Electrodynamics II: Assignment 2.
Due January 25 at 11:00am in class or 10:45am in the instructor's mailbox.

1. Two nearly monochromatic current densities $\mathrm{J_a}(r,t) = \mathrm{J_a}(r)e^{i\omega t}$ and $\mathrm{J_b}(r,t) = \mathrm{J_b}(r)e^{i\omega t}$ produce corresponding fields $\mathbf{E_a}$, $\mathbf{B_a}$ and $\mathbf{E_b}$, $\mathbf{B_b}$. In vacuum, for any volume and its corresponding surface, show that
$\mu_0 \iiint (\mathrm{E}_b \cdot \mathrm{J}_a - \mathrm{E}_a \cdot \mathrm{J}_b) dV = \oiint (\mathrm{E}_a \times \mathbf{B}_b - \mathrm{E}_b \times \mathbf{B}_a) \cdot \hat{\mathbf{n}}\, dS$.
This is a reciprocity theorem relating a first's source and a second's fields to the first's fields and the second's source. It means that certain properties of systems are unchanged when radiating fields are received and or receiving fields are radiated. This is discussed in Landau & Lifshitz, *Electrodynamics*. It's also an important topic in any text on radiating or receiving systems. This can also be derived from time-reversal invariance. A very simple application of this is to embed a voltage source and an ammeter anywhere in a circuit. On swapping the voltage source and ammeter, the ammeter reading is unchanged. Some common electromagnetic systems, typically containing magnetic materials, violate time-reversal invariance, and are called "non-reciprocal systems".

2. Show that the retarded potentials (Jackson eqns 6.48) obey the Lorentz (Lorenz) condition (Jackson eqn 6.14). This is a problem in Jackson and many other texts. They typically start by deriving the identity $\nabla \cdot \left(\frac{\mathrm{J}}{\mathrm{R}}\right) = \frac{1}{\mathrm{R}}(\nabla \cdot \mathrm{J}) + \frac{1}{\mathrm{R}}(\nabla' \cdot \mathrm{J}) - \nabla' \cdot \left(\frac{\mathrm{J}}{\mathrm{R}}\right)$**,** where **R**=**r**'-**r** and ∇ and ∇' act on **r** and **r**' respectively. This leads to an expression for $\nabla \cdot \mathbf{A}$**.**

4. Before quantum theory, some pictured the electron's structure as a sphere of radius R on which a total charge e is uniformly pasted. (This picture requires a new force to hold the sphere together.)
a. For what radius R is the field energy equal to the electron mass m? You might calculate what radius you get if you use the actual electron mass.
b. Suppose the electron moves at a constant non-relativistic velocity. Qualitatively describe the magnetic field. Is it sensible in this model to ascribe the magnetic field energy to the electron's kinetic energy?
c. For the electron in (b), what in general is the direction of the Poynting vector? Does this make sense?

4. We don't usually think of magnetic currents since there's no magnetic charge. But, for magnetized materials, we could think of effective magnetic charge (Jackson 5.96). Hence, for magnetic materials, there are effective currents and therefore an effective electric vector potential **F** that when acted on by the curl, gives the electric field. Suppose a magnetization $\mathbf{M}_\omega(\mathbf{r})$ is oscillating at a nearly monochromatic frequency ω. In terms of **M**, find an expression for the electric vector potential. (This is a problem in development.)