

# Electrodynamics II: Assignment 1.

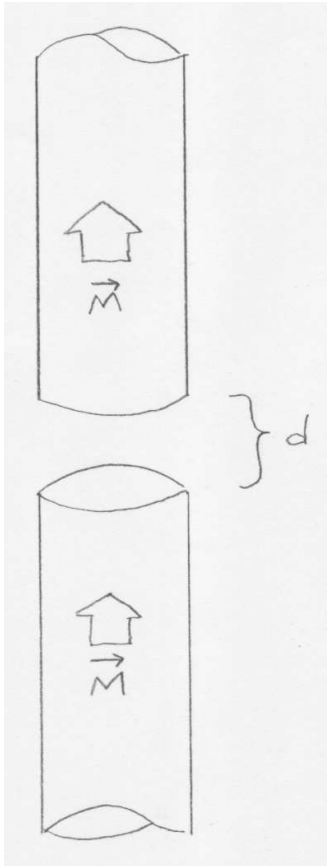
**Due January 17 at 11:00 am in class  
(don't submit it to the instructor's mailbox).**

1. Space charge. Consider two closely-spaced (so you can ignore fringe fields) parallel conducting plates in vacuum. The plates are maintained at a potential difference  $\Phi_0$ . Suppose the work function of the electrons in the plates is zero (a condition obtained, say, by heating the plates). What's the current density between the plates?

2. Recall we introduced a magnetic scalar potential  $\mathbf{B} = -\mu_0 \nabla \Phi_m$  where the scalar potential had a geometric interpretation  $\Phi_m = \frac{I}{4\pi} d\Omega$  representing the change in the solid angle subtended by a current loop as seen at a field point due to a small translation of the loop. Suppose the loop is a circle of radius  $R$  carrying current  $I$ . Use this formalism to find the magnetic field along the loop axis.

3. Two semi-infinite permanent magnets of radii  $R$  are coaxially aligned so as to have a gap  $d$ , as shown.

- Using equivalent magnetization currents, find the magnetic induction  $\mathbf{B}$  at the center point of the gap. Express this field in terms of the opening angle, relative to the axis,  $\theta_0$  of the edge of the pole face as seen at the field point at the center of the gap.
- Using equivalent "magnetic charge" as the source of magnetic scalar potential, Jackson eqn 5.95, find the magnetic induction  $\mathbf{B}$  at the center point of the gap. From our discussion in class, this is a good application of magnetic scalar potential since there are no true currents.



4. Consider a permeable (with permeability  $\mu$ ) conducting infinitely-long cylinder of radius  $R$  carrying a uniform current  $\mathbf{J}$  in the axial direction. In addition, there's a uniform external field  $\mathbf{B}_0$  at right angles to the cylinder's axis. Find the vector potential everywhere. This problem is similar to the dielectric cylinder in an external  $\mathbf{E}_0$  field from electrostatics. Some thoughts on the expansion solution: Because of the symmetry and 2D nature of the problem, Laplace's and Poisson's equations for  $\mathbf{A}$  are relatively simple. As usual, there are separate solutions inside and outside the cylinder and you'll match boundary conditions to find the expansion coefficients. A subtlety: You should include the particular solution of the vector potential outside a wire  $-\mu J \frac{r^2}{4}$  to solutions of Laplace's equation inside the cylinder. To the potential outside the cylinder, you should add the term corresponding the uniform applied field. Also, note the potential at points along the axis is non-singular.

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