Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
May 29, 2020, 11am
On-line lecture

Administrative:

1. HW#8 due now (with some exceptions).
2. You should be getting your homework back; if not let me know.

Lecture:

J. Chapter 14: Collisions, Energy Loss, and Scattering of Charged Particles; Cherenkov and Transition Radiation. (With a few topics from C. 13 and C. 15.)
1. J. C. 14.3. Synchrotron radiation; charge in circular orbit.
2. J. C. 14. Miscellaneous comments: Thomson cross section, ultra-relativistic charged-particle interactions.
3. J. C. 15.4 Weizsäcker-Williams approximation, virtual quanta.
4. J. C. 13.4 Cherenkov radiation II.

BACK TO THE LARMOR RADIATION FORMULA AND THE SPECIAL CASE $\vec{\beta}$ IS PARALLEL TO $\dot{\vec{\beta}}$.

- FIND THE RADIATED, OUTWARD POYNTING VECTOR.

RECALL J, EQN. 14,13-14, THEN TAKE THE RADIATED (ACCELERATION) FIELDS

$$\vec{E}(\vec{r},t) = \frac{e}{c}\left[\frac{\hat{r}\times\{\hat{r}-\vec{\beta}\}\times\dot{\vec{\beta}}}{(1-\hat{r}\times\vec{\beta})^3\, r}\right]_{RET} \quad (CGS)$$

$$\vec{B}(\vec{r},t) = \left[\hat{r}\times\vec{E}\right]_{RET} \quad (CGS)$$

RECALL THE POYNTING VECTOR

$$\vec{S} = \frac{c}{4\pi}\,\vec{E}\times\vec{B} \quad (\text{VACUUM}) \quad (CGS).$$

HENCE THE OUTWARD ($\hat{r}$) POYNTING VECTOR IS

$$\left[\vec{S}\cdot\hat{r}\right]_{RET} = \frac{e^2}{4\pi c}\left[\frac{1}{r^2}\left|\frac{\hat{r}\times\left[(\hat{r}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{(1-\hat{r}\times\vec{\beta})^3}\right|^2\right]_{RET}$$

J, EQN 14.35

AS JACKSON NOTES,

THE NUMERATOR ENCODES THE RELATIONSHIP BETWEEN $\vec{\beta}$ AND $\dot{\vec{\beta}}$,

THE DENOMINATOR COMES FROM THE BOOST TO THE MOVING FRAME.

AS $\beta \to 1$, THE DENOMINATOR BECOMES RELATIVELY MORE IMPORTANT.

$\vec{S}$ HAS UNITS OF POWER PER AREA. SUPPOSE WE ASK THE QUESTION OF HOW MUCH ENERGY PER AREA AN OBSERVER (WHERE $\vec{S}\cdot\hat{n}$ IS EVALUATED) SEES DURING A FINITE INTERVAL OF THE CHARGES ACCELERATION?

OUR VARIABLE t IS THE OBSERVER'S TIME. THE VARIABLE t' IS THE RETARDED TIME, WITH AS USUAL $t' = t - r/c$; r THE RETARDED POSITION.

THE ENERGY PER AREA FOR A FINITE ACCELERATION INTERVAL IS

$$E = \int_{t_1 - r(t_1')/c}^{t_2 - r(t_2')/c} \left[\vec{S}\cdot\hat{r}\right]_{RET} dt$$

WE CAN REWRITE THIS IN TERMS OF THE RETARDED TIME

$$E = \int_{t_1'}^{t_2'} \left[\vec{S}\cdot\hat{r}\right]_{RET} \frac{dt}{dt'}\, dt'$$

JACKSON EQN. 14.36

HENCE $\left[\vec{S}\cdot\hat{r}\right]_{RET}\frac{dt}{dt'}$ IS THE POWER PER AREA IN THE CHARGES TIME.

WE CAN THEN WRITE THE DIFFERENTIAL POWER EMISSION AS

$$\frac{dP(t')}{d\Omega} = r^2\,\vec{S}\cdot\hat{r}\,\frac{dt}{dt'}$$

$$= r^2\,\vec{S}\cdot\hat{r}\,(1-\hat{r}\cdot\vec{\beta})$$

WHERE WE USED $t' = t - r(t')/c$

J. EQN. 14.37

WITH $\vec{S}\cdot\hat{r}$ FROM BEFORE.

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c}\,\frac{\left|\hat{r}\times\{(\hat{r}-\vec{\beta})\times\dot{\vec{\beta}}\}\right|^2}{(1-\hat{r}\cdot\vec{\beta})^5}$$

JACKSON EQN 14.38

BACK TO THE PROBLEM AT HAND, $\vec{\beta}$ PARALLEL TO $\dot{\vec{\beta}}$, SO THE $\vec{\beta} \times \dot{\vec{\beta}}$ TERM VANISHES, AND

$$|\hat{r} \times (\hat{r} \times \dot{\vec{\beta}})|^2 = \dot{\beta}^2 \sin^2\theta$$

WITH θ THE ANGLE BETWEEN $\vec{\beta}$ (AND $\dot{\vec{\beta}}$) AND THE OBSERVER. (STRAIGHT AHEAD IS $\theta = 0$).

THIS GIVES

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c} \dot{\beta}^2 \frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

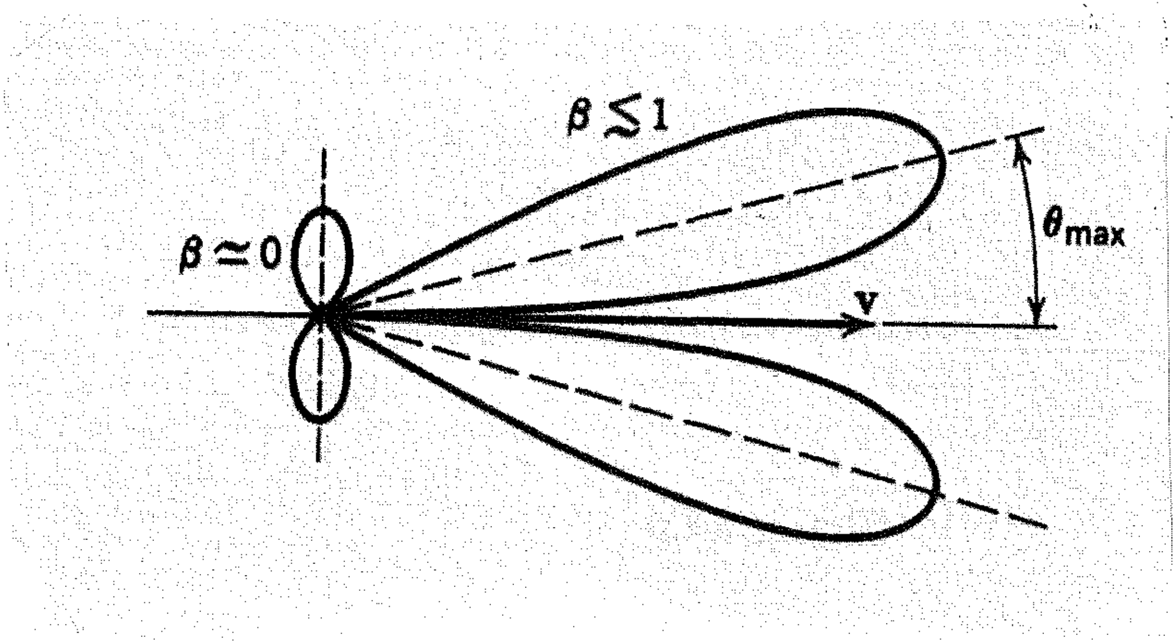
JACKSON EQN 14.39.

FOR $\beta \ll 1$ THE DENOMINATOR DOESN'T CONTRIBUTE MUCH AND WE HAVE THE FAMILIAR DIPOLE RADIATION PATTERN, AND $\frac{dP}{d\Omega}$ REDUCES TO THE LARMOR RESULT

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \dot{\beta}^2 \sin^2\theta.$$

BUT FOR $\beta \to 1$, THE DENOMINATOR CAN BE SIGNIFICANT. FURTHER, ITS ASYMMETRIC IN β: FORWARD (+) AND BACKWARD (−) DIRECTIONS CAN BE VERY DIFFERENT

THE RADIATION PATTERN IS



$\vec{\beta} \parallel \dot{\vec{\beta}}$ J. FIG. 14.4

JACKSON COMPUTES THE EXTREMUM IN θ OF $dP/d\Omega$

$$\theta_{MAX} = \cos^{-1}\left\{\frac{1}{3\beta}\left(\sqrt{1+15\beta^2} - 1\right)\right\} \xrightarrow[\beta \to 1]{} \frac{1}{2\gamma}$$

J. EQN 14.40.

JACKSON ALSO NOTES

$$\left(\frac{dP}{d\Omega}\right)_{PEAK} \sim \gamma^{8}$$ (YIKES!) {BEAMING}
(J. PAGE 669).

THIS IS OBVIOUSLY OF GREAT INTEREST FOR MATERIALS SCIENCE.

THE EFFECT CAN BE ENHANCED BY EMPLOYING "WIGGLERS" AND "UNDULATORS", NOT COVERED HERE.

ALSO NOT COVERED ARE DETAILS OF THE FREQUENCY STRUCTURE AND DETAILED ANGULAR STRUCTURE OF THE RADIATION.

IF THE CHARGED PARTICLE IS SLOWING DOWN, THIS IS ANOTHER WORD FOR BREMSTRAHLUNG RADIATION.

Q: CAN YOU GUESS HOW THE RADIATION IS POLARIZED?

NOW, THE CASE WHERE $\vec{\beta}$ IS PERPENDICULAR TO $\dot{\vec{\beta}}$.

WE NEED TO EVALUATE

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{\left|\hat{r} \times \{(\hat{r} - \vec{\beta}) \times \dot{\vec{\beta}}\}\right|^2}{(1 - \hat{r}\cdot\vec{\beta})^5}$$

EVALUATE

$$\hat{r} \times \{(\hat{r} - \vec{\beta}) \times \dot{\vec{\beta}}\}$$

$$= (\hat{r} - \vec{\beta})(\dot{\vec{\beta}} \cdot \hat{r}) - \dot{\vec{\beta}}(1 - \vec{\beta} \cdot \hat{r})$$

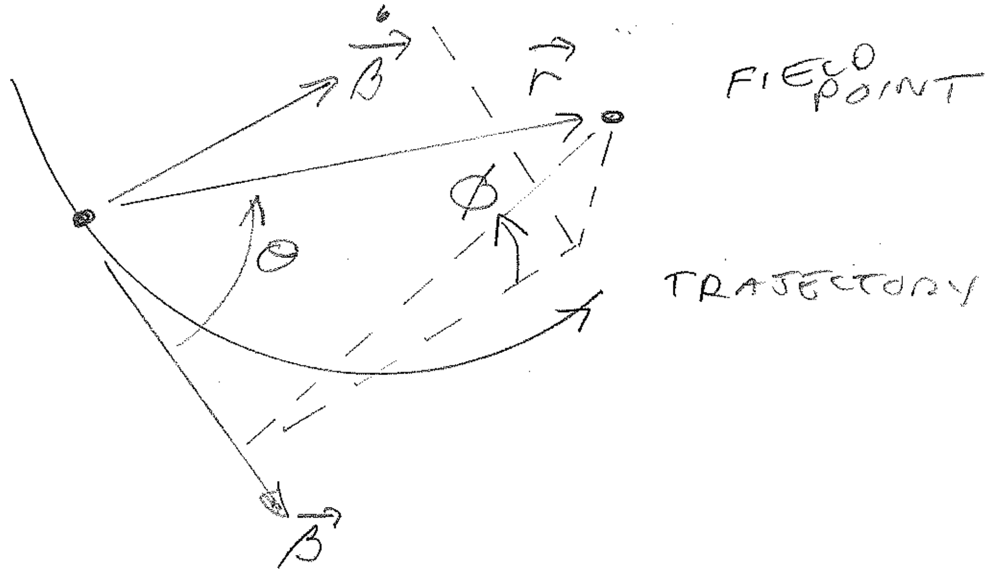
$\{$FROM $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}).\}$

YOU SQUARE THIS (WITH $\vec{\beta} \cdot \dot{\vec{\beta}} = 0$) TO GET THE NUMERATOR

$$\dot{\beta}^2 (1 - \vec{\beta} \cdot \hat{r})^2 - (1 - \beta^2)(\dot{\vec{\beta}} \cdot \hat{r})^2.$$

WE NEED TO DEFINE SOME ANGLES.

θ IS THE POLAR ANGLE (FORWARD = 0);
ϕ IS THE ANGLE OUT OF THE PLANE.



WE EVALUATE DOT PRODUCTS TO GIVE

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\dot{\beta}^2 \frac{1}{(1-\beta\cos\theta)^3}$$

$$\times \left[1 - \frac{1}{\gamma^2}\frac{\sin^2\theta\cos^2\phi}{(1-\beta\cos\theta)^2}\right]$$

Q: SHOW THIS REDUCES TO THE LARMOR FORMULA.

A: THAT'S $\beta \to 0$, AND THE ANGLE "θ" USED IN THE LARMOR FORMULA HAS ITS COSINE EQUAL TO THESE $\sin\theta\cos\phi$.

NOTICE FOR $(\beta \neq 0)$ $\frac{dP}{d\Omega}$ DOES NOT VANISH ALONG THE DIRECTION OF ACCELERATION $(\theta = \pi/2,\ \phi = 0,\ \pi)$.

THIS IS SOMETHING TO PONDER.

FOR $\beta \to 1$,

$$\frac{dP}{d\Omega} \sim \gamma^6 \quad \text{(YIKES!) (BEAMING)}$$

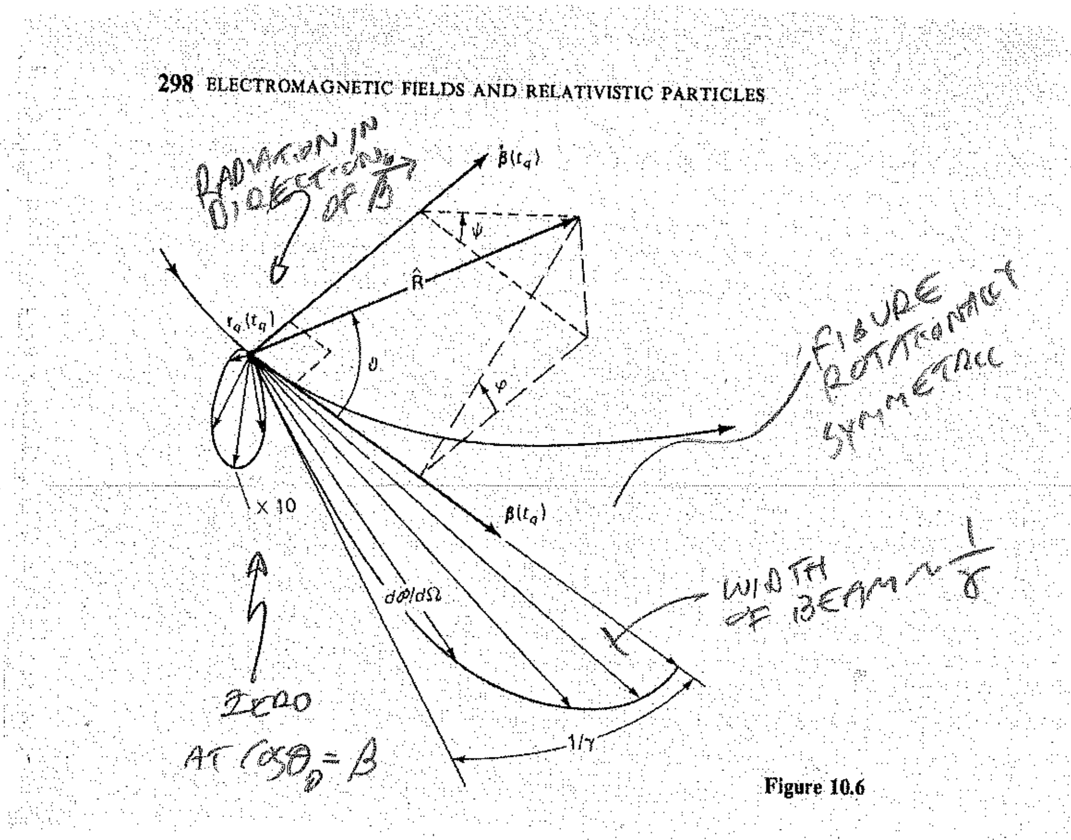
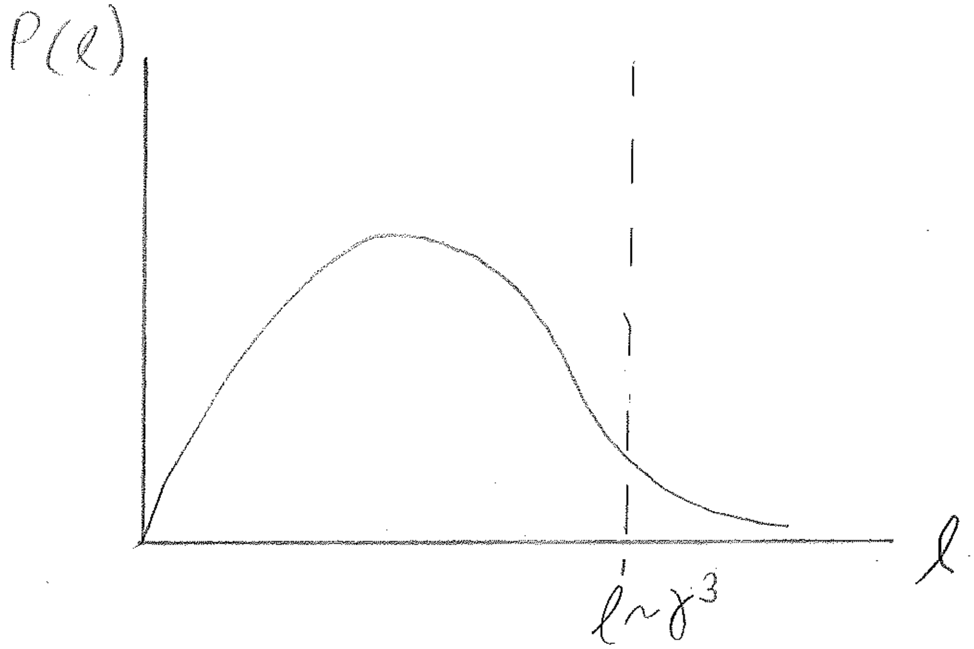


Figure 10.6

RADIATION SPECTRUM IN A CIRCULAR STORAGE RING! (SEE LANDAU AND LIFSCHITZ "CLASSICAL FIELDS" P216)

USUALLY EXPRESSED AS POWER PER ROTATIONAL HARMONIC: $\omega, 2\omega, 3\omega, \ldots$

$\ell = 1, 2, 3, \ldots$



FOR e^+, e^- RINGS, γ^3 IS HUGE.

THIS CORRESPONDS TO A "CRITICAL FREQUENCY" $\left(\frac{c}{r}\right)\gamma^3$, OR A "CRITICAL ENERGY" $\hbar\left(\frac{c}{r}\right)\gamma^3$.

FINISH UP J.C. 14.

COMMENTS ON LIÉNARD - WIECHERT POTENTIALS AND RADIATION.

- CONSIDER MORE THAN 1 CHARGE MOVING IN A CIRCLE. THERE'S SUPERPOSITION. FOR ORBITAL HARMONICS WHERE THE PHASE DIFFERENCE BETWEEN CHARGES IN A TIGHT "BUNCH" IS SMALL, THE RADIATION IS COHERENT! N^2 TIMES THAT OF A SINGLE CHARGE.

HOWEVER, IF CHARGES UNIFORMLY OCCUPY THE CIRCLE, RADIATION FIELDS VANISH AND ONLY $1/r^3$, $1/r^2$ TERMS REMAIN IN POTENTIALS. HENCE NO RADIATION.

A BIT OF HISTORY. IT WAS OBSERVED IN A "BETATRON" THAT A CONTINUOUS CIRCULAR CURRENT DOES INDEED RADIATE. THIS WAS EVENTUALLY UNDERSTOOD AS DUE TO FLUCTUATIONS $\sim \sqrt{N}$ (WITH $N^2 \sim$ THE COHERENT INTENSITY! SO FLUCTUATIONS RADIATE AS THE INCOHERENT SUM OF N FREE ELECTRONS.

- J.C.14 IS WHERE JACKSON INTRODUCES THOMSON (FREE ELECTRON) SCATTERING. WE FOUND THIS RESULT EARLIER VIA THE "ELECTRON MODEL"

$$\ddot{\vec{r}} = \frac{e}{m}\vec{E} \qquad \text{FOR } v \ll c.$$

THE EXTERNAL $\vec{E}$ FIELD INDUCED A DIPOLE RADIATION FIELD WITH A DIPOLE-LIKE INTENSITY AND TOTAL RADIATED POWER.

(N.B., WE FOUND THIS RESULT AGAIN VIA "POLARIZATION POTENTIALS"!)

THE CLASSICAL THOMSON CROSS SECTION WAS FOUND BY DIVIDING THE TOTAL POWER BY INTENSITY!

$$\sigma_0 = \frac{8}{3}\pi r_0^2$$

r_0 = CLASSICAL ELECTRON RADIUS

$$= \frac{e^2}{mc^2} \text{ (CGS)}, \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2} \text{ (MKS)}.$$

(J. EQN. 14.126).

A COMMENT ON ULTRA-RELATIVISTIC MOTION IN EXTERNAL FIELDS. (IF THERE ARE NO EXTERNAL FIELDS OR INTERACTIONS, "ULTRA RELATIVISTIC" ISN'T MUCH OF A COMPLICATION.)

HOWEVER, IF, e.g.,

$$\frac{H}{H_0}\gamma \gtrsim 1$$

WITH H THE EXTERNAL FIELD,

γ THE BOOST OF THE ELECTRON,

H_0 THE "SCHWINGER" FIELD OR "CRITICAL FIELD"

$$H_0 = \frac{m_e^2}{e}\frac{c^2}{\hbar} \quad (\text{CGS})$$

$$\approx 4.4\times 10^{13}\ \text{GAUSS}.$$

... THEN COMPLETELY NEW BEHAVIOR AND EFFECTS ARISE

YOU COULD GUESS CLASSICAL ELECTRODYNAMICS GETS INTO TROUBLE IN THIS REGIME SINCE THE CRITICAL ENERGY ≫ PARTICLE ENERGY.

THERE IS ENORMOUS DEBATE ABOUT IN THIS REGIME SYNCHOTROTRON RADIATION IS SUPPRESSED. THIS COULD HAVE BIG EFFECTS IN ASTROPHYSICS (RADIATION FROM AROUND NEUTRON STARS) AND PARTICLE PHYSICS (DESIGN OF ULTRA-HIGH ENERGY ACCELERATORS). 1F

A COMMENT ON NOMONCLATURE: WEIZSÄCHER-WILLIAMS VIRTUAL QUANTA. (J.C. 15.4)

THIS APPLIES PLANCK'S HYPOTHESIS

$$E_\omega = \hbar\omega$$

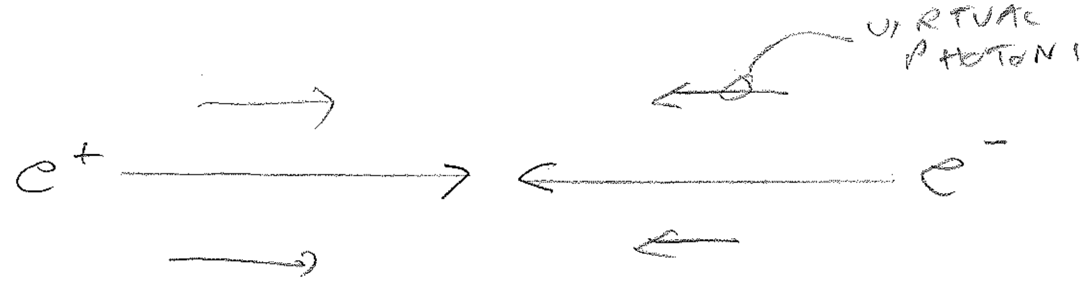
TO THE BOHR IMPACT-PARAMETER FORMALISM.

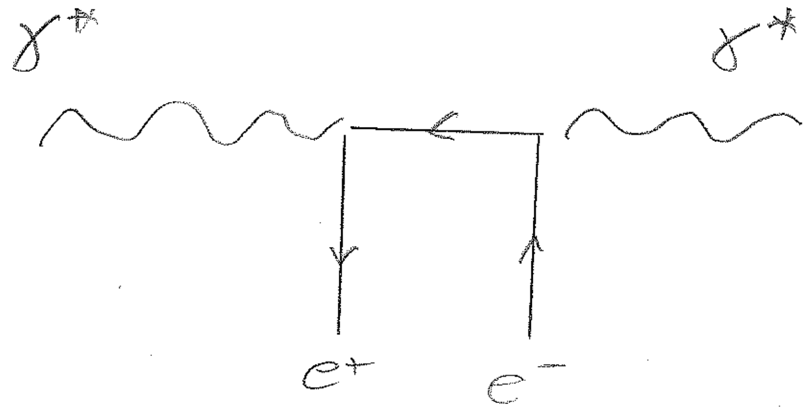
THEN FOURIER-DECOMPOSES THE TIME DEVELOPMENT OF THE FIELD INTO FREQUENCY COMPONENTS. THE FREQUENCY COMPONENTS ARE "TURNED INTO" VIRTUAL PHOTONS.

ALAS... WE DON'T HAVE TIME TO GO OVER "BREMSSTRAHLUNG DURING COLLISIONS" J.C. 15.1-2.

THIS IS CRUDE, BUT REMARKABLY ACCURATE.

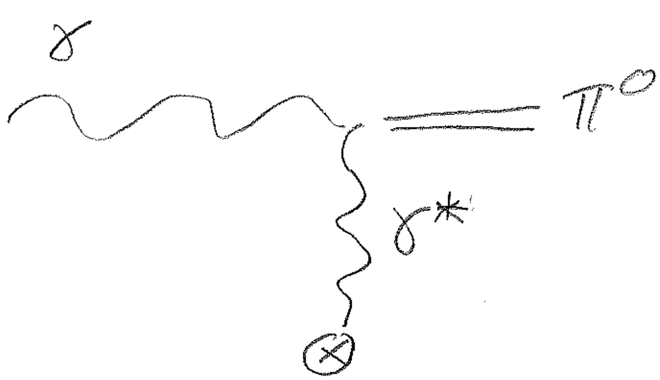
EXAMPLE: "2 PHOTON" COLLISIONS IN e^+e^- COLLIDERS





AT LOW OUTGOING e^+e^- ENERGIES, THIS WORKS REMARKABLY WELL

ALSO, e.g.



FINDING THE SPECTRUM OF WEIZSÄCKER-WILLIAMS PHOTONS IS COMPLICATED. (SEE J. C. 15 FOR A PARTIAL SOLUTION.) BUT WE CAN GET PART-WAY THERE USING RESULTS FROM J. C. 13.

UNDER CERENKOV RADIATION J. C. 13, 4, THERE'S THE ENERGY LOST PER LENGTH EXPRESSED AS AN INTEGRAL OVER FREQUENCY. WE EXTRACT THE INTEGRAND, AND WRITE IT AS

$$U_\omega \, d\omega \sim e^2 \Delta x \left(1 - \frac{c^2/n^2}{v^2}\right) \omega \, d\omega$$

WITH $U_\omega \sim \frac{dE_\omega}{dx}$. J. EQN. 13.48

THEN, APPLY THE PLANCK HYPOTHESIS $E_\omega = \hbar\omega$:

$$dN_\omega \, d\omega \sim \alpha \, \Delta x \left(1 - \frac{c^2/n^2}{v^2}\right) d\omega$$

IT SEEMS THIS AS AN INTEGRAL OVER FREQUENCY DIVERGES.

ČERENKOV RADIATION II.

(FILL IN FROM J.C. 13).

WE SAW A CHARGE IN UNIFORM MOTION DOES NOT RADIATE. WE SAW THE CORRESPONDING POYNTING VECTOR FOR THE $1/r^3$ TERMS ("INDUCTION OR "REACTIVE" TERMS") FALLS TOO FAST IN r TO CONTRIBUTE TO OUTWARD ENERGY FLOW AT LARGE DISTANCES.

THE SITUATION IS DIFFERENT IN DIELECTRIC MEDIA. WE FIND POTENTIALS FROM, e.g.,

$$\nabla^2 \vec{A} - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = -\frac{1}{c}\vec{J}.$$

THIS HAS THE SAME SOLUTIONS AS THE VACUUM SOLUTIONS BY THE SUBSTITUTION $c \longrightarrow c/n$, INCLUDING IN THE RETARDED TIME

$$t_{RET} = t - r\frac{1}{c} \rightarrow t - r\frac{n}{c}$$

WE ALSO MAKE THE SUBSTITUTION TO THE LIÉNARD-WIECHERT DENOMINATOR S

$$S = r - \vec{r}\cdot\vec{v}\frac{1}{c} \rightarrow r - \vec{r}\cdot\vec{v}\frac{n}{c}$$
$$= r\left\{1 - v\frac{n}{c}\cos\theta\right\}$$

FOR $1 - v\frac{n}{c}\cos\theta$ VANISHING, THE OBSERVER SEES INFINITE FIELD STRENGTH. FOR $1 - v\frac{n}{c}\cos\theta$ TO VANISH (OR BE IMAGINARY),

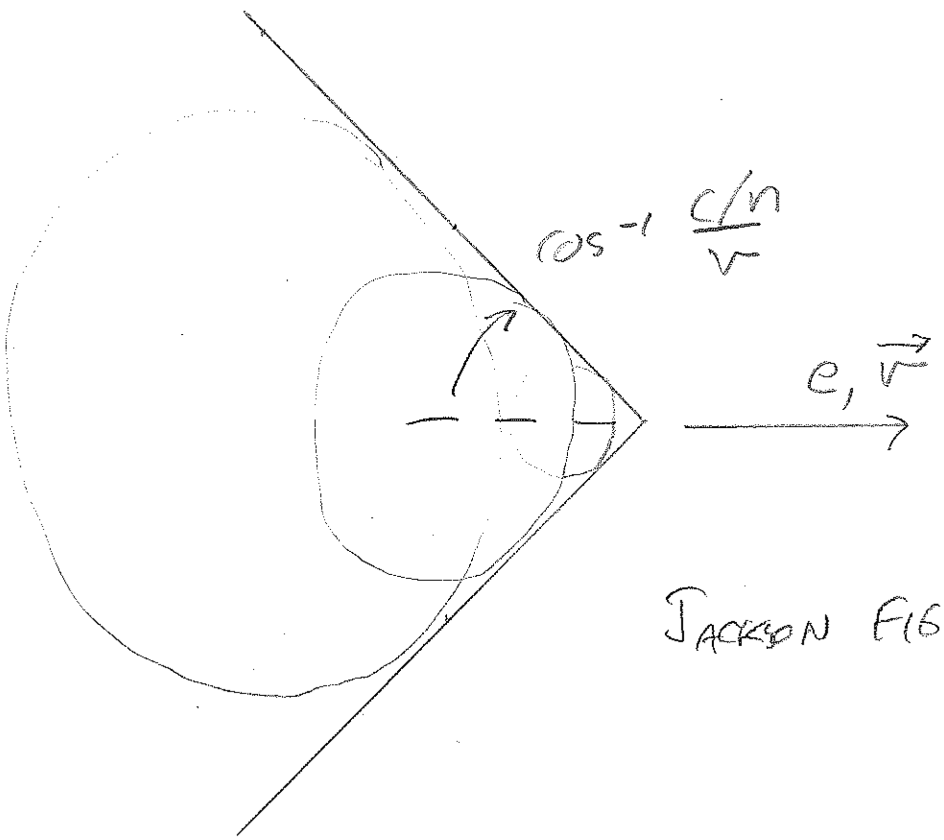
$$v > \frac{c}{n}.$$

IN THIS CASE THE CONE DEFINED BY $\cos\theta = v\frac{n}{c}$ HAS A LARGE FIELD STRENGTH IN RADIATION.

YOU MIGHT PONDER WHY FOR $v > \frac{c}{n}$ YOU CAN'T SIMPLY WRITE WAVE EQUATIONS IN Φ AND $\vec{A}$ IN TERMS OF STATIC SOLUTIONS (A REPEAT OF OUR COVARIANT DERIVATION). IT'S SAID "THE DIRECT EVALUATION OF THE LIÉNARD-WIECHERT POTENTIALS IN UNIFORM MOTION THROUGH A MEDIUM FAILS".

HOWEVER RECALL IN MATERIALS $\varepsilon(\omega)$ AND THEREFORE $n(\omega)$ APPROACH 1 FOR LARGE ω.

THE TOTAL POWER AND NUMBER OF PHOTONS IS FINITE (THIS LAST REQUIRES SOME THOUGHT).



JACKSON FIG 13.5