don't submit it to the instructor's mailbox).

1. Two nearly monochromatic current densities $\mathbf{J_a}(r,t) = \mathbf{J_a}(r)e^{i\omega t}$ and $\mathbf{J_b}(r,t) = \mathbf{J_b}(r)e^{i\omega t}$ produce corresponding fields **E_a**, **B_a** and **E_b**, **B_b**. In vacuum, for any volume and its corresponding surface, show that
$\mu_0 \iiint (\mathrm{E}_b \cdot \mathbf{J}_a - \mathrm{E}_a \cdot \mathbf{J}_b) dV = \oiint (\mathrm{E}_a \times \mathbf{B}_b - \mathrm{E}_b \times \mathbf{B}_a) \cdot \hat{\mathbf{n}}\, dS$.
This is a "reciprocity theorem" relating a first's source and a second's fields to the first's fields and the second's source. It means that certain properties of systems are unchanged when radiating fields are instead received and *vise versa*. This is discussed in Landau & Lifshitz, *Electrodynamics*. It's also an important topic in any text on radiating or receiving systems, and it's fundamental to circuit theory. This can also be derived from time-reversal invariance. A simple application of this is to embed a voltage source and an ammeter anywhere in an arbitrarily-complicated linear circuit. On swapping the voltage source and ammeter, the ammeter reading is unchanged. Some common electromagnetic systems, typically containing magnetic materials, violate time-reversal invariance, and are called "non-reciprocal systems", such systems are important in the microwave and optical. That time-reversal seems violated in a purely electromagnetic system is a subject beyond this course.

2. Consider the "loaded half-dipole" antenna from the mid-term exam. (a) Find the time-average radiated power. (b) What's the "radiation resistance" of the antenna? The radiation resistance is the real part (that is, the dissipative part, or the "resistive part") of the impedance "looking into" the antenna terminals.

3. Consider a linearly polarized plane wave in free space impinging on a free electron. Use, e.g., the results from problem 3, to find the total cross section of radiation scattering into all angles. Scattering of free electrons is Thomson scattering, arrived-at by different means in Jackson chapter 14.

4. Another loose end from lecture: Duality. (a) Show that the electromagnetic duality transformation (Jackson eqns 6.151-2) indeed leaves Maxwell's equations (Jackson eqn 6.150) in the same form. (b) Show that the duality transformation leaves quadratic forms, for instance $\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}$ form invariant; why is this important?

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