Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\,\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t = \frac{\partial t}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\,\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl: $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\,\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$	(permeability of free space)
$c = 3.00 \times 10^{8}\ \mathrm{m}/s$	(speed of light)
$e = 1.60 \times 10^{-19}\ \mathrm{C}$	(charge of the electron)
$m = 9.11 \times 10^{-31}\ \mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{r}$ Coulomb's Law $\quad \mathbf{F} = Q\mathbf{E} + Q\mathbf{v}\times\mathbf{B}$ Lorentz force

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\iiint \dfrac{\rho(\mathbf{r}')}{r^2}\hat{r}\, d\tau'$ electric field from continuous charge distribution

$\oint\!\!\oint \mathbf{E}\bullet d\mathbf{a} = \dfrac{1}{\varepsilon_0}Q_{\text{encl}}$ Gauss' Law (integral) $\quad \oint \mathbf{E}\bullet d\mathbf{l} = 0$ (statics)

$V(\mathbf{r}) = -\int_{\wp}^{\mathbf{r}} \mathbf{E} \bullet d\mathbf{l}$ Electrostatic potential, and $\mathbf{E} = -\nabla V$ (statics)

$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ and $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')}{r} d\tau'$ (setting reference point at infinity)

$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$ and $\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\varepsilon_0}\sigma$ boundary conditions

$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$

$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q}$ $\quad W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\mathbf{r}_i)$ and $W = \frac{1}{2}\iiint \rho V \, d\tau$

$Q = CV \quad W = \frac{1}{2}CV^2 \quad W = \frac{1}{2}Q^2 / C$ capacitors

$\nabla^2 V = -\frac{1}{\varepsilon_0}\rho$ Poisson's equation $\quad \nabla^2 V = 0$ Laplace's equation

$V(x,y) = \sum_{n=0}^{\infty}(A_n e^{+kx} + B_n e^{-kx})(C_n \sin ky + D_n \cos ky)$ solution to Laplace's equation in Cartesian coordinates in two dimensions

$$\int_0^a \sin(n\pi \frac{y}{a})\sin(n'\pi \frac{y}{a})\,dy = \begin{cases} 0 & n \neq n' \\ \frac{a}{2} & n = n' \end{cases}$$ orthogonality of sines

$$\int_0^a \sin(n\pi \frac{y}{a})\,dy = \begin{cases} 0 & \text{n even} \\ \frac{2a}{n\pi} & \text{n odd} \end{cases} \qquad e^{+kx} + e^{-kx} = 2\cosh kx$$

$$\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = 2/3$$

$P_0(x)$	$= 1$
$P_1(x)$	$= x$
$P_2(x)$	$= (3x^2 - 1)/2$
$P_3(x)$	$= (5x^3 - 3x)/2$
$P_4(x)$	$= (35x^4 - 30x^2 + 3)/8$
$P_5(x)$	$= (63x^5 - 70x^3 + 15x)/8$

TABLE 3.1 Legendre Polynomials.

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

solution to Laplace's equation in spherical coordinates with azimuthal symmetry

$$\int_0^\pi P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\,d\cos\theta = \begin{cases} 0 & \ell \neq \ell' \\ \dfrac{2}{2\ell+1} & \ell = \ell' \end{cases}$$ orthogonality of Legendre polynomials

$$V(r,\theta) = -E_0\left(r - \frac{R^3}{r^2}\right)\cos\theta$$ potential outside a neutral conducting sphere in uniform field

$$\frac{1}{\Re} = \frac{1}{r}\sum_0^\infty \left(\frac{r'}{r}\right)^n P_n(\cos\alpha)$$ 1/r expansion in Legendre polynomials

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\iiint (r')^n P_n(\cos\alpha)\rho(\mathbf{r}')d\tau'$$ multipole expansion

$$\vec{p} = \iiint \vec{r}'\rho(\vec{r}')d\tau' \quad \vec{p} = \sum_1^n q_i\vec{r}_i' \quad V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\bullet\hat{r}}{r^2}$$ dipole moment

$$\vec{E}_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \qquad \mathbf{P} = \varepsilon_0\boldsymbol{\chi_e}\mathbf{E} \qquad \mathbf{D} = \varepsilon\mathbf{E}$$

$$\mathbf{F}_{\text{mag}} = \int I\,d\mathbf{l}\times\mathbf{B}$$

$\mathbf{K} = d\mathbf{I}/d\ell_\perp \quad \mathbf{J} = d\mathbf{I}/da_\perp$ surface and volume currents

$\nabla\bullet\mathbf{J} + \dfrac{d\rho}{dt} = 0$ conserved current

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}I\int\frac{d\mathbf{l}'\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2}da' \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}I\int\frac{\mathbf{J}\times\hat{r}}{|\mathbf{r}-\mathbf{r}'|^2}d\tau'$$

$$\oint\mathbf{B}\bullet d\mathbf{l} = \mu_0 I_{\text{enc}}$$

$\nabla\bullet\mathbf{A} = 0$ "Coulomb gauge" convention $\quad \nabla^2\mathbf{A} = -\mu_0\mathbf{J}$ (for Coulomb gauge)

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d\tau' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}da' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{I}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{l}'$$

$$\oint\mathbf{A}\bullet d\mathbf{l} = \Phi_m$$

$$\mathbf{A}_{\text{above}} - \mathbf{A}_{\text{below}} = 0 \qquad \frac{\partial\mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial\mathbf{A}_{\text{below}}}{\partial n} = -\mu_0\mathbf{K}$$

$$\mathbf{m} = I\iint\hat{n}\,da = I\mathbf{a}$$

$$\mathbf{m} = \frac{1}{2}\oint\mathbf{r}\times\mathbf{I}\,d\ell \quad \mathbf{m} = \frac{1}{2}\iint\mathbf{r}\times\mathbf{K}\,da \quad \mathbf{m} = \frac{1}{2}\iiint\mathbf{r}\times\mathbf{J}\,d\tau$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{r}}{r^2}$$

$$\mathbf{N} = \mathbf{m}\times\mathbf{B} \quad \mathbf{F} = \nabla(\mathbf{m}\bullet\mathbf{B}) \quad U = -\mathbf{m}\bullet\mathbf{B}$$

$$\mathbf{J}_b = \nabla\times\mathbf{M} \quad \mathbf{K}_b = \mathbf{M}\times\hat{n}$$

$$\nabla\times\mathbf{H} = \mathbf{J}_f \qquad \oint \mathbf{H}\bullet d\mathbf{l} = I_{\text{f,encl}}$$

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \qquad B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}} = \mu_0 K \qquad \mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 \mathbf{K}\times\hat{n}$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}) \qquad \mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f\times\hat{n}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad \mathbf{M} = \chi_m \mathbf{H} \qquad \mathbf{B} = \mu\mathbf{H}$$

$$U = \frac{1}{2}\iiint \mathbf{H}\cdot\mathbf{B}\, dv \qquad U = \frac{1}{2}\iiint \mathbf{D}\cdot\mathbf{E}\, dv \quad U = -\mathbf{m}\cdot\mathbf{B} - \mathbf{p}\cdot\mathbf{E}$$

$$\Phi_b = M_{ba} I_a \quad M_{ba} = \frac{\mu_0}{4\pi}\oint\oint \frac{dl_a\cdot dl_b}{r} \quad \mathcal{E}_b = -M_{ba}\frac{dI_a}{dt} \quad \Phi = LI \quad \mathcal{E} = -L\frac{dI}{dt}$$

Non-statics:

$$\mathbf{B} = \nabla\times\mathbf{A} \quad \mathbf{E} = -\nabla\Phi - \frac{d\mathbf{A}}{dt} \quad \mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\Lambda \quad \Phi \to \Phi' - \frac{d\Lambda}{dt}$$

$$\nabla^2\Phi + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\rho/\varepsilon_0 \quad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{dt^2} - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{d\Phi}{dt}\right) = -\mu_0\mathbf{J}$$

Lorentz (Lorenz) gauge:

$$\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{d\Phi}{dt} = 0 \quad \nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\rho/\varepsilon_0 \quad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{dt^2} = -\mu_0\mathbf{J}$$

Coulomb (transverse, radiation) gauge:

$$\nabla\cdot\mathbf{A} = \mathbf{0} \quad \nabla^2\Phi = -\rho/\varepsilon_0 \quad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{dt^2} = -\mu_0\mathbf{J} + \frac{1}{c^2}\nabla\frac{\partial\Phi}{\partial t}$$

Retarded solutions:

$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = -4\pi f$ has solution $\Psi(\mathbf{r},t) = \iiint \frac{[f(\mathbf{r}',t')]_{ret}}{\mathbf{r}-\mathbf{r}'}dv'$

Poynting's formalism:

$$\frac{\partial u}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{J}\cdot\mathbf{E} \quad \mathbf{S} = \mathbf{E}\times\mathbf{H} \quad \langle\mathbf{S}\rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E}\times\mathbf{H}^*]$$

Momentum density $\mathbf{g} = \frac{1}{c^2}\mathbf{E}\times\mathbf{H}$

Angular momentum density $\mathcal{L} = \frac{1}{c^2}\mathbf{r}\times(\mathbf{E}\times\mathbf{H})$

Plane waves $\mathbf{E}(\mathbf{r},t) = \mathbf{E_0}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ $\mathbf{B}(\mathbf{r},t) = \mathbf{B_0}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Helmholtz equation $(\nabla^2 + \mu\epsilon\omega^2)\binom{\mathbf{E}}{\mathbf{B}} = 0$

Snell's laws: $\theta_i = \theta_r$ $n_1 sin\theta_i = n_2 sin\theta_t$

Fresnel equations (non-magnetic media),
E polarized perpendicular to the plane of incidence:

$$\frac{E_{0r}}{E_{0i}} = \frac{(n_1/n_2)cos\theta_i - cos\theta_t}{(n_1/n_2)cos\theta_i + cos\theta_t} \quad \frac{E_{0t}}{E_{0i}} = \frac{2(n_1/n_2)cos\theta_i}{(n_1/n_2)cos\theta_i + cos\theta_t}$$

Fresnel equations (non-magnetic media),
E polarized in the plane of incidence:

$$\frac{E_{0r}}{E_{0i}} = \frac{-cos\theta_i + (n_1/n_2)cos\theta_t}{cos\theta_i + (n_1/n_2)cos\theta_t} \quad \frac{E_{0t}}{E_{0i}} = \frac{2(n_1/n_2)cos\theta_i}{cos\theta_i + (n_1/n_2)cos\theta_t}$$

Oscillator model of bound charges:

$$\frac{\epsilon(\omega)}{\epsilon_0} = \varepsilon_b(\omega) = i\frac{Ne^2 f_0}{m\omega(\gamma_0 - i\omega)} \quad \sigma = \frac{f_0 N e^2}{m(\gamma_0 - i\omega)}$$

High-frequency limit, plasma frequency

$$\frac{\epsilon(\omega)}{\varepsilon_0} \approx 1 - \frac{\omega_p^2}{\omega^2} \quad \omega_p^2 = \frac{NZe^2}{\epsilon_0 m} \quad ck = \sqrt{\omega^2 - \omega_p^2}$$

Group velocity

$$v_g = \left.\frac{d\omega}{dk}\right|_0$$

Skin depth, surface resistance, power loss

$$\delta = \sqrt{\frac{2}{\mu_c \omega \sigma}} \quad R_s = 1/\sigma\delta \quad \frac{dP_{\text{loss}}}{da} = \frac{R_s}{2}|\mathbf{K}_{\text{eff}}|^2$$

Wave equation

$$[\nabla_t^2 + (\mu\epsilon\omega^2)]\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix} = 0 \quad \text{boundary conditions } E_z|_S = 0 \quad \left.\frac{\partial B_z}{\partial n}\right|_S = 0$$

TEM solutions

$$k_{\text{TEM}} = \omega\sqrt{\epsilon\mu} \quad \mathbf{B}_{\text{TEM}} = \pm\sqrt{\varepsilon\mu}\,\hat{\mathbf{z}} \times \mathbf{E}_{\mathbf{TEM}}$$

TM, TE solutions

$$\mathbf{H}_t = \frac{\pm 1}{Z}\hat{\mathbf{z}} \times \mathbf{E}_t, \text{ with wave impedance } Z = \begin{cases} \dfrac{k}{\varepsilon\omega} = \dfrac{k}{k_0}\sqrt{\dfrac{\mu}{\varepsilon}} & TM \\ \dfrac{\mu\varepsilon}{k} = \dfrac{k_0}{k}\sqrt{\dfrac{\mu}{\varepsilon}} & TE \end{cases}$$

Longitudinal field components Ψ satisfy

$$(\nabla_t^2 + \gamma^2)\psi = 0 \text{ with } \gamma^2 = \mu\varepsilon\omega^2 - k^2$$

$$\mathbf{E}_t = \pm\frac{ik}{\gamma^2}\nabla_t\psi \text{ (TM)}, \quad \mathbf{H}_t = \pm\frac{ik}{\gamma^2}\nabla_t\psi \text{ (TE)}$$

boundary conditions $\psi|_S = 0$ (TM), $\left.\frac{\partial\psi}{\partial n}\right|_S$ (TE) (repeated from above)

Cutoff frequency

$$\omega_\lambda = \frac{\gamma_\lambda}{\sqrt{\varepsilon\mu}}$$

Perturbation to cavity surface (Slater's first equation; also called Bethe-Schwinger formula, or Waldron's formula)

$$\frac{\Delta\omega}{\omega_0} = \frac{\int_{\Delta V}(\mu H^2 - \varepsilon E^2)dV}{\int_V (\mu H^2 + \varepsilon E^2)\,dV} = \frac{\Delta U_H - \Delta U_E}{U}$$

Power losses in cavity

$$Q = \omega_0 \frac{\text{stored energy}}{\text{power loss}} = \frac{\omega_0}{\delta\omega} = \frac{\omega_0}{\Gamma}$$

Radiation

$\mathbf{H} = \frac{1}{\mu_0}\nabla\times\mathbf{A}$ $\mathrm{E} = \frac{\mathrm{i}Z_0}{\mathrm{k}}\nabla\times\mathbf{H}$ with impedance of free space $Z_0 = \sqrt{\mu_0/\epsilon_0}$

Electric dipole radiation

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\iiint \mathbf{J}(\mathbf{r}')\, dV' = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}\frac{e^{ikr}}{r}$$

Magnetic dipole radiation

$$\mathbf{A}(\mathbf{r}) = \frac{ik\mu_0}{4\pi}(\mathbf{n}\times\mathbf{m})\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right)$$

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint \mathbf{A}\cdot d\mathbf{a}$

Curl Theorem: $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint \mathbf{A}\cdot d\mathbf{l}$