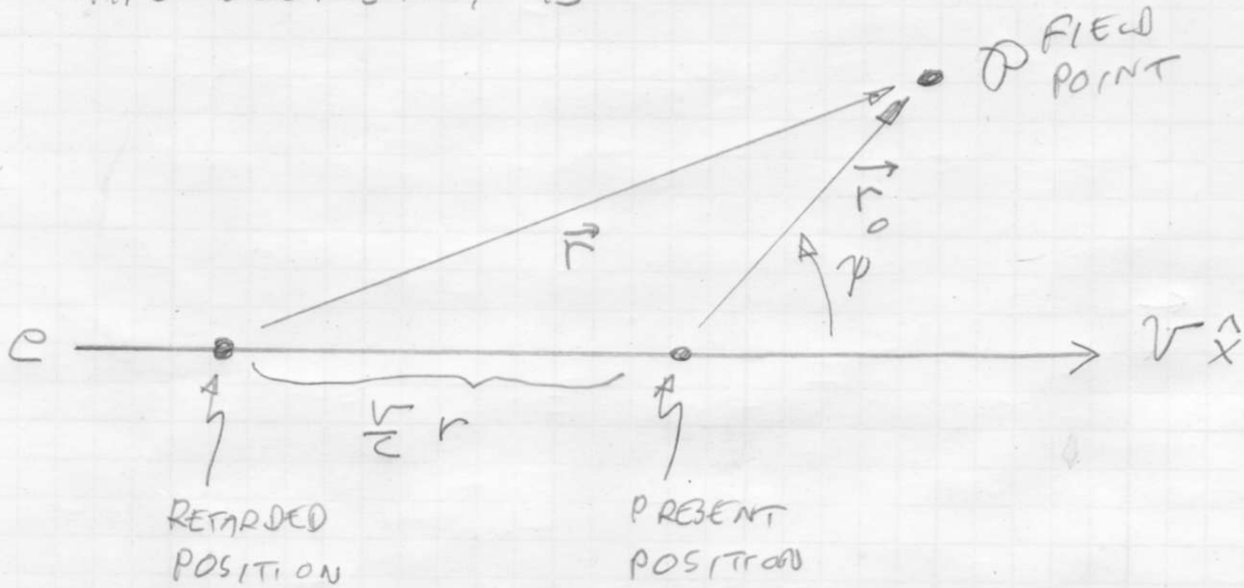
SPECIAL LECTURE.

FIELDS OF A CHARGE MOVING AT CONSTANT VELOCITY.

THE GEOMETRY IS



THE LIÉNARD-WIECHERT POTENTIALS ARE

$$A^{\mu} = e\left(\frac{v/c}{s}, \frac{1}{s}\right)$$

$$\text{WITH} \quad s = r - \vec{r}\cdot\frac{\vec{v}}{c}.$$

THE INTERESTING THING ABOUT THIS EXAMPLE IS THE POTENTIALS A^{μ} AND THE FIELDS CAN BE EXPRESSED IN TERMS OF THE "PRESENT" POSITION OF THE CHARGE.

THE POSITION INFORMATION IS EMBEDDED IN THE LIÉNARD-WIECHERT DENOMINATOR s.

SO, WE'LL EXPRESS s IN TERMS OF THE PRESENT POSITION OF THE CHARGE.

WE START WITH $s = r - \vec{r} \cdot \frac{\vec{v}}{c}$.

FROM THE FIGURE,

$s^2 = r_0^{\,2} - \left(\vec{r}_0 \times \frac{\vec{v}}{c} \right)$, WITH $\vec{r}_0$ THE "PRESENT" POSITION. TAKING THE SQUARE-ROOT OF s^2 AND NOTING

$\vec{r}_0 = x_0 \hat{x} + y_0 \hat{y} + z_0 \hat{z}$,

$$s = \left\{ x_0^{\,2} + y_0^{\,2} + z_0^{\,2} - \frac{v^2}{c^2}\left(y_0^{\,2} + z_0^{\,2}\right) \right\}^{1/2}$$

$$= \left\{ x_0^{\,2} + \left(1 - \frac{v^2}{c^2}\right)\left(y_0^{\,2} + z_0^{\,2}\right) \right\}^{1/2}$$

FROM THE FIGURE $\sin^2 \psi = y_0^{\,2} + z_0^{\,2}$, SO

$$s = r_0 \left\{ 1 - \frac{v^2}{c^2} \sin^2 \psi \right\}.$$

WE HAVE LIENARD-WIERCHERT POTENTIALS

$$\Phi = \frac{e}{s}, \quad \vec{A} = \frac{1}{c^2} \frac{e\vec{v}}{s} \qquad (c65)$$

SO Φ AND $\vec{A}$ ARE IN TERMS OF THE "PRESENT" POSITION COORDINATES r_0 AND ψ.

THE ELECTRIC FIELD IS GIVEN BY

$$\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A} \qquad (165)$$

THE TIME DERIVATIVE IS EVALUATED BY RECALLING THE FIELD IS ESTABLISHED BY THE UNIFORMLY-MOVING CHARGE. A STATIONARY OBSERVER SEES AT TIME $t+dt$ THE FIELD AT THE POSITION $-\vec{v}\,dt$ AT TIME t. HENCE $\frac{\partial}{\partial t} \to -\vec{v}\frac{\partial}{\partial x}$.

HENCE

$$E_x = e\frac{x_0}{s^3}\left(1-\frac{v^2}{c^2}\right)$$

$$E_y = e\frac{y_0}{s^3}\left(1-\frac{v^2}{c^2}\right)$$

$$E_z = e\frac{z_0}{s^3}\left(1-\frac{v^2}{c^2}\right)$$

THESE COMBINE TO

$$\vec{E} = \frac{e}{s^3}\vec{r}_0\left(1-\frac{v^2}{c^2}\right)$$

NOTICE THE $\vec{E}$ FIELD POINTS TO THE "PRESENT" POSITION OF THE CHARGE.

RECALLING $s^2 = r_0^2\left\{1-\frac{v^2}{c^2}\sin^2\psi\right\}$,

$$\vec{E} = e\frac{\vec{r}_0}{r_0^3}\frac{1-\frac{v^2}{c^2}}{\left\{1-\frac{v^2}{c^2}\sin^2\psi\right\}^{3/2}}$$

THIS LAST EXPRESSION FOR $\vec{E}$ IS SOLELY IN TERMS OF "PRESENT" COORDINATES.

THE MAGNETIC FIELD $\vec{B}$ COMES FROM $\vec{B} = \vec{\nabla} \times \vec{A}$. THIS IS NON-TRIVIAL TO COMPUTE. I'LL ASSERT

$$\vec{B} = \frac{e}{c^2} \frac{\vec{v} \times \vec{r}_0}{s^3} \left(1 - \frac{v^2}{c^2}\right)$$

WHICH IS SENSIBLE; WE COULD JUST HAVE GUESSED (OR FIGURED OUT) WE STILL HAVE $\vec{B} = \frac{1}{c^2} \vec{v} \times \vec{E}$ IN THIS FRAME.

THE $\vec{E}$ AND $\vec{B}$ FIELDS, AS EXPECTED, ARE COULOMB AND BIOT-SAVART FIELDS IN THE $v \to 0$ (THUS $s \to r$) LIMIT.