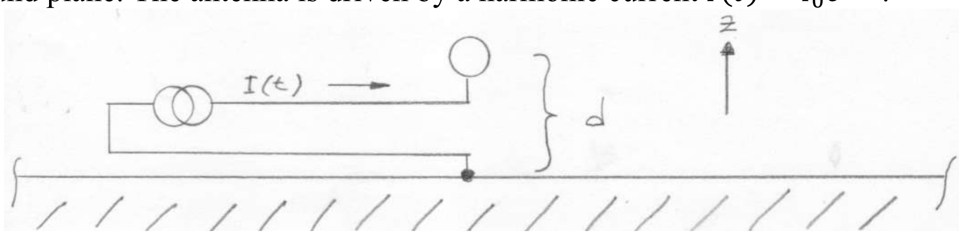
I. (40 points) Hertz vector (polarization potential) formalism.

As shown, the end of a "loaded half-dipole" antenna is located a distance d above a ground plane. The antenna is driven by a harmonic current $I(t) = I_0 e^{i\omega t}$.



a. Fields via Hertz vector. (10 points) Show, for field points above the ground plane far from the antenna, fields can be expressed in terms of the electric Hertz vector as

$$\mathbf{E} = \nabla \times (\nabla \times \Pi_e) \quad \text{and} \quad \mathbf{B} = \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \times \Pi_e)$$

b. Dipole moment. (10 points) Find the electric dipole moment of the antenna.

c. Hertz vector. (10 points) For field points above the ground plane far from the antenna, find the electric Hertz vector for the antenna.

d. Radiation fields. (10 Points) From the Hertz-vector formalism, find the radiation fields above the ground plane far from the antenna. (As we briefly discussed in class, for a "loaded" antenna, like this, where the size of the loading element is at least comparable to the antenna size, the radiation is dominated by the charge build-up at the ends.)