Physics 514, Winter Quarter 2018
Electrodynamics: Homework Assignment 2
Due Jan. 19, 5:00pm either 11:00am in class or 10:45am in the instructor's mailbox.

1. Show that the retarded potentials (Jackson eqns 6.48) obey the Lorentz (Lorenz) condition (Jackson eqn 6.14). This is a problem in Jackson and a challenge problem in several undergraduate texts. They derive the identity $\nabla\cdot\left(\frac{\mathrm{J}}{\mathrm{R}}\right)=\frac{1}{\mathrm{R}}(\nabla\cdot\mathrm{J})+\frac{1}{\mathrm{R}}(\nabla'\cdot\mathrm{J})-\nabla'\cdot\left(\frac{\mathrm{J}}{\mathrm{R}}\right)$, where **R**=**r**'-**r** and ∇ and ∇' act on **r** and **r**' respectively. This leads to an expression for $\nabla\cdot\mathbf{A}$.

2. A point in space has zero charge for times t < 0 and charge Q for times t > 0.
a. Find the electric field from Gauss's Law. The answer seems to violate elementary notions of causality; explain the source of this. Now apply symmetry to also find the magnetic field.
b. Now find electric and magnetic fields from potentials. The Coulomb gauge seems sensible for this problem, where the potentials are those of Jackson eqn 6.23 and the retarded potential from the solution of Jackson eqn 6.30.
The (non-physical) sudden appearance of a point charge is the subject of a discussion in Peter Berman, *Am. J. Physics* **76** (2008) 48.

3. Two nearly monochromatic current densities $\mathrm{J_a}(r,t)=\mathrm{J_a}(r)e^{i\omega t}$ and $\mathrm{J_b}(r,t)=\mathrm{J_b}(r)e^{i\omega t}$ produce corresponding fields $\mathbf{E_a}$, $\mathbf{B_a}$ and $\mathbf{E_b}$, $\mathbf{B_b}$. For any volume and its surface, show that
$$\mu_0\iiint(\mathrm{E}_b\cdot\mathrm{J}_a-\mathrm{E}_a\cdot\mathrm{J}_b)dV=\oiint(\mathrm{E}_a\times\mathbf{B}_b-\mathrm{E}_b\times\mathbf{B}_a)\cdot\hat{\mathbf{n}}dS\ .$$
This is a reciprocity theorem that relates a first's source and a second's fields to the first's fields and the second's source. This is important: it means that certain properties of systems are unchanged when radiating fields or receiving fields. This is discussed in Landau & Lifshitz, *Electrodynamics*. It is also an important topic in any text on radiating or receiving systems. This can also be derived from time-

reversal invariance. Some common electromagnetic systems violate time-reversal invariance, and are called “non-reciprocal systems”.

4. Before quantum theory, some thought the electron’s structure was a sphere of radius R to which a total charge *e* is uniformly pasted. (This theory of course requires a new force to hold the sphere together.)
a. For what radius R is the field energy equal to the electron mass m? You might calculate what radius you get if you use the actual electron mass.
b. Suppose the electron moves at a constant non-relativistic velocity. Qualitatively describe the magnetic field. Is it sensible in this model to ascribe the magnetic field energy to the electron’s kinetic energy?
c. For the electron in (b), what in general is the direction of the Poynting vector? Does this make sense?