Electrodynamics II: Assignment 5.
Due February 22 at 11:00am in class or 10:45am in the instructor's mailbox.

1. A loose end from lecture. The wave equation for **E** in Jackson is in the absence of transport currents. Show that if there is finite conductivity, the wave equation acquires a term $\sigma\mu\frac{\partial}{\partial t}\mathbf{E}$. Similarly, show the wave equation for **B** acquires a term $\sigma\mu\frac{\partial}{\partial t}\mathbf{B}$. Show that there are plane wave solutions with $k^2 = \varepsilon\mu\omega^2 + i\mu\sigma\omega$.

2. It's usual to find the classical Thomson scattering cross section (Jackson eqn. 14.126) in the "electron theory" by finding the acceleration of a free (unbound) electron due to an external electric field (Jackson eqn 7.49), then finding the radiated power due to radiation fields from the electron's acceleration. Instead, find the Thomson cross section in "electron theory" from the electron's induced dipole moment. Hints: (1) You might recall the result of homework #3, problem 2. (2) The "electron radius" in Jackson eqn 14.126 is in Gaussian units; in MKSA units it's $e^2/4\pi\varepsilon_0 mc^2$. (In quantum mechanics, the differential Thomson cross section acquires a forward-backward asymmetry and the frequency of the scattered photons depends on the scattering angle. See the discussion of the Klein-Nishina formula in Heitler "The quantum theory of radiation".)

3. Consider a plane wave making an incident angle θ_i on a semi-infinite collissionless ionized gas. If the gas had a definite boundary and a uniform value of the ion density N throughout, this would be a simple problem where we could treat the gas a dielectric with $n < 1$. However, these assumptions are rarely valid, and solving the resulting non-linear wave equations in which the conductivity is a function of position is difficult. Let's instead assume the index of refraction n varies a negligible amount over one wavelength and the ion density N slowly increases with increasing depth into the gas.
a. Qualitatively sketch the path in the gas of a plane-wave ray with incident angle θ_i.

b. At each point on the ray's trajectory, the ray makes an angle θ with the normal direction; find θ.
c. Find the index of refraction $n_{\pi/2}$ at the "turn-around" point of the ray within the gas. As a special case, suppose the incident angle θ_i is normal incidence; find the index of refraction n and the phase velocity of the ray at the "turn-around" point within the gas.

4. Simple waveguide. Consider the parallel-plate waveguide in vacuum we discussed in class. It consists of two parallel conductors separated by a distance *d*, with the width of the waveguide $w \gg d$ (no fringing fields, etc.).
a. Find the characteristic impedance of the waveguide for the TEM mode (the ratio of the voltage amplitude between the conductors to the current amplitude down the guide).
b. For the TEM mode, find the phase velocity.
c. For the TM modes, find the cutoff frequencies.
d. For the TM modes, find the wave impedance (the ratio of the amplitude of **E** to the amplitude of **H**).
e. For the TM modes, find the phase velocity.
f. For the TM modes, find the guided wavelength λ_g.