FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$	(permeability of free space)
$c = 3.00 \times 10^{8}\,\mathrm{m}/s$	(speed of light)
$e = 1.60 \times 10^{-19}\,\mathrm{C}$	(charge of the electron)
$m = 9.11 \times 10^{-31}\,\mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Larmor formula: $P = \frac{\mu_0}{6\pi c} q^2 a^2$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem: $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}};\quad d\tau = dx\,dy\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}};\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl: $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}};\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

$\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{r}$ Coulomb's Law, $\mathbf{F} = Q\mathbf{E} + Q\mathbf{v}\times\mathbf{B}$ Lorentz force

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\iiint \dfrac{\rho(\mathbf{r}')}{r^2}\hat{r}\, d\tau'$

$\oiint \mathbf{E}\bullet d\mathbf{a} = \dfrac{1}{\varepsilon_0}Q_{\text{encl}}$ Gauss' Law (integral) $\oint \mathbf{E}\bullet d\mathbf{l} = 0$ (statics)

$\Phi_{ab} = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$ Electrostatic potential, and $\mathbf{E} = -\boldsymbol{\nabla\Phi}$ (statics)

$\boldsymbol{\Phi}(\mathbf{r}) = \frac{\mathbf{1}}{\mathbf{4}\boldsymbol{\pi\epsilon_0}}\frac{\boldsymbol{q}}{\boldsymbol{r}}$ and $\boldsymbol{\Phi}(\mathbf{r}) = \frac{\mathbf{1}}{\mathbf{4}\boldsymbol{\pi\epsilon_0}}\iiint \frac{\boldsymbol{\rho}(\mathbf{r}')}{\boldsymbol{r}}d\tau'$

$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$ and $\frac{\partial\Phi_{\text{above}}}{\partial n} - \frac{\partial\Phi_{\text{below}}}{\partial n} = \frac{1}{\varepsilon_0}\sigma$ boundary conditions

$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$

$\Phi(b) - \Phi(a) = \frac{W}{q}$ $W = \frac{1}{2}\sum_i q_i\Phi_i$ $W = \frac{1}{2}\iiint \rho\Phi\, d\tau$

$Q = CV$ $W = \dfrac{1}{2}CV^2$ $W = \dfrac{1}{2}Q^2/C$ capacitors

$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}$ Poisson's equation $\nabla^2\Phi = 0$ Laplace's equation

$\nabla'^2 G(\mathbf{r},\mathbf{r}') = -4\pi\delta(\mathbf{r}-\mathbf{r}')$

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\iiint \rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d^3x' + \frac{1}{4\pi}\oiint \left[G(\mathbf{x},\mathbf{x}')\frac{\partial\Phi}{\partial n'} - \Phi(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial n'}\right]da'$$

$\Phi(x,y) = \sum_n (a_n e^{+kx} + b_n e^{-kx})(c_n \sin ky + d_n \cos ky)$ A solution to Laplace's equation in rectangular coordinates in two dimensions

$\int_0^a \sin(n\pi\frac{y}{a})\sin(n'\pi\frac{y}{a})dy = \begin{cases} 0 & n \neq n' \\ \dfrac{a}{2} & n = n' \end{cases}$ orthogonality, e.g., of sines

$\int_0^a \sin(n\pi\frac{y}{a})dy = \begin{cases} 0 & \text{n even} \\ \dfrac{2a}{n\pi} & \text{n odd} \end{cases}$ $e^{+kx} + e^{-kx} = 2\cosh kx$

$\int_0^\pi \cos^2\theta\sin\theta\, d\theta = 2/3$

$P_0(x)$	$= 1$
$P_1(x)$	$= x$
$P_2(x)$	$= (3x^2 - 1)/2$
$P_3(x)$	$= (5x^3 - 3x)/2$
$P_4(x)$	$= (35x^4 - 30x^2 + 3)/8$
$P_5(x)$	$= (63x^5 - 70x^3 + 15x)/8$

TABLE 3.1 Legendre Polynomials.

$\Phi(r,\theta) = \sum_l (a_l r^l + \frac{b_l}{r^{l+1}}) P_l(\cos\theta)$

A solution to Laplace's equation in spherical coordinates with azimuthal symmetry

$\int_{-1}^{+1} P_{l'}(x) P_l(x)\, dx = \frac{2}{2l+1}\delta_{ll'}$ orthogonality

$\frac{1}{R} = \frac{1}{r}\sum_n \left(\frac{r'}{r}\right)^n P_n(\cos\theta)$ 1/R expansion azimuthal symmetry

Expansion into spherical harmonics $\Phi(r,\theta,\phi) = \sum_{\ell,m}\left[a_{\ell,m} r^\ell + \frac{b_{\ell,m}}{r^{\ell+1}}\right] Y_{\ell,m}(\theta,\phi)$

$\frac{1}{R} = 4\pi \sum_{\ell,m} \frac{1}{2\ell+1}\frac{r_<^\ell}{r_>^{\ell+1}} Y^*_{\ell,m}(\theta',\phi') Y_{\ell,m}(\theta,\phi)$ 1/R expansion spherical harmonics

Expansion into Bessel regular functions

$$\Phi(\rho,\phi,z) = \sum_{m,n} J_m(k_{m,n}\rho) \times \left(a_{m,n} e^{+k_{m,n}z} + b_{m,n} e^{-k_{m,n}z}\right) \times \left(c_{m,n} e^{+im\phi} + d_{m,n} e^{-im\phi}\right)$$

Orthogonality $\int_0^a \rho J_\nu\left(x_{\nu n'}\frac{\rho}{a}\right) J_\nu\left(x_{\nu n}\frac{\rho}{a}\right) d\rho = \frac{a^2}{2}[J_{\nu+1}(x_{\nu n})]^2 \delta_{nn'}$

Multipole expansion

$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{\ell m}\frac{4\pi}{2\ell+1} q_{\ell m}\frac{Y_{\ell m}(\theta,\phi)}{r^{\ell+1}}$ $\quad q_{\ell m} = \iiint Y^*_{\ell m}(\theta',\phi') r'^\ell \rho(\mathbf{r}')\, dv'$

$W = \frac{1}{2}\iiint \rho(\mathbf{r})\,\Phi(\mathbf{r})\, dv$ $\quad W = \frac{\epsilon_0}{2}\iiint E^2\, dv$ $\quad W = \frac{1}{2}\iiint \mathbf{E}\cdot\mathbf{D}\, dv$

Boundary conditions $(\mathbf{D}_2 - \mathbf{D}_1)\cdot\mathbf{n}_{21} = \sigma$ $\quad (\mathbf{E}_2 - \mathbf{E}_1)\times\mathbf{n}_{21} = 0$

$\mathbf{F}_{\text{mag}} = \int I\, d\mathbf{l}\times\mathbf{B}$

$\mathbf{K} = d\mathbf{I}/d\ell_\perp$ $\quad \mathbf{J} = d\mathbf{I}/da_\perp$ surface and volume currents

$\nabla\bullet\mathbf{J} + \frac{d\rho}{dt} = 0$ conserved current

$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I\int\frac{d\mathbf{l}'\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2}$ $\quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} da'$ $\quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I\int\frac{\mathbf{J}\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} d\tau'$

$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{\text{enc}}$

$\nabla \bullet \mathbf{A} = 0$ "Coulomb gauge" convention $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ (for Coulomb gauge)

$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau'$ $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da'$ $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}'$

$\oint \mathbf{A} \bullet d\mathbf{l} = \Phi_m$

$\mathbf{A}_{\text{above}} - \mathbf{A}_{\text{below}} = 0$ $\frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$

$\mathbf{m} = I \iint \hat{n}\, da = I\mathbf{a}$

$\mathbf{m} = \frac{1}{2} \oint \mathbf{r} \times \mathbf{I}\, d\ell$ $\mathbf{m} = \frac{1}{2} \iint \mathbf{r} \times \mathbf{K}\, da$ $\mathbf{m} = \frac{1}{2} \iiint \mathbf{r} \times \mathbf{J}\, d\tau$

$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{r}}{r^2}$

$\mathbf{N} = \mathbf{m} \times \mathbf{B}$ $\mathbf{F} = \nabla(\mathbf{m} \bullet \mathbf{B})$ $U = -\mathbf{m} \bullet \mathbf{B}$

$\mathbf{J}_b = \nabla \times \mathbf{M}$ $\mathbf{K}_b = \mathbf{M} \times \hat{n}$ $\rho_m = -\nabla \cdot \mathbf{M}$ $\sigma_m = \mathbf{n} \cdot \mathbf{M}$

$\nabla \times \mathbf{H} = \mathbf{J}_f$ $\oint \mathbf{H} \bullet d\mathbf{l} = I_{\text{f,encl}}$

$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0$ $B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}} = \mu_0 K$ $\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{n}$

$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$ $\mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{n}$

$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$ $\mathbf{M} = \chi_m \mathbf{H}$ $\mathbf{B} = \mu \mathbf{H}$

$U = \frac{1}{2} \iiint \mathbf{H} \cdot \mathbf{B}\, dv$ $U = \frac{1}{2} \iiint \mathbf{D} \cdot \mathbf{E}\, dv$ $U = -\mathbf{m} \cdot \mathbf{B} - \mathbf{p} \cdot \mathbf{E}$

$\Phi_b = M_{ba} I_a$ $M_{ba} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_a \cdot dl_b}{r}$ $\mathcal{E}_b = -M_{ba} \frac{dI_a}{dt}$ $\Phi = LI$ $\mathcal{E} = -L \frac{dI}{dt}$

Non-statics:

$\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{E} = -\nabla\Phi - \frac{d\mathbf{A}}{dt}$ $\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\Lambda$ $\Phi \to \Phi' - \frac{d\Lambda}{dt}$

$\nabla^2\Phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\rho/\varepsilon_0$ $\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{dt^2} - \nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{d\Phi}{dt}\right) = -\mu_0 \mathbf{J}$

Lorentz (Lorenz) gauge:

$\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{d\Phi}{dt} = 0$ $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\rho/\varepsilon_0$ $\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{dt^2} = -\mu_0 \mathbf{J}$

Coulomb (transverse, radiation) gauge:

$\nabla \cdot \mathbf{A} = 0$ $\nabla^2\Phi = -\rho/\varepsilon_0$ $\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{dt^2} = -\mu_0 \mathbf{J} + \frac{1}{c^2}\nabla\frac{\partial\Phi}{\partial t}$

Retarded solutions:

$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = -4\pi f$ has solution $\Psi(\mathbf{r}, t) = \iiint \frac{[f(\mathbf{r}', t')]_{ret}}{\mathbf{r} - \mathbf{r}'} dv'$

Polarization vectors (in vacuum; note consistent units):

$$\nabla^2 \mathbf{\Pi_e} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{\Pi_e} = -\frac{\mathbf{P}}{\epsilon_0} \qquad \nabla^2 \mathbf{\Pi_m} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{\Pi_m} = -\mu_0 \mathbf{M}$$

$$\mathbf{A} = \frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{\Pi_e} + \nabla \times \mathbf{\Pi_m} \qquad \Phi = -\nabla \cdot \mathbf{\Pi_e}$$

Poynting's formalism:

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E} \qquad \mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Momentum density $\mathbf{g} = \frac{1}{c^2}\mathbf{E} \times \mathbf{H}$

Angular momentum density $\mathcal{L} = \frac{1}{c^2}\mathbf{r} \times (\mathbf{E} \times \mathbf{H})$