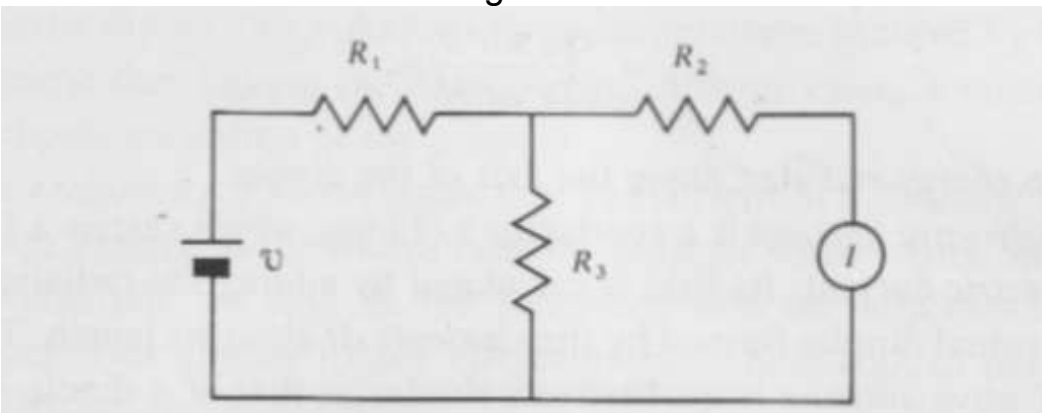
Physics 514, Winter Quarter 2018
Electrodynamics: Homework Assignment 8
Due March 2, either 11:00am in class or 10:45am in the instructor's mailbox.

1. Magnetic-dipole radiation, done in several texts. In class we discussed radiation from a small electric-dipole antenna. Now consider a loop of radius R (with $R \ll$ the distance to the field point and the free-space wavelength) carrying harmonic current $I_0 e^{i\omega t}$.
a. Find the vector potential **A** at the field point and verify it reduces to the static-limit at zero frequency.
b. Now find the corresponding **E** and **H** fields.
c. Find the time-average radiated power.
d. Find the radiation resistance (notice the high power of the dimensionless length).

2. The reciprocity theorem states: Given two antennas *a* and *b*, $I_{ab}/V_a = I_{ba}/V_b$, where I_{ab} is the current induced in antenna *a* due to antenna *b*, I_{ba} is the current induced in antenna *b* due to antenna *a*, and V_a and V_b are the voltages applied to antennas *a* and *b*.
a. This is a general theorem: Show it's true for the equivalent circuit where the ammeter and voltage source are *a* and *b*.



b. (challenge problem, done in several texts) Derive this theorem. You may want to start with the identity

$$\nabla \cdot (E_a \times H_b - E_b \times H_a)$$
$$= H_b \cdot \nabla \times E_a - E_a \cdot \nabla \times H_b - H_a \cdot \nabla \times E_b + E_b \cdot \nabla \times H_a$$

3. Consider the lowest TM mode of a cylindrical resonator of radius R and length L. A small conducting "dimple" of volume ΔV projects into the cavity from the cavity end-wall axially at the bottom-center of the cavity. Find the shift in resonant frequency due to this "dimple". For the math: Integrals containing rJ_0^2 often become J_1^2.

4. Consider a thin infinite sheet carrying surface current $K_0 e^{i\omega t}$.
a. Find the time-average power radiated per unit area from the sheet.
b. Find the radiation resistance per unit area for the current-sheet antenna.

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