Physics 514, Winter Quarter 2018
Electrodynamics: Homework Assignment 5
Revised due date: Feb. 12, by 11:00am in the instructor's mailbox.

1. An isotropic point light source is embedded within a semi-infinite dielectric characterized by index of refraction n. Find the fraction of the point-source power escaping the dielectric. (This is meant to be a simple problem: ignore the reflections when there's accompanying transmission through the boundary.)

2. It's usual to find the classical Thomson scattering cross section (Jackson eqn. 14.126) in the "electron theory" by finding the acceleration of a free (unbound) electron due to an external electric field (Jackson eqn 7.49), then finding the radiated power due to radiation fields from the electron's acceleration. Instead, find the Thomson cross section in "electron theory" from the electron's induced dipole moment. Hints: (1) You might recall the result of homework #3, problem 1. (2) The "electron radius" in Jackson eqn 14.126 is in Gaussian units; in MKSA units it's $e^2/4\pi\varepsilon_0 mc^2$. (In quantum mechanics, the differential Thomson cross section acquires a forward-backward asymmetry and the frequency of the scattered photons depends on the scattering angle. See the discussion of the Klein-Nishina formula in Heitler "The quantum theory of radiation".)

3. Reflection, refraction and topological insulators. Certain properties of topological insulators result from the constitutive relations $\mathbf{D}=\varepsilon\mathbf{E}-(\alpha/Z_0)\mathbf{B}$ and $\mathbf{H}=\mathbf{B}/\mu+(\alpha/Z_0)\mathbf{E}$, where α is the fine-structure constant and Z_0 is the impedance of free space.

a. In a source-free region, show that the **E** and **B** field solutions to Maxwell's equations are the usual transverse plane waves.

b. If a linearly-polarized plane wave in vacuum is normally incident on this topological insulator, find the angle by which the transmitted wave is rotated in its polarization.

c. Similarly find the angle by which the reflected wave is rotated in its polarization.

(For more details, see, e.g., Qi, Hughes and Zhang, “Topological Field Theory of Time-Reversal Invariant Insulators,” Phys.Rev. B78 (2008) 195424; erratum Phys.Rev. B81 (2010) 159901.)

4. Consider a plane wave making an incident angle θ_i on a semi-infinite collissionless ionized gas. If the gas had a definite boundary and a uniform value of the ion density N throughout, this would be a simple problem where we could treat the gas a dielectric with $n < 1$. However, these assumptions are rarely valid, and solving the resulting non-linear wave equations in which the conductivity is a function of position is difficult. Let’s instead assume the index of refraction n varies a negligible amount over one wavelength and the ion density N slowly increases with increasing depth into the gas.
a. Qualitatively sketch the path in the gas of a plane-wave ray with incident angle θ_i.
b. At each point on the ray’s trajectory, the ray makes an angle θ with the normal direction; find θ.
c. Find the index of refraction $n_{\pi/2}$ at the “turn-around” point of the ray within the gas. As a special case, suppose the incident angle θ_i is normal incidence; find the index of refraction n and the phase velocity of the ray at the “turn-around” point within the gas.