Electrodynamics II: Assignment 2.
Due January 24 at 11:00 am in class
(don’t submit it to the instructor’s mailbox).

1. Work out the in-class examples of problems involving gauge transformations.
a. Show it’s always possible to transform into Coulomb gauge (Jackson eqn 6.21).
b. Show it’s always possible to transform into Lorentz (Lorenz) gauge (Jackson eqn 6.14).
c. Show it’s always possible to transform into Weyl gauge Φ=0.
d. Show it’s always possible to transform into axial gauge $\mathbf{n}\bullet\mathbf{A}$ =0, with constant unit vector **n**.
e. Can there be a gauge with gauge condition **A**=0? Explain.
f. Demonstrate what I asserted in class: The restricted gauge transformations (Jackson eqn 6.20) preserve the Lorentz condition (Jackson eqn 6.14).

2. Consider a concentric spherical capacitor that’s slowly self-discharging. Use the Lorentz condition, then direct integration, to find the magnetic field, if any. We briefly talked about this in class. There are integration constants to be ignored; explain why these constants can be ignored. (This is a standard problem, but I find it troublesome. The slow-discharge condition admits a quasi-static assumption, but the Lorentz condition contains a time derivative which is non-zero; hence the quasi-static assumption isn’t consistently applied.) (There are also many other ways to attack this problem. You could, for instance, show that the total current--true current plus displacement current--vanishes, so there’s no source of field.)

3. In class I stated you can all but ignore displacement current in a good conductor. Suppose there's a harmonic electric field given by the real part of $\mathbf{E}_0 e^{i\omega t}$ in a material of conductivity σ. Find the ratio of the magnitudes of the true and displacement currents. For high purity copper, find the frequency where the two current magnitudes are equal. A couple of comments: You may need the permittivity of the conductor. It turns out to be conceptually difficult to measure it for a good conductor. But you can assume it's not too far from its vacuum value. Also, the two currents are not in phase, as we'll discover later.

4. A sphere centered on the origin contains a uniform charge density ρ. The sphere is then spun up to angular velocity ω. (a) Find the vector potential at a point **r** outside the sphere. (b) Thereby find the resulting magnetic induction **B**. In working out part (a), for one troublesome integral I required help from Mathematica.